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Short Communication

Method for calculating the transmitter pulse energy for a laser altimeter

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Abstract

We present a method for calculating the minimum detectable power and the transmitter pulse energy for an air-/spaceborne laser altimeter with an avalanche photodiode receiver. A sample calculation based on GLARS (Cohen, S. C. *et al, IEEE Trans.*, 1987, **GE-25**, 589–592) specifications shows that the pulse energy should be 94 mJ for a worst-case signal-to-noise ratio of unity.

Key words: Laser altimeter, avalanche photodiodes, signal-to-noise ratio.

1. Introduction

Cohen *et al*[1] have proposed the Geoscience Laser Altimetry/Ranging System (GLARS) on the Earth Observing System (EOS) for various applications. They obtain an (S/N) of 19 to 690 (worst-case to best-case) for the mode-locked pulsed Nd:YAG laser with 120 mJ pulse energy and 100 ps pulse width at 1064 nm. We present the theory for the calculation of the minimum detectable power and the transmitter pulse energy and apply it to the system specifications and parameters given by Cohen *et al*[1].

2. Theory

For an Si APD[2],

$$(S/N) = \frac{\left(\frac{eq}{hv}\right)^2 P_r^2}{\left\{2e\left[\langle I_d\rangle + \frac{eq}{hv}(P_b + P_r)\right]F\Delta f + \frac{4kT\Delta f}{G^2 R_L}\right\}} \tag{1}$$

where e is the electronic charge, q, the detector quantum efficiency, h, the Planck's constant, v, the frequency of the laser line, I_d, the average noise current (dark current) of the APD, P_b, the background optical power incident on the APD, P_r, the received signal optical power on the APD, Δf, the bandwidth of the receiver, k, the Boltzmann's constant, T, the temperature of the APD load resistor, F, the APD excess noise factor, G, the APD gain, and R_L, the load resistance.

The minimum detectable number of photoelectrons $N_{\min}$ is obtained from equation (1) as

$$N_{\min}=\Delta f F \Delta T_r\left\{1+\left[1+\frac{1}{\Delta f F^2}\left\{\left(\frac{2\langle I_d\rangle}{e}+\frac{q}{h\nu}P_b\right)F+\frac{4kT}{G^2e^2R_L}\right\}\right]^{0.5}\right\} \tag{2}$$

where ΔT_r is the pulse width of the received pulse, and the other terms are as defined above.

The vale of ΔT_r (the width of the received pulse) is given by

$$\Delta T_r=\Delta T+\Delta T_s \tag{3}$$

where ΔT is the transmitted pulse width and ΔT_s the pulse spread (ignoring spreading due to atmospheric turbulence). ΔT_s is given by

$$\Delta T_s=\frac{2}{c}\tan(\theta+S)RB \tag{4}$$

(equation (5) from Cohen *et al*[1]) where c is the velocity of light, θ, the angular offset of the transmitted beam from nadir, S, the surface slope, R, the range to the surface, and B, the laser beamwidth (beam divergence).

For an ideal resistor-capacitor (RC)-based low-pass filter with a flat frequency response $H(f)=1$ between $0<f<f_M$ and $=0$ for $f>f_M$, where f_M is the maximum frequency contained in the original received pulse input, and the cutoff frequency f_2 is given by[3] as

$$f_2\Delta T_r=1.0. \tag{5a}$$

If the RC network provides both low- and high-pass characteristics (as is usually the case in practical APD receivers), the lower cutoff frequency f_1 for negligible over/under-shoot is

$$f_1\Delta T_r=0.02. \tag{5b}$$

Since the bandwidth $\Delta f=f_2-f_1$, we have

$$\Delta f\Delta T_r=0.98. \tag{5c}$$

The value of the load resistance R_L is chosen as follows:

Since

$$f_2=(2\pi R_{|}C)^{-1} \tag{6}$$

assuming

$$R_{|}=R_{\|}=\left(\frac{1}{R_L}+\frac{1}{R_b}\right)^{-1} \tag{7}$$

where R_b is the APD bias resistance, and $C = C_d$, the APD junction capacitance. From equation (6) we have

$$R_{\parallel} = (2\pi f_2 C_d)^{-1} \tag{8a}$$

and, from equation (7), it follows that

$$R_L = \left(\frac{1}{R_{\parallel}} - \frac{1}{R_b}\right)^{-1}. \tag{8b}$$

The term $(q/hv)p_b$ involving the background power; occurring in equation (1) is obtained from Cohen *et al*[1] and is as follows:

$$\frac{q}{hv}P_b = \frac{q}{hv}E_b(r/\Omega)_{c1}R_0\tau_r F_b A_r \tag{9}$$

(based on equation (2) of Cohen *et al*[1]) where E_b is the solar spectral radiance ($\mathrm{Wm^{-2}\,sr^{-1}\,\AA^{-1}}$), $(r/\Omega)_{c1}$, the cloud backscatter coefficient, R_0, the receiver field of view (sr), τ_r, the receiver optical transmission, F_b, the filter bandpass (A), and A_r, the area of the receiver ($\mathrm{m^2}$).

Finally, the minimum laser transmitter pulse energy E_{tm} (for a worst-case $(S/N) = 1$) is given by:

$$E_{tm} = N_{\min}\frac{hvR^2}{qA_r T_0 T_{\mathrm{cm}}^2 T_{\mathrm{am}}^2 (r/\Omega)_m} \tag{10}$$

where $(r/\Omega)_m$ is the minimum target backscatter coefficient, T_0, the system optical transmission, T_{cm}^2, the minimum (worst-case) two-way cirrus cloud transmission, and T_{am}^2, the minimum (worst-case) two-way atmospheric aerosol transmission.

3. Calculation

We now calculate the various terms, starting with the bandwidth.

The value of ΔT_r (width of the received pulse) = 10.777 ns, substituting the values of c, θ, S, R and B in equations (3–4).

The bandwidth $\Delta f = 90.934$ MHz.
The load resistor $R_L = 490.236$ Ohms.
We assume $R_L = 490$ Ohms for further calculation.

In practice, R_L must be less than this value because $C > C_d$ is due to other stray capacitances. This will increase the thermal noise contribution (the last term in the denominator of equations (1) and (2)), thereby increasing $N_{\min}$ and E_{tm}.

Substituting the following values from Cohen *et al*[1] for the various other terms in equations (2–10):

$I_d = 2 \times 10^{-12}\,\mathrm{pA/(Hz)^{0.5}}$

$E_b = 590 \times 10^{-4}\,\mathrm{Wm^{-2}sr^{-1}\,Å^{0-1}}$ at 1064 nm

$(r/\Omega)_{c1} = 1$ (assumed)

$R_0 = B^2 = 9 \times 10^{-8}\,\mathrm{sr}.$

$\tau_r = \sqrt{0.2}$ (assuming equal contributions to the system optical transmission of 0.2 from the transmitter and the receiver optics) = 0.4472136.

$F_b = 10\,\mathrm{Å}$

$A_r = \pi \times (0.25)^2 = 0.19635\,\mathrm{m}^2$

$T = 25\,^\circ\mathrm{C} = 298\,^\circ\mathrm{K}$ (assumed)

$T_0 = 0.2$

$T_{\mathrm{cm}}^2 = 0.56$

$T_{\mathrm{am}}^2 = 0.2$

$(r/\Omega)_m = 0.1/\pi$

$G = 250$

$F = 5.7$

we obtain,

$N_{\min} = 32.1356$ photoelectrons, or 33 photoelectrons. (In practice, $N_{\min}$ will be larger, as pointed out above, because the value of R_L must be smaller to avoid a bandwidth limitation).

Taking the value of $N_{\min} = 33$ photoelectrons, the minimum laser transmitter optical pulse energy is calculated from equation (10), as

$$E_{tm} = 93.9226\,\mathrm{mJ} \text{ or } 94\,\mathrm{mJ} \text{ (for a worst-case } (S/N) = 1).$$

(In practice, E_{tm} should be greater than this value for a practical value of worst-case (S/N), which should be greater than 1. For the GLARS system with a transmitter pulse energy E_t of 120 mJ[1], the worst-case (S/N) is only equal to 1.278. E_t should be increased to have a sufficient margin for the worst-case (S/N)).

4. Conclusion

This paper presents the procedure for calculating the transmitter pulse energy requirements for a laser altimeter which can be used in the design of air-/spaceborne systems with avalanche photodiode receivers. Equations for the signal-to-noise ratio, minimum detectable power, bandwidth, and load resistor selection are presented for representative avalanche photodiode parameters. The minimum detectable power for the GLARS specifications given by Cohen *et al*[1] is 33 photoelectrons and the transmitter pulse energy required is 94 mJ for a worst-case $(S/N) = 1$. The GLARS system design value of 120 mJ for the transmitter pulse energy at 1064 nm should be increased, to have a sufficient margin for the worst-case (S/N).

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