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Dynamics of stick-slip

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Abstract

We discuss the mathematical mechanism leading to 'stick-slip behaviour' seen in very different situations 'Stick-slip' is seen in systems which are subjected to a fixed response and force developed in the sample is measured by dynamically coupling the system to a measuring device We discuss in some detail a model for a type of plastic instability and a model for earthquakes In both these cases, we demonstrate that the mathematical mechanism operating is the Hopf bifurcation Similar conclusions have been arrived at more recently in theoretical and experimental analysis on frictional sliding The possibility of modelling several apparently different phenomena is outlined

1. Introduction

A number of instabilities of dynamic origin bear formal resemblance to each other They exhibit what may be generically referred to as 'stick-slip behaviour' This is observed in situations where systems are subjected to a fixed response and force developed in the sample is measured by dynamically coupling the system to a measuring device. Stick-slip is intrinsically dynamical in nature and is characterized by the system spending most part of the time in the stick mode and a very short time in the slip mode Jerky flow or the Portevin–Le Chatelier effect or serrated yielding[1–7] is one such phenomenon which we have studied in great detail.[8–13] There are a number of other examples that have been studied in some detail in literature Stick-slip-like behaviour seen in sliding of a block of material on another due to the frictional force acting between them is an example known for a long time [14–16] The instability here is expected to arise from the velocity-weakening friction law [17–18] Seismic events resulting from the movement of tectonic plates is considered to be another example Here again, models use a velocity-weakening friction law to be operating between the tectonic plates. Both these problems have been recently studied from a dynamical point of view [19–21] Another example which bears a great resemblance to the central theme of the paper is the peeling of an adhesive tape When the tape is pulled at a constant velocity, the experimental strain energy released shows an 'unstable region' as a function of the drive velocity [22] Domain wall movement under the action of external field is yet another example.[23–24] Nonlinear conduction in some of the CDW materials is yet another example [25] One common feature of these cases is that these systems exhibit what can be generically called 'negative flow rate characteristic' (see Kubin and Poirior in Ref. 3) In two of the cases mentioned above[9–18], we have shown that this is a consequence of Hopf bifurcation. Recently, in the case of friction studies, similar result has been proved both experimentally and theoretically.[19–21] We conjecture that Hopf bifurcation is at the root of

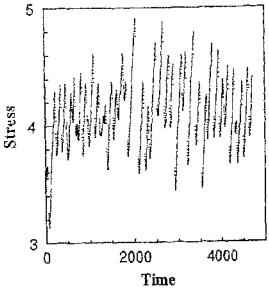

FIG 1 A typical stress-time plot for AlCu alloy

every case of stick-slip [26] Our interest in these seemingly different phenomenon is to evolve a mean field dynamical approach along the lines of the model for the PLC effect In the following, we will deal with jerky flow in some detail and show the formal similarity with other phenomena mentioned above

2. Portevin–Le Chatelier Effect

First, we recall the principal features of the jerky flow Metals subjected to a constant rate of tensile deformation normally exhibit a single yield drop However, under some metallurgical conditions, successive or repeated yield drops are seen This is referred to as the PLC effect or the jerky flow The phenomenon has been observed in several metals[1–2] such as iron with impurities (e g. C), commercial aluminium, stainless steel, brass, alloys of aluminium and magnesium, etc A typical plot of stress vs strain (time) is shown in Fig 1, for single crystal of AlCu As mentioned in the introduction, these kind of plots are typical to stick-slip phase. (for instance, compare Fig 5, p 6 of Ref 16). Repeated yield drops occur only within a certain window of strain rates and temperature The occurrence of repeated yield drops is also sensitive to the concentration of solute atoms Each of these yield drops is accompanied by the formation and propagation of dislocation bands

The basic mechanism of repeated yield drop is as follows [6] Dislocations move under the action of stress There are solute atoms which are present in these metallic alloys If dislocations are static or moving very slow, solute atoms have a tendency to aggregate at the core of the dislocations thereby making them more difficult to move, i e more stress will be required to make them move than without the solute atoms at the core This is called as *ageing* Now, consider the case when the dislocations are moving faster They will still gather solute atoms which would eventually arrest them But if dislocations are moving very fast, solute atoms may not find sufficient time to aggregate at the dislocation cores to slow them down Thus, there is a competition between the rate at which dislocations move and the rate at which solute atoms arrive at dislocation cores The first time scale is dependent on the rate of deformation of the specimen and the second on the diffusion constant of the solute atoms which is temperature dependent The negative flow rate characteristic of the yield stress as a function of applied strain rate is a direct consequence of these two competing effects [6] Another important feature of plastic deformation is that its exhibits hysteresis

2 1 *A dynamical model for the PLC effect*

From the above discussion, it is clear that the PLC effect results from the interaction of dislocations with mobile point defects This induces a negative strain rate sensitivity triggering the onset of the PLC effect The first effort to model this phenomena at a dynamical level was undertaken by us [8–9] The approach is dynamical The method seeks to establish a connection between interaction of the defects participating at the microscopic level to the macroscopically measured quantities We proposed a theory involving three types of dislocations and some transformations between them Even though the spatial inhomogeneous structure was ignored and only the temporal oscillatory state was sought to be described, the model proved to be very successful in that it could explain most of the experimentally observed features [1–2] The basic idea could be summarized by stating that limit-cycle solutions arise due to nonlinear interaction among the three different types of dislocations, suggesting a new mathematical mechanism for the PLC Recently, an extension of the model to account for the formation and propagation of dislocation bands has also been attempted[13]

We incorporate several mechanisms such as multiplication, immobilization and annihilation of dislocations into the rate of change of densities of dislocations of different types Some terms are used in simplified form to facilitate the use of mathematical framework of dynamical systems Thus, our attempt is to include typical mechanisms which describe generic features of the PLC effect rather than to reproduce properties of any particular material. Our model consists of three types of dislocations—mobile dislocations with density N_m, immobile dislocations with density N_{im}, and dislocations having clouds of solute atoms with density N_i The rate equations for the densities of dislocations are

$$\dot{N}_m = \theta V_m N_m - \beta N_m^2 - \beta N_m N_{im} + \gamma N_{im} - \alpha N_m, \quad (1)$$

$$\dot{N}_{im} = k\beta N_m^2 - \beta N_m N_{im} - \gamma N_{im} + \alpha' N_i, \quad (2)$$

$$\dot{N}_i = \alpha N_m - \alpha' N_i, \quad (3)$$

where the over dot refers to the time-derivative The first term in eqn (1) is the rate of production of dislocations due to cross-glide with a rate constant θ In the first term, V_m denotes the velocity of mobile dislocations, which depends on some power of the applied stress σ_a. The second term refers to two mobile dislocations either annihilating or immobilizing with rate constants $(1-k)\beta$ and $k\beta$, respectively (see the term $k\beta N_m^2$ in eqn (2)) The third term represents the annihilation of a mobile dislocation with an immobile one. The fourth term represents the remobilization of immobile dislocations due to stress or thermal activation with rate constant γ (see the term γN_{im} in eqn (2)) The last term represents the process of solute atoms gathering around the mobile dislocations, thereby slowing down such dislocations We consider such dislocations as new entities with density N_i. This process results in an outgoing term in eqn (1) and an incoming term in eqn (3). As more and more solute atoms gather, the dislocations eventually stop We represent this by a loss term in eqn (3) and a gain term in eqn (2) Thus, α refers to the concentration of the solute atoms which participate in slowing down the dislocations, and $(\alpha')^{-1}$ refers to the time constant for slowing down to occur The above equations should be coupled to the machine equation describing the rate of change of the stress developed in the sample This has the form

$$\sigma_a = \kappa(\varepsilon - bN_m V_m) \tag{4}$$

where κ is the effective stiffness, b, the magnitude of the Burgers' vector, and ε, the applied strain rate Using a power-law dependence for $V_m = V_0(\sigma_a/\sigma_0)^m$, eqns (1–4) can be cast into a dimensionless form by introducing scaled variables

$$x = N_m \frac{\beta}{\gamma}, y = N_{im} \frac{\beta}{\theta V_0}, z = N_i \frac{\beta\alpha'}{\gamma\alpha}, \phi = \frac{\sigma_a}{\sigma_0}, \text{ and } \tau = \theta V_0 t \tag{5}$$

The rescaled form of the dynamical equations are

$$\dot{x} = \phi^m x - b_0 x^2 - xy + y - ax, \tag{6}$$

$$y = b_0(kb_0 x^2 - xy - y + az), \tag{7}$$

$$z = c(x - z), \tag{8}$$

$$\phi = d(e - \phi^m x), \tag{9}$$

where the over dot now refers to a derivative with respect to the scaled time τ The new parameters are defined as follows

$$b_0 = \frac{\gamma}{\theta V_0}, a = \frac{\alpha}{\theta V_0}, c = \frac{\alpha'}{\theta V_0},$$

$$d = \frac{\kappa b\gamma}{\theta\beta\sigma_0}, \text{ and } e = \frac{\varepsilon\beta}{bV_0}. \tag{10}$$

There is a range of parameters a, b_0, c, d, e, k and m for which the dislocation densities are oscillatory in time It is obvious that whenever the quantity ϕ_x^m exceeds e, there will be an yield drop The theoretically calculated multiple yield drop curve is shown in Fig 2 Surprisingly, many qualitative results such as the negative strain rate behaviour of the flow stress, the existence of two critical strain rates within which the phenomenon is observed, the dependence of the amplitude of serrations on the strain rate and strain, and bounds on the concentration of solute atoms are correctly predicted *The most important prediction is that the negative strain rate sensitivity of the flow stress arises naturally as the consequence of a Hopf bifurcation* The negative strain rate sensitivity of the flow stress is shown in the inset of Fig 2 *The dependence*

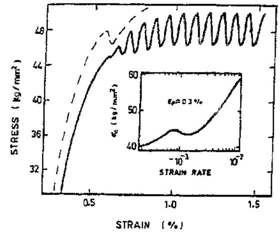

FIG 2 Theoretically obtained [illegible] yield drop plot (denoted by solid line) Th[illegible] experimental result showing a single yield drop The inset shows the stress σ_a vs strain rate

of the critical strain on the strain rate has a 'V' shape. Ours is the only model which predicts this experimentally observed feature Finally, our model also predicts chaotic stress drops in an intermediate regime of applied strain rates [12, 27] The above equations can be reduced to an order parameter equation in the form of Stewart–Landau equation [28] The nature of bifurcation in the entire parameter space (a, b) has been analysed to show the existence of both supercritical and subcritical bifurcation

Due to the fact that the model is dynamical in nature, the model predicts *chaotic stress which has been recently verified* [13, 29–32] To check the prediction that the PLC stress drops are of dynamical origin, analysis of the experimental data has to be carried out by available methods for time series developed in the area of dynamical systems The first attempt in this direction was undertaken by Ananthakrishna and coworkers.[29] (see also Ananthakrishna[13] where the value of the correlation dimension has been mentioned without giving details of the calculations) We have carried out an examination of the stress signals obtained from single crystals of Cu–10%Al and Cu–14%Al loaded under constant strain-rate conditions The results, though preliminary, suggested that the time series could be chaotic The methodology used was the simplest algorithm for implementation, namely, that due to Grassberger and Procaccia [33] Recently, there have been three more attempts[30–32], one of which is from a different group [32] In these studies, sophisticated methods such as singular value decomposition (SVD) method coupled to GP algorithm[34] and calculation of the positive Lyapunov exponent were used [35–36] Even so, the length of the time was short For this reason, several experiments were performed on polycrystals of Al Mg Recently, a detailed analysis of these long stress signals using correlation dimension, SVD with GP algorithm, and the Lyapunov spectrum have shown unambiguously that stress signals are chaotic [32h] The estimated number of degrees of freedom required for a dynamical description of the phenomenon that emerge from this analysis turns out to be the same as in the model Since the system is spatially extended, these modes represent collective degrees of freedom of dislocations as was used in the model

3. Similarities of the PLC effect with frictional sliding

The case of stick-slip in friction studies has considerable similarities with the PLC effect We shall start recapitulating some known facts The first known study on friction goes by the name of Amontons–Coulomb law (as early as 1699 and 1781, respectively) This states that the frictional resistance is independent of the surface area S and that static friction coefficient μ_s is the ratio of the pulling (tangential) force (f_s) to the normal force (f_n) In a dynamical situation, one can define the dynamic friction coefficient μ_d under a constant pulling velocity v If one looks at the area of contact under magnification, one finds that the contact is actually at a few micro contact points. Thus, the effective area of contact S_{eff} is much smaller than the actual area of contact Due to this, the normal force acts only on the microcontacts This also implies that there will be creep of the material resulting in increase of S_{eff} This is reflected in the experimentally measured fact that the static friction coefficient increases as a function of time In other words, there is *ageing* of contacts. Equivalently, $\mu_s = \mu_s(\tau_s)$, where τ_s is the stick time Thus, if the pulling speed is low, the *ageing* is higher Clearly, when these two time scales, namely, the ageing time scale and the loading time scale are of the same order, there will be interesting effects which result in stick-slip behaviour. Recently, there has been considerable experimental and theoretical investigations in this area [19-21]

In addition, experiments performed on rock samples show a striking similarity between the stress–strain curves in jerky flow and force vs displacement curves in experiments on rock samples Here again, the repeated drops in force as a function of displacement occur only within a window of displacement rates and temperature (see, for example, Ref 18).

As in the case of the PLC effect, where most theoretical descriptions use the 'negative flow rate characteristic', the Burridge–Knopoff model[37] for earthquake instability uses a simple velocity-weakening friction law [17] This law has a negative slope from the start as in the case of dynamic friction coefficient [19–21] This law is assumed to operate along the region of contact of tectonic plates which are in relative motion. It is due to this that earthquake-like events mimicking the Gutenberg–Richter[17] law are seen Recent investigation using an 'N'-shaped friction law in Burridge–Knopoff model (which is valid for the PLC effect) shows rich spatio-temporal dynamics.[38] *However, the dynamical origin of this velocity-weakening law has not been investigated Thus, it would be interesting to construct a model for stick-slip on faults and to demonstrate that the velocity weakening friction law used in earthquake models is not merely a convenient assumption, but one that can be derived on the basis of nonlinear interaction* Similar considerations appear to hold in the case of stick-slip dynamics in friction studies on 'solid-on-solid' experiments also

3.1 *A dynamical model for the stick-slip behaviour on faults*[18]

In this part, we outline a model along the lines similar to the dynamical model of the PLC effect We expect that this line of attack will help us derive an 'N'-shaped curve for the frictional law. Earthquakes are often described as slip events on the interface of two tectonic plates that are in relative motion caused by mantle-wide convection. We confine our attention to the interfaces of the plates They have complicated inhomogeneities Some recent studies indicate that the fault topology is self-affine In an idealized, one-dimensional representation of the fault

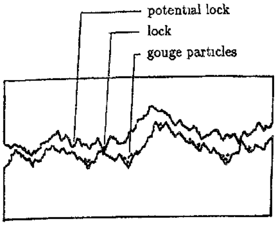

FIG 3 A schematic representation of a fault

(Fig 3), we picture the two plates as rough surfaces with protrusions As the plates move past each other, some of the protrusions of one plate may get entangled with those on the other, to form 'locks' that are immobile Protrusions that are not locked, but are free to move without hindrance are termed 'potential locks' As some regions are locked, the imposed drive velocity, v_d, produces strain This accumulates along the region of contact, being large at some places and small at others Each part of the fault has a velocity depending on the local shear stress $v(\sigma)$ The locked regions will get 'unlocked' and slip when the local stress exceeds a critical value The 'unlocking' may occur either by elastic or plastic deformation of the protrusions, or by fracture The last of these leads to the formation of gouge particles which accumulate at the interface Thus, regions that are moving at one point of time may be locked and stationary at a later time, and vice versa In the most simplified mean-field approach where the spatial aspects are ignored, we consider a system with the following three entities—potential locks that are in motion (h), locks that are immobile (L) as long as the stress is below a critical value, and the gouge particles (g) that are produced due to wear and tear The much smoother surface (s) produced due to wear and tear is not explicitly considered Let ρ_h, ρ_L and ρ_g be the densities of the three entities, respectively The following are the simplest transformations that can be envisaged

$$h + h \to L \tag{11}$$

$$L \to Ag + k_1 h + s \tag{12}$$

$$h + h + g \to L \tag{13}$$

$$g \to s \tag{14}$$

The first transformation refers to two potential locks meeting and forming a lock The second represents the fact that a lock may fracture giving rise to gouge particles with a rate constant A, with the remaining fraction forming a potential lock (representing a much less dominant asperity). The third transformation denotes the fact that gouge particles can be sandwiched between the two colliding potential locks leading to the formation of locks The last equation refers to the loss of the gouge material, which can happen in several ways For instance, it may be lost as it falls into a deep crevice or it may get stuck to the smooth surface (This surface that plays the role of pool chemicals in chemical reactions is not explicitly considered in the model) The rate equations corresponding to these are

$$\rho_h = -\beta v(\sigma)\rho_h^2 + k_1\psi(\sigma)\rho_L - \Lambda v(\sigma)\rho_h^2\rho_g, \tag{15}$$

$$\rho_h = +\beta v(\sigma)\rho_h^2 - \psi(\sigma)\rho_L + 2\Lambda v(\sigma)\rho_h^2\rho_g, \tag{16}$$

$$\rho_g = \varsigma\psi(\sigma)\rho_L - \Lambda v(\sigma)\rho_h^2\rho_g - \chi\rho_g \tag{17}$$

The coefficients of each term represent the rate constants There are two stress-dependent factors $v(\sigma)$ and $\psi(\sigma)$ The first refers to the local velocity, while the second represents the breaking stress at which the locks break. Both of these could have a power law dependence on stress. We need to include the influence of normal stress on the 'ageing' process In addition, these equations should be coupled to the rate of change of the shear stress developed at the interface due to the constant macroscopic drift velocity v_d Under some reasonable assumptions this can be written as:

$$\sigma = G\left[v_d - bv(\sigma)\rho_h\right], \tag{18}$$

where G is the shear modulus of the fault As in the previous case, one can cast these equations in scaled form which will be easier to handle Making use of power law representation for the stress-dependent functions, it is possible to show that these set of equations exhibit limit cycle solutions Following the analysis of the PLC model, we have calculated the dependence of the flow stress on driving velocity. In the region of the instability, we find the negative flow rate characteristic

These equations can be improved, in particular, it is important to consider the normal component of the stress Further studies are being carried out

4. Peeling of adhesive tape, domain wall movement and charge density waves

4 1 *Peeling of adhesive tape*

It is a common experience to find that peeling of of an adhesive is jerky if the rate of peeling is in an appropriate range In fact, the strain energy released is audible Clearly, this phenomenon falls in the category of 'stick-slip' Some detailed experiments and theoretical analysis has been carried out by Maugis and Barquins.[22] These experiments are simple but the physics is rich As the velocity of peeling is increased, the authors find that stick-slip appears abruptly with the amplitude jumping between the two branches corresponding to the crack initiation and crack arrest This amplitude decreases as the velocity of the peel is increased and eventually, stick-slip disappears. As in the case of the PLC effect, the two distinct time scales are clear One corresponds to the velocity of the drive and the other to the viscoelastic time scale Again, the ageing effect that we stressed in earlier section is manifest due to the viscoelastic medium, i.e. once the crack is opened up to a point, if now one imagines that the tape is held static, the viscoelasticity of the fluid tends to heal the crack to a certain extent depending on the amount of force acting between the two faces of the crack the and the elasticity of the tape The same is true as long as the time scale associated with the velocity of drive is small These authors have also attempted to explain this phenomenon by assuming an 'N'-shaped force versus velocity relationship, which is actually measured by them There is also a recent analysis of this problem showing chaotic behaviour [39]

4 2 *Motion of* 180° *Bloch wall*

The motion of a Bloch wall in a magnetically soft material under the action of a magnetic field has considerable resemblance with the motion of dislocation in the presence of solute atoms Usually, the mobility of domain walls is controlled by lattice defects such as dislocations, inclusions, etc However, in magnetically soft materials, diffusion of impurity atoms can become important in a certain range of temperatures For instance, carbon atoms in α-iron are such defects Martin *et al.*[24] have studied the motion of a single 180° Bloch wall in a single crystal of Si-Fe Consider two adjacent antiparallel magnetic domains In equilibrium, the proportion of interstitials vary in the region of the domain wall which is a few thousand angstroms width The domain wall is pinned by these interstitials However, the rate at which equilibrium is reached depends on the time scale associated with the diffusion of the carbon atoms Now, assume that the domain wall is made to move at a constant velocity v by an applied field Clearly, there are two competing time scales The analogy with a dislocation motion in a medium of solute atoms is clear In the regime when the two time scales are of the same order interesting effects of 'stick-slip' arise In fact, experimentally, the authors find periodic motion, two frequency regime and even chaotic regime as a function of the velocity of the domain wall

4 3 *Charge density waves*

Finally, let us consider the last example of fluctuations in voltage seen in charge density wave (CDW) compounds Charge density waves have been studied extensively. Even so, certain aspects of CDW such as the transition from ohmic to non-ohmic regime is not well understood. Under the action of an electric field, one finds transition from the ohmic regime ($E < E_t$) to the non-ohmic one ($E > E_t$) Above the threshold, the current–voltage characteristic is non-ohmic and nonlinear In addition, one also finds extremely low-frequency voltage pulses (~ 1Hz) in certain compounds such as blue bronzes $K_{0.30}MoO_3$. At low value of the electric fields, it is clear that the CDW as a whole cannot move, yet one does find current in the sample In order to explain this, Lee and Rice have suggested that phase dislocations (PDL) of the CDW could carry current at electric fields too low to depin the CDW as a whole This situation is quite analogous to plastic deformation where plastic flow occurs by generating dislocations which further move under the effect of an applied stress This analogy is useful and allows for interpretation of many qualitative features of the nonlinear and hysteresis phenomena observed in CDW transport The threshold E_t can be viewed as the onset of plastic flow of the charged PDL in the CDW lattice Below the threshold, the PDLs are unable to move, being pinned by impurities of crystal defects. Above the threshold, the motion of the PDLs is cooperative and nonlinear The very low-frequency voltage pulses (~ 1Hz) observed in many systems are quite similar to the serrations observed in the Portevin–Le Chatelier effect. Thus, they can be attributed to successive and cooperative unlocking and relocking of PDLs in the CDW lattice This analogy has been brought out by Dumas and Fienberg.[25]

The analogy between a tensile test at a constant stress rate and voltage measurement at constant current sets up a correspondence between stress σ_a and voltage V and strain ε and the current I As in plastic strain rate, ε_p is proportional to the number, N_m, of mobile dislocations and their velocity, V_m, the nonlinear current, J_{CDW}, is proportional to the number, N_d, and velocity V_d of moving PDLs Again, as in the case of plasticity where N_m and V_m, being functions of applied stress σ_a, N_d and V_d depend nonlinearly on the applied field

The features mentioned earlier show that it is possible to model the phenomenon of transition from ohmic to non-ohmic regime in a manner similar to the PLC effect Just like dislocations that are produced and move under the action of stress, newly charged PDLs can be generated through the crystal under the effect of an electric field possibly by a Frank–Read-like mechanism as suggested by Lee and Rice The repeated generation of new loops (slipped phase planes) by Frank–Read sources will produce a current Further, the interaction of such PDLs with impurities may lead, depending on the field, to the slow motion (dragging impurities) or fast (free from impurities) motion. This is again similar to the interaction of solute atoms with moving dislocations in the PLC effect. The PDLs can also form dipoles providing a cause of locking. Some of the striking similarities between CDW and jerky flow have been shown by the studies of Dumas and Frienberg[25], and Fienberg and Friedel [25] We refer the interested reader to the original papers [23]

5. Conclusions

In this paper, we have shown that the basic cause of 'stick-slip' is very general The stick- slip feature arises due to the 'negative flow rate characteristic'. This feature has been measured in a good number of situations In two instances, we have shown that this feature can be shown to result from Hopf bifurcation based on dynamical models for the physical phenomenon Similar attempts have been reported more recently We conjecture that Hopf bifurcation is at the root of the stick-slip nature of all these phenomena.

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