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Optical solitons in nonlinear fiber optics

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Abstract

We explain the concept of optical solitons in nonlinear fiber optics with brief reference to all the important underlying physical concepts The possibility of optical soliton in presence of important higher-order effects like third-order dispersion, self-steepening and stimulated inelastic scattering is discussed as well as the concept of wavelength division multiplexing using solitons The coexistence of optical solitons and self-induced transparency solitons in erbium-doped fibers is also explained

Keywords: Fiber optics, optical solitons, optical communications, networks, light pulses

1. Introduction

The world's communications infrastructure is poised to undergo a revolution Networks that send light pulses through optical fibers and optical amplifiers will change the way we communicate across the planet. But fiber-optic links already carry information across the globe, so what is new? First, the new networks have much greater capacities and will be able to transfer more information more quickly Second, these systems are genuine networks: the links connect many different nodes and can easily be reconfigured. We can thus envisage a world wide network in which distance does not represent a technical or economic barrier to communication. When compared to other types of communication, optical communication using optical fibers has more advantages like.[1]

- higher channel-handling capacity,
- low loss,
- no electro-magnetic interference,
- small dimension,
- abundant availability of raw material

In spite of these advantages, optical modes of communication also have constraints due to dissipation and dispersion. To avoid these problems, repeaters are placed at constant periodic intervals to reshape pulses and to provide necessary amplification

Recently, in many fields of science, a new concept of permanent wave propagation, namely, *solitons* have been reported Solitons are solitary waves which are formed by exact balancing between linear dispersive and nonlinear effects. The exact mathematical description of these pulses requires the solution of a nonlinear partial differential equation. With respect to the system involved these counterbalancing effects vary. A particularly exciting prospect for future communication systems is the possibility of using optical solitons to encode data In optical fibers, the (linear) group velocity dispersion (GVD) exactly balances with (nonlinear effect) self-phase modulation (SPM) to form optical solitons We can understand their nature from simple physical arguments. Nonlinearity creates a frequency sweep or 'chirp' across the pulse, with lower frequencies at the trailing side Dispersion also creates a chirp that is in the opposite sense if it is positive These two chirps can therefore cancel out, and hence the two distorting mechanisms can offset each other. The effect of nonlinearity and dispersion depends on the format of the optical pulses The conventional digital format in optical communication is non-return-to-zero (NRZ), so called because the optical power is on for 1 and off for 0, and does not return to zero for consecutive 1s With these two effects the pulse propagation in pure silica fiber is governed by the nonlinear Schrodinger (NLS) equation, as was theoretically predicted by Hasegawa and Tappert in 1973[2], and subsequently confirmed experimentally by Mollenauer *et al* [3]

For increased channel-handling capacity it is necessary to transmit ultrashort pulses at the order of sub-picosecond and femtosecond But the propagation of ultrashort pulses experience higher-order effects like third-order dispersion (TOD), self-steepening (SS) and stimulated Raman scattering (SRS) With the inclusion of TOD and SS the fiber system is described by the Hirota equation, which also allows soliton-type pulse propagation [4, 5] The pulse propagation in presence of all the higher-order effects is described by the higher-order NLS (HNLS) equation. Sasa and Satsuma[6] proved that the HNLS fiber system also has soliton solution for a particular choice for parametric restrictions among the parameters related to TOD, SS and SRS Very recently, we have also derived[7] the parametric conditions involving all the effects which the HNLS fiber system allows excepting soliton-type pulse propagation, and showed that HNLS equation can have a simple single peaked solution.

The concept of wavelength division multiplexing (WDM) is also used for transmitting larger number of channels in multimode fibers [1] An optical soliton is possible only in monomode fibers However, a monomode fiber can handle two orthogonally polarized waves [8] Even in monomode fibers, two fields of slightly different group velocities can be transmitted to achieve WDM [9] In either case, two fields are to be transmitted Dynamics of the pulse propagation with two fields is described by the coupled NLS (CNLS) equation, or on including TOD and SS, by the coupled Hirota equation and so on Both CNLS[10] and coupled Hirota equation[11] allow soliton-type pulse propagation In presence of all the higher-order effects, the two-field propagation is governed by coupled HNLS (CHNLS) equation,[12] which we introduced, and obtained the soliton solution [13]

To compensate the optical losses, either Raman pumping or continuous wave pumping in erbium (Er)-doped fibers is utilized[4, 5]; both these methods need an external pump source for amplification In 1967, McCall and Hahn[14] proposed yet another type of solitons in two-level resonant atoms, due to self-induced transparency (SIT) effect This type of soliton is described by the Maxwell–Bloch (MB) equations. Maimistov and Manykin,[15] in 1983, derived coupled NLS and MB equations to describe wave propagation of optical pulses in fibers with two-level resonant impurities Many researchers[15–19] have extended this work and we have recently rederived the parametric condition for the possibility of NLS–MB soliton in Er-doped fibers using Painlevé analysis.[16]

For ultrashort pulse propagation in fibers doped with two-level resonant atoms one has to include higher-order effects Recently, we have proposed coupled system of Hirota and MB equations (H–MB) to describe pulse propagation in Er-doped fibers with TOD and SS,[20] and also shown the possibility of soliton pulse propagation [21–23] In presence of all higher-order effects, the system is governed by HNLS–MB equations We have explicitly obtained parametric condition to study soliton propagation in this system through Painlevé analysis.[24]

In this paper, we discuss the concept of optical soliton by explaining all the physical concepts attached to fiber system We show the possibility of soliton in presence of all higher-order effects Then we briefly explain the WDM concept using solitons Finally, we discuss the coexistence of optical and SIT solitons in Er-doped fibers

2. Fiber characteristics

According to the intensity of the pulse, a fiber can act linearly or both linearly and nonlinearly. Optical losses and material dispersion are linear effects in pure silica fibers As silica fibers are made up of centro-symmetric molecules, the lowest dominant nonlinear effect is due to the third-order susceptibility in the form of Kerr nonlinearity For ultrashort pulses, higher-order effects such as TOD, SS and SRS are also important In this section, we discuss these concepts, and detailed mathematical and physical ideas can be found in Keiser and Hasegawa [1,4,5]

2 1 *Optical losses*

Optical losses are mainly due to the following two reasons

- presence of OH ion impurities
- Rayleigh scattering

The OH ions are absorbed by silica fibers during the time of manufacture itself Great care is taken to make the concentration of OH ion very low Since OH ion impurities have many resonant vibrational energy levels, there will be heavy attenuation when optical frequencies are transmitted in this resonant region

Rayleigh scattering is due to the presence of random density fluctuations of the frozen fused silica materials The amount of scattering varies inversely proportional to the fourth power of the wavelength of light propagated

2.2 *Material dispersion*

It is also very important to control dispersion, which is caused by variation of the fiber's refractive index with wavelength. This means that different wavelengths travel with different velocities Each optical signal contains a band of wavelengths and although dispersion in fiber can be small this effect can accumulate to broaden individual pulses leading to a loss of information.

Also, the important parameter of any optical dielectric material is the index of refraction. The index of refraction depends on the frequency of the optical signal being propagated For digital optical communication, pulses are propagated. An optical pulse consists of a band of

Fourier frequency components, each frequency component experiencing a different index of refraction. Since the velocity at which the light signal propagating is determined by the index of refraction, each Fourier component of a pulse will travel with a different velocity, termed group velocity. This causes temporal spreading of the pulse due to group delay This is called the group velocity dispersion (GVD)

GVD is considered as a major problem in the field of optical communication, due to which the energy of the pulse will fall away from the time gap of a pulse, inducing cross talk Error detection is also possible due to lowering of peak power because of GVD.

GVD is measured as a parameter in the fiber, and depending on the sign, there can be two types of dispersion The value of the GVD parameter can be negative or positive with respect to frequency, and the wavelength at which the GVD parameter tends to zero is called the zero dispersion wavelength Fortunately, fibers can be designed to have zero dispersion at the desired operating wavelength The problem is that the effects of nonlinearity are more severe at zero dispersion, mainly because both the signal and the amplifier noise then have the same phase velocity This leads to strong mixing between signal and noise which cause distortion in the pulse Nonlinearity can be reduced by decreasing signal power, but a power of at least a few milliwatts is needed to maintain the signal-to-noise ratio The solution is to design fibers with dispersion that alternate in sign the dispersion of the whole system is then close to zero but the local dispersion is non-zero. At this point the entire dispersion is taken care of by third-order dispersion (TOD) Dispersion below the zero dispersion wavelength is called normal or positive dispersion In normal dispersion regime, red-shifted (lower) frequency components travel faster than blue-shifted (higher) ones. The opposite occurs on the other side of zero dispersion wavelength and the dispersion is called anomalous or negative dispersion. Solitons in normal and anomalous dispersion regime are called dark and bright solitons, respectively Only the bright soliton is very useful in the field of optical communication

2 3. *Kerr nonlinearity*

The most important and difficult one to control is nonlinearity which arises as the refractive index of the fiber depends on intensity This should be ignored in shorter systems because it is intrinsically small, but the transmission lengths of the new systems are so long that the effect accumulates Nonlinearity can then cause new frequencies to be generated in the optical signal making it unstable and distorted

As the new communication systems are inherently nonlinear a new approach to system design has been adopted. For example, in linear systems this signal-to-noise ratio can be increased simply by increasing the input power, but in nonlinear systems the distortion of the signal depends on the power it carries. The signal power must therefore be optimized for each system. Fortunately, there are accurate mathematical descriptions of the processes involved, and extensive simulations have helped in the design of new system.

An example is the separation of the amplifiers. These are relatively costly and can be unreliable, so we would like to place them as far apart as possible However, both the amplifier-induced noise and nonlinearity depend on amplifier spacing. The noise injected by each amplifier is proportional to the gain, and the gain needed at each amplifier increases exponentially with separation Increasing separation reduces the number of amplifiers but also increases exponentially the noise injected at each one. The total noise in the system is thus higher for larger separations Nonlinearity, on the other hand, is generally reduced at larger separations, since it depends on the average power across the system

When the intensity of the light pulse propagated exceeds a certain threshold level then the fiber parameters not only depend on frequency but also on its intensity. In materials like pure silica fiber, the dominant nonlinearity is mainly due to third-order susceptibility. (All the even-order susceptibilities vanish because of the centro-symmetric nature of the SiO_2 molecule) Most of the nonlinear effects due to third-order susceptibility need phase matching condition to be satisfied for enhancement. Unless special care is taken in general, the phase matching condition is not satisfied The important nonlinear effect due to third-order susceptibility without phase matching is the Kerr nonlinearity or the intensity-dependent index of refraction

In any signal propagation, there will be a generation of phase shift among different frequency components proportional to the index of refraction. Since the index of refraction depends on the intensity, which is a time-varying quantity, the induced phase shift will also vary in time. This can be considered as a generation of newer frequency components on both sides of the bandwidth As this phase modulation to the pulse is due to its intensity, it is called the self-phase modulation (SPM). SPM can be considered as a spread in the frequency domain.

In anomalous dispersion regime, the two effects, GVD and SPM, exactly balance each other. That is, the temporal broadening due to GVD is balanced by spectral broadening due to SPM. This is the basic reason for the occurrence of optical solitons. Once this type of balancing is achieved in the fiber, then the signal can be transmitted to very long distance without any loss in signal.

2.4. *Self-steepening*

Self-steepening (SS), also called Kerr dispersion, is a nonlinear effect due to third-order susceptibility when there is a propagation of ultrashort pulses As the index of refraction depends on intensity, the group velocity will also vary with the intensity of the pulse, so the peak of the pulse will travel slower than the wings. Hence, the pulse gets steeper and steeper during the course of propagation It has the possibility of breaking after propagation over considerable distance. Physically, SS also produces spectral broadening and unlike SPM produces asymmetrical broadening.

2.5. *Stimulated Raman scattering*

Stimulated Raman scattering (SRS) is another nonlinear effect due to third-order susceptibility which gives a self-frequency shift to ultrashort pulses. The self-frequency shift is a self-induced red shift in the pulse spectrum arising from SRS: the long wavelength components of the pulse experience Raman gain at the expense of short wavelength components, resulting in an increasing red shift as the pulse propagates. It has been recognized that self-frequency shift is a potentially detrimental effect in soliton communication systems because power fluctuations at the source translate into frequency fluctuations in the fiber through power dependence of the soliton self-frequency shift and hence into timing jitter at the receiver.[25] Spectral broadening due to SRS is also asymmetrical

2.6. *Wavelength division multiplexing*

A simple way of increasing the capacity of these systems is to add new signals at other optical wavelengths, a technique known as wavelength division multiplexing (WDM). The amplifiers have a broad enough bandwidth to amplify over a range of wavelengths close to 1 5 μm, the usual transmission wavelength A second wavelength, for instance, that doubles the capacity of the system up to 16 different wavelengths can be used So huge data capacities can be achieved using WDM Nonlinearity is again a problem, however, because it allows separate channels to interact; for example, the presence of a pulse in one channel can alter the refractive index and phase velocity of another channel. These interactions cause the information in both channels to be distorted although these problems can be reduced by using dispersion management and controlling gain separation of the amplifiers. In this case, pulse dynamics is governed by coupled nonlinear partial differential equation

3. Optical solitons in pure silica fiber

The electromagnetic field vectors that characterize the light wave are generated by Maxwell equations which we shall write in the form.

$$\nabla\times\vec{E}=-\frac{1}{c}\frac{\partial\vec{B}}{\partial t}, \tag{1a}$$

$$\nabla\times\vec{B}=\vec{J}-\frac{1}{c}\frac{\partial\vec{D}}{\partial t}, \tag{1b}$$

$$\nabla\cdot\vec{E}=\rho \tag{1c}$$

$$\nabla\cdot\vec{B}=0, \tag{1d}$$

in which $\vec{D}=\vec{E}+\vec{P}$, where $\vec{P}$ is the polarization of the medium.

Since we are concerned with propagation through spatially homogeneous systems that possess charge neutrality, we may set $\vec{J}=\rho=0$ The only source for the light wave in the medium is then the polarization term $\vec{P}$ Polarization is due to departure from complete spherical symmetry in the shape of atoms while the departure itself is due to electromagnetic field of light wave in the medium. It is the interaction between light wave and medium that introduces nonlinearity into the problem. The light wave satisfies a wave equation that may be obtained by taking the curl of (1a) and using (1b). We obtain

$$\nabla^2\vec{E}-\frac{1}{c^2}\frac{\partial^2\vec{E}}{\partial t^2}=\frac{1}{c^2}\frac{\partial^2\vec{P}}{\partial t^2} \tag{2}$$

This equation is found to be nonlinear when the dependence of $\vec{P}$ upon $\vec{E}$ that is described above is used.

We now confine our attention to the propagation of plane wave fronts and a plane polarized wave Reasonable correspondence between theory and experiment has been achieved with this simplification and at the present time soliton propagation has only been treated satisfactorily in one dimension

Pulse propagation with the effect of GVD and SPM in pure silica fiber (here the effect of loss is not considered) is governed by nonlinear Schrodinger equation. The dimensionless form of NLS equation is given by,[8]

$$E_z = i(1/2E_{tt} + |E|^2E), \tag{3}$$

where E is the slowly varying envelope of electric field and subscripts z and t denote spatial and temporal derivatives

Certain solutions of nonlinear partial differential equations like eqn (3) are called soliton solutions. The soliton solution of eqn (3) will give the idea of the shape, width and intensity of the pulse to be transmitted for soliton-type pulse propagation The identification of linear eigenvalue problem (Lax pair) confirms its complete integrability and soliton-type pulse propagation in the system The NLS equation has a Lax pair and N-soliton solution,

$$\begin{matrix} \Psi_t = U\Psi, \\ \Psi_z = V\Psi \end{matrix} \qquad \Psi = [\Psi_1 \Psi_2]^T, \tag{4}$$

where

$$U = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\lambda + \begin{pmatrix} 0 & E \\ -E^* & 0 \end{pmatrix},$$

$$V = i\left[\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\lambda^2 + \begin{pmatrix} 0 & E \\ -E^* & 0 \end{pmatrix}\lambda + \frac{1}{2}\begin{pmatrix} |E|^2 & E_t \\ E_t^* & -|E|^2 \end{pmatrix}\right]$$

and $\lambda(= \alpha + i\beta)$ is the eigenvalue parameter, so that the consistency condition $U_z - V_t + [U, V] = 0$ leads to NLS equation. The one soliton solution of eqn (3) takes the form,

$$E = \mathrm{sech}(t)\exp\left(\frac{iz}{2}\right).$$

The possibility of soliton-type pulse propagation in nonlinear fiber system was first proposed by Hasegawa and Tappert[2] and experimentally confirmed by Mollenauer *et al.*[3]

3 2. *Experiments on generation of optical solitons*

For the propagation of a soliton in an optical fiber to be verified experimentally, it is necessary to generate a short optical pulse with sufficiently large power and use a fiber which has a loss rate less than 1 dB/km. It was only in the late 1970s that optical fiber loss could be brought to this level For the generation of optical solitons, it is further required that the spectral width of the laser be narrower than the inverse of the pulse width in time This requires the generation of a pulse with a narrow spectrum known as Fourier transform limited pulse.

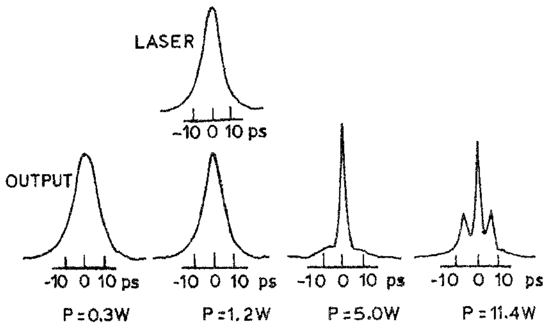

FIG 1 Experimental formation of soliton formation

It was for these reasons that the theoretical prediction of the transmission of a soliton was successfully demonstrated experimentally only after seven years In 1980, Mollenauer *et al.*[3] at AT&T Bell Labs succeeded for the first time in verifying soliton transmission in an optical fiber experimentally, by utilising an F^{2+} color center laser which is pumped by an Nd·YAG laser Using a fiber with a relatively large cross-section (10^6 cm^2) and a length of 700 m, they transmitted an optical pulse with a 7ps pulse width and measured the output pulse shape by means of autocorrelation For this particular fiber, the theoretically derived peak power for the formation of an optical soliton was 1 2 W Such a large power level was chosen in order that the autocorrelation measurement could easily be made at the output side

Figure 1 illustrates this famous experimental result Here, for different power levels at the input side, the pulse shape is measured by autocorrelation at the output side of the fiber It is clear from this figure that while the output pulse width increases for a power below the threshold of 1 2 W, it continuously decreases for input power above 1.2 W The appearance of two peaks in the case of an input power of 11 4 W is a consequence of the pulse interference of three solitons, which are generated simultaneously in this instance The periodic behaviour of the higher-order solitons was confirmed by Stolen *et al*[26] in a later experiment

4. Optical soliton in the presence of higher-order effects

For more channel-handling capacity it is necessary to propagate ultrashort pulses Mitschke and Mollenauer[27] reported in 1986, the self-frequency shift for solitons due to SRS In 1985, Kodama[28] and Kodama and Hasegawa[29] derived the HNLS equation which governs the ultrashort pulse propagation in pure silica fiber with higher-order effects like TOD, SS and SRS The HNLS equation takes the form,

$$E_z = i(\alpha_1 E_{tt} + \alpha_2|E|^2E) + \varepsilon[\alpha_3 E_{ttt} + \alpha_4(|E|^2E)_t + \alpha_5 E(|E|^2)_t], \quad (5)$$

where α_1, α_2, α_3, α_4 and α_5 are the parameters related to GVD, SPM, TOD, self-steepening and SRS, respectively

Equation (5) for $\varepsilon = 0$ reduces to the well-known nonlinear Schrödinger (NLS) equation.[2] For $\alpha_3 = \alpha_5 = 0$, eqn (5) describes the derivative NLS equation[30] which governs the propagation of NLS soliton in the presence of Kerr dispersion. Generally, Kerr dispersion is treated as a perturbation to the soliton propagation but the derivative NLS system also allows soliton-type propagation and the linear eigenvalue problem will be in the form of Kaup Newell or WKI type.[30–31]

Recently, we have proved that the HNLS equation allows soliton-type pulse propagation only for the following parametric restrictions,[7]

$$\alpha_1 . \alpha_2 = \frac{1}{2} \text{ and } \alpha_3\ \alpha_4 : \alpha_5 = 1.6\ 3$$

With the above conditions, the HNLS equation takes the following form,

$$E_z = i\left(\tfrac{1}{2} E_{tt} + |E|^2 E\right) + \varepsilon\left[E_{ttt} + 6|E|^2 E_t + 3E\left(|E|^2\right)_t\right] \tag{6}$$

Equation (6) was proposed by Sasa and Satsuma,[6] in 1991, who transformed the HNLS equation to the modified complex KdV equation and derived the 3^3 Lax pair for the same. They also derived the soliton solution using inverse scattering transform (IST). The soliton solution they have presented is a peculiar one with two peaks which is very difficult to generate in a soliton laser

5. Wavelength division multiplexing using solitons

Eventhough eqns (3) and (5) may adequately describe the propagation in single-mode waveguides, routing and switching operations which involve soliton pulses require interaction between two or more modes In general, the coupled mode approach still permits description of the pulse propagation in a multimode waveguide by means of vector versions of the above equations Although these systems of equations are no longer integrable, one may obtain quantitative information about pulse propagation by restricting to numerical or perturbative methods.

Here, first we will deal with the minimum number, i.e. two, of interacting modes and then will extend our results to many modes Situations of physical interest that can be described by two coupled NLS equations include two parallel waveguides coupled through evanescent field overlap, the coupling of two polarization modes in uniform guides, or in structures with a periodic variation of the parameters. It is also interesting to note that the study of the propagation of optical solitons in two or more mode nonlinear couplers, besides being important from the theoretical point of view, is important in view of their possible potential applications. Recent advances have permitted proof that solitons are ideal candidates for performing all optical switching operations in nonlinear couplers In fact, their stability leads to the possibility of controlling of coupling of the whole pulse, by means of changing the input power of a single pulse, or the phase of a superimposed signal

On the other hand, the concept of WDM is also used for transmitting an increased number of channels For this, more than one field has to be transmitted Solitons with two fields are described by CNLS equations,[31] and the effects of TOD and SS, by coupled Hirota equation As already mentioned, both CLNS[10] and coupled Hirota[11] equations allow soliton-type pulse propagation. In the presence of all the higher-order effects two-field propagation is governed by coupled HNLS (CHNLS) equation which takes the form,

$$E_{1z} = i\left[\alpha_1 E_{1,tt} + \alpha_2\left(|E_1|^2 + |E_2|^2\right)E_1\right] + \varepsilon\left\{\alpha_3 E_{1,ttt} + \alpha_4\left[\left(|E_1|^2 + |E_2|^2\right)E_1\right]_t + \alpha_5 E_1\left(|E_1|^2 + |E_2|^2\right)_t\right\},$$

$$E_{2z} = i\left[\alpha_1 E_{2,tt} + \alpha_2\left(|E_1|^2 + |E_2|^2\right)E_2\right] + \varepsilon\left\{\alpha_3 E_{2,ttt} + \alpha_4\left[\left(|E_1|^2 + |E_2|^2\right)E_2\right]_t + \alpha_5 E_2\left(|E_1|^2 + |E_2|^2\right)_t\right\} \quad (7)$$

Similar to HNLS equation, CHNLS equations also admit soliton solution only for the following form.

$$E_{1z} = i\left(\frac{1}{2}E_{1,tt} + \left(|E_1|^2 + |E_2|^2\right)E_1\right) + \varepsilon\left[E_{1,ttt} + 6\left(|E_1|^2 + |E_2|^2\right)E_{1t} + 3E_1\left(|E_1|^2 + |E_2|^2\right)_t\right]$$

$$E_{2z} = i\left(\frac{1}{2}E_{2,tt} + \left(|E_1|^2 + |E_2|^2\right)E_2\right) + \varepsilon\left[E_{2,ttt} + 6\left(|E_1|^2 + |E_2|^2\right)E_{2t} + 3E_2\left(|E_1|^2 + |E_2|^2\right)_t\right] \quad (8)$$

Very recently, we have derived the Lax pair for eqns (8) and obtained the soliton solutions using Bäcklund transformation [11] We also derived the N-soliton solution for the CHNLS equation using the Hirota bilinear method [13] The one soliton solution of eqns (8) takes the form,

$$E_1 = \sqrt{2}\beta \operatorname{sech}\left(2\beta t + 8\varepsilon\beta^3 z\right)\exp\left(2i\beta^2 z\right), \quad (9a)$$

$$E_2 = \sqrt{2}\beta \operatorname{sech}\left(2\beta t + 8\varepsilon\beta^3 z\right)\exp\left(2i\beta^2 z\right). \quad (9b)$$

Thus, it has been shown that the CHNLS system equations allow soliton-type pulse propagation.

6. Solitons in erbium-doped fibers

The interaction of intense light radiation with various forms of matter is a fruitful source of problems in nonlinear wave propagation Different modes of an atomic medium have been devised to isolate numerous types of phenomena that can occur. In particular, if the frequency of the light wave is almost exactly equal to a transition frequency between two populated energy levels of the atoms that comprise the material, then strongly resonant interactions between light and matter can take place

In the theoretical description of this strongly resonant situation it is frequently possible to ignore all other energy levels of the atoms and treat the interaction of light with a so-called two-level atom. By assuming classical treatment of light, resonant interaction of intense light with matter can be treated quite thoroughly

Extensive investigation of this strongly resonant situation led to the observation of soliton behaviour both in experiments and numerical solutions of the governing equations. The effect is known as self-induced transparency

If the fiber contains two-level resonant impurities or erbium atoms, doped to the core for the purpose of pulse amplification, this will induce SIT Since optical pulse propagation in a two-level resonant medium is governed by Maxwell–Bloch (MB) equations,[14] the nonlinear fiber system doped with erbium atoms will be described by the coupled system of the NLS–MB equations,[15, 16]

$$E_z = i\alpha_1 E_{tt} - i\alpha_2|E|^2E + \langle p\rangle,$$
$$p_t = i\omega p - fE\eta,$$
$$\eta_t = f(Ep^* + E^*p), \tag{10}$$

where p and η are given by $v_1v_2^*$ and $|v_1|^2 - |v_2|^2$, respectively Here v_1 and v_2 are the wave functions of the two energy levels of the resonant atoms and f the character describing the interaction between resonant atoms and optical field. The bracketed term < > means averaging over the entire frequency range,

$$\langle p(z,t,\omega)\rangle = \int_{-\infty}^{\infty} p(z,t)h(\omega)d\omega,$$
$$\int_{-\infty}^{\infty} h(\omega)d\omega = 1, \tag{11}$$

where $h(\omega)$ is the uncertainty in the energy level.

Because of the averaging term in eqn (11), it will be very difficult to analyse the equation. To overcome this mathematical complication, we assume the sharp line limit in which we can write $<p> = p$.

Through Painlevé analysis, we find that the NLS–MB fiber system allows soliton-type propagation only when the following parametric condition is satisfied,

$$-2f^2\alpha_1 = \alpha_2 \tag{12}$$

Similar condition has also been derived by Maimistov and Manykin [15]

Substituting condition (12) into eqn (11), the NLS–MB equations modify to

$$E_z = i\left(\tfrac{1}{2}E_{tt} + |E|^2E\right) + 2\langle p\rangle,$$
$$p_t = 2i\omega p + 2E\eta,$$
$$\eta_t = -(Ep^* = E^*p). \tag{13}$$

The Lax pair is found to be,

$$\begin{aligned}\Psi_t &= U\Psi \\ \Psi_z &= V\Psi\end{aligned} \qquad \Psi = [\Psi_1 \;\; \Psi_2]^T, \tag{14}$$

where

$$U = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\lambda + \begin{pmatrix} 0 & E \\ -E^* & 0 \end{pmatrix},$$

$$V = i\left[\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\lambda^2 + \begin{pmatrix} 0 & E \\ -E^* & 0 \end{pmatrix}\lambda + \frac{1}{2}\begin{pmatrix} |E|^2 & E_t \\ E_t^* & -|E|^2 \end{pmatrix}\right] + \begin{pmatrix} \left\langle \dfrac{\eta}{\lambda - i\omega} \right\rangle & \left\langle \dfrac{-p}{\lambda - i\omega} \right\rangle \\ \left\langle \dfrac{-p^*}{\lambda - i\omega} \right\rangle & \left\langle \dfrac{-\eta}{\lambda - i\omega} \right\rangle \end{pmatrix}$$

and $\lambda(=\alpha + i\beta)$ is the eigenvalue parameter so that the consistency condition $U_z - V_t + [U, V] = 0$ leads to NLS–MB equations (13)

The single soliton solution of NLS–MB equations is also constructed using Darboux–Bäcklund transformation (BT)[16] and is found to be

$$E(z, t) = 2\beta \operatorname{sech}(\rho_1)\exp(i\sigma_1 - i\theta_1),$$

$$p(z,t,\omega) = \frac{2\beta\{\beta \sinh(\rho_1) + i(\alpha - \omega)\cosh(\rho_1)\}\exp(i\sigma_1 - i\theta_1)}{\beta^2 \sinh(\rho_1) + (\alpha - \omega)^2 \cosh^2(\rho_1) + \beta^2/4},$$

$$\eta(z,t;\omega) = \frac{\beta^2 \sinh^2(\rho_1) + (\alpha - \omega)^2 \cosh^2(\rho_1) - \beta^2/4}{\beta^2 \sinh^2(\rho_1) + (\alpha - \omega)^2 \cosh^2(\rho_1) + \beta^2/4}, \tag{15}$$

where ρ_1 and σ_1 are functions of z, t and soliton velocity parameters given by,

$$\rho_1(z,t) = 2\beta t + \left[-4\alpha\beta + \int_{-\infty}^{\infty} \frac{2\beta h(\omega)d\omega}{\beta^2 + (\alpha - \omega)^2}\right]z + \rho_1^{(0)},$$

$$\sigma_1(z,t) = 2\alpha t + \left[2(\beta^2 - \alpha^2) - \int_{-\infty}^{\infty} \frac{2(\alpha - \omega)h(\omega)d\omega}{\beta^2 + (\alpha - \omega)^2}\right]z + \sigma_1^{(10)},$$

($\rho_1^{(0)}$ and $\sigma_1^{(0)}$ are independent of both z and t), and θ_1 is a real constant

Many research workers[16–18] have reported that a stable $2\pi/N = 1$ NLS–MB soliton exists Also, the multiple soliton structure proved that the higher-order NLS–MB solitons always split into multiple of $2\pi/N = 1$ solitons The phase change of the new soliton is governed solely by the NLS component while the pulse delay is determined solely by SIT component when detuning from resonance is zero

6 1 *Experiments on SIT solitons in erbium-doped fibers*

Nakazawa *et al*[19] have successfully transmitted optical solitons with the effect of SIT in Er-doped fibers They used a waveguide of 8,900 ppm Er ion concentration, cooled to 4 2 K The

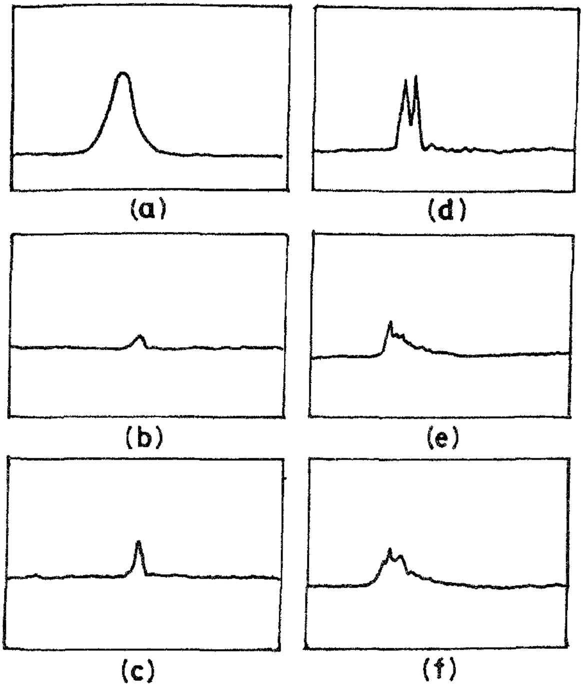

FIG 2 Experimental results for SIT in a 1 5m-long Er fiber The longitudinal axis in each figure is in arbitrary unit and the transverse axis is 500ps/div 2π and multiple 2π soliton are clearly observed

pulse source was a 1 5 μm mode-locked Er-doped glass laser with a pulse width of 500 ps Stable 2 and 4π, and multiple soliton pulses that broke up were clearly observed Figure 2 shows the experimental results for SIT in a 1 5m-long Er fiber The linear absorption was as large as 179 dB, corresponding to $\alpha L = 41\ 2$ Figure 2a is an input pulse with a width of 500 ps At a peak intensity of 50–70 W (Fig 2b), it was possible to transmit the pulse although it had a low intensity For a peak power of 100–130 W, a considerable pulse narrowing is observed from 500 to 120ps, as shown in Fig 2c This situation corresponds to an excitation of

$2\pi < \theta < 3\pi$ pulses, having input pulse with decreased duration and increased intensity. It is important to note here that the coupled power of 100 –130 W agrees well with the estimation of 107 W for the generation of an SIT soliton By increasing the coupled peak intensity to 170–250 W, a double peak pulse was clearly observed (Fig 2d) which corresponds to a 2π pulse (two separated 2π pulses) due to the excitation of a $3\pi < \theta < 5\pi$ input pulse By further increasing the input peak power to above 300 W, pulse breakups with more than four 2π pulses were observed (Fig. 2e and f)

This is attributed to the fact that multiples solitons were excited by the high power. These breakups, and pulse narrowing with an increase in intensity, are proofs of the SIT Similar experiments were also done for 3- and 6m-long Er fibers

7. Solitons with Kerr dispersion in erbium-doped fibers

As in pure silica fiber, ultrashort pulses in Er-doped fibers also suffer from higher-order effects like TOD, SS and SRS Doktorov and Shchesnovich[33] considered only the SS higher-order effect, and showed the coupled system of derivative NLS–MB fiber system allows soliton-type pulse propagation By considering the above effects, we introduced the coupled system of the Hirota and MB equations (H–MB system) which governs the wave propagation of ultrashort pulses in Er-doped fibers with TOD and SS [20] The H–MB system equations are

$$\begin{aligned} E_z &= i\left(\tfrac{1}{2}E_{tt} + |E|^2 E\right) + \left(E_{ttt} + 6|E|^2 E_t\right) + \langle p\rangle \\ p_t &= 2i\omega p + E\eta \\ \eta_t &= -\left(Ep^* + E^* p\right) \end{aligned} \tag{16}$$

We have analysed[21, 23] the possibility of soliton-type pulse propagation in H–MB system using Painlevé analysis and with the help of Lax pair have also obtained the explicit one-soliton solution using BT. The Lax pair is given by

$$\begin{aligned} \Psi_t &= U\Psi, \\ \Psi_z &= V\Psi, \end{aligned} \qquad \Psi = (\Psi_1 \ \ \Psi_2)^T \tag{17}$$

where

$$U = \begin{pmatrix} -i\lambda & E \\ -E^* & i\lambda \end{pmatrix},$$

$$V = \begin{pmatrix} A & B \\ -B^* & -A \end{pmatrix} + i\begin{pmatrix} \left\langle \dfrac{\eta}{\lambda - i\omega} \right\rangle & \left\langle \dfrac{-p}{\lambda - i\omega} \right\rangle \\ \left\langle \dfrac{-p^*}{\lambda - i\omega} \right\rangle & \left\langle \dfrac{-\eta}{\lambda - i\omega} \right\rangle \end{pmatrix}$$

Here

$$A = -4i\lambda^3 - 2i\lambda^2 + 2i\lambda|E|^2 + i|E|^2 + i|E|^2 + \left(EE_t^* - E_t E^*\right),$$

$$B = -4\lambda^2 E + 2\lambda E + 2i\lambda E_t - 2|E|^2 E + iE_t - E_{tt}$$

The H–MB equations are obtained from the consistency condition $U_z - V_t + [U, V] = 0$. The single soliton solution of H–MB equation can be obtained as

$$E(z,t) = 2\beta \operatorname{sech}(\rho_2)\exp(i\sigma_2 - i\theta_2), \tag{18}$$

where ρ_2 and σ_2 are functions of z, t and soliton velocity parameters given by

$$\rho_2(z,t) = 2\beta t + \left[8\alpha\beta \int_{-\infty}^{\infty} \frac{2\beta h(\omega)d\omega}{\beta^2 + (\alpha-\omega)^2} + 8\beta(3\alpha^2 - \beta^2)\right]z + \rho_2^{(10)},$$

$$\sigma_2(z,t) = 2\alpha t + \left[4(\beta^2 - \alpha^2) - \int_{-\infty}^{\infty} \frac{2(\alpha-\omega)h(\omega)d\omega}{\beta^2 + (\alpha-\omega)^2} + 8\alpha(\alpha^2 - 3\beta^2)\right]z + \sigma_2^{(10)},$$

$\rho_2^{(0)}$ and $\sigma_2^{(0)}$ are independent of both z and t and θ_2 is a real constant.

7.1 *Coexistence of HNLS and SIT solitons*

With inclusion of all the higher-order effects, the wave dynamics in Er-doped fiber is described by the coupled system of the HNLS and MB equations (HNLS–MB equations). Maimistov[34] considered the HNLS–MB equation and linearized the same, although he did not study the soliton possibility. We analysed the possibility of soliton-type pulse propagation in the HNLS–MB equation using Painlevé analysis and presented the Lax pair for the same.[24] The HNLS–MB equation reads as

$$E_z = i\left[\alpha_1 E_{tt} + \alpha_2 |E|^2 E\right] - \epsilon\left[\alpha_3 E_{ttt} + \alpha_4 |E|^2 E_t + \alpha_5 E\left(|E|^2\right)_t\right] + \langle p \rangle,$$

$$p_t = 2i\omega p + E\eta,$$

$$\eta_t = -\left(Ep^* + pE^*\right). \tag{19}$$

Using Painlevé analysis we derived that the HNLS–MB equation is integrable for choices of parameters given by,

$$\alpha_1 \cdot \alpha_2 = 1:2 \text{ and } 3\alpha_3 = \alpha_4 = 2\alpha_5. \tag{20}$$

For the system of eqn (19 with (20)) we have obtained the Lax pair and discussed the soliton solutions.[24]

8. Solitons in random nonlinear erbium-doped fibers

We have treated the randomness to the Er-doped fiber system using Painlevé analysis and framed the inhomogeneous NLS–MB system which allows soliton-type pulse propagation.[34] The INLS–MB system is governed by the following set of evolution equations,

$$E_z = i\left[\tfrac{1}{2} E_{tt} + |E|^2 E \frac{1}{2(z+z_0)} E\right],$$

$$p_t = 2i\omega p + E\eta,$$

$$\eta_t = -\left(Ep^* + Ep^*\right) \tag{21}$$

8.1 *Experimental results and advantages in soliton-based communication system*

Recognizing the potential of soliton communication, more than ten research groups worldwide are currently conducting soliton transmission experiments, attempting to push the transmission capacity to higher limits

In order to use solitons in our communication networks, a soliton must be created by injecting a pulse of the right size (intensity) and shape into a fiber that has dispersion of the appropriate sign. The pulse must be 'bell-shaped' to have the required relationship between temporal profile and frequency spectrum The intensity must be large enough for the nonlinearity to counteract dispersion, and this depends on the level of dispersion in the fiber and the inverse square of the pulse width Once achieved, the balance between dispersion and nonlinearity is stable, if, say, the intensity is too small, the pulse will broaden to allow stable propagation This balance can be maintained over extremely long distances, and solitons can travel millions of kilometers without significant distortion

The design of systems based on solitons must take additional factors into account. To begin with, these systems need an entirely new approach to generate data, for example, the soliton must be kept separated by at least five times their pulse width to prevent any overlap between them Laboratories worldwide are also addressing the question of how much data can be transmitted using solitons and over what distances The first problem that has emerged is that solitons are easily displaced in time, even though they still retain their individual integrity. Noise injected by the amplifiers can 'kick' the solitons from their starting positions in a random way This so-called Gordon–Haus jitters is the primary limiting factor in soliton system, and it is this that prevented solitons from being used in the first generation of optical networks

Researchers have now developed many schemes for combating Gordon–Hauss jitter The simplest and most effective of these is to insert filters at each amplifier stage that only allow certain frequencies through Such filters control the central frequency of the solitons and prevent it from moving in time These schemes should allow solitons to be transmitted over extremely long distances and with data rates well in excess of those achieved in first generation systems

The second problem with solitons is loss in the fiber which reduces signal intensity and thus destroys the balance with dispersion It turns out, however, that loss is not a serious problem if the initial power of the soliton is chosen so that the average power over an amplifier link equals the power needed to balance dispersion. The accumulated dispersion in each link must also be kept within certain limits

Solitons can also be multiplexed at several wavelengths, without the interaction between channels suffered by NRZ systems This issue is the focus of much current soliton research It turns out that in the real world, where loss in the fibers leads to energy dissipation during transmission, there is some interaction between the channels. The interaction causes a frequency shift in the solitons which gives rise to timing jitter. This effect can be achieved if the wavelengths used by each channel are close enough together. This is quite different from the NRZ systems in which the interaction is reduced if the wavelength separation is made larger

There is still hot debate as to whether solitons or NRZ format will be used in our future communication systems. NRZ systems based on wavelength multiplexing have operated on ten channels over distances of 10,000 km. A soliton system using seven channels at 10 Gbit s^{-1} has also been demonstrated by Linn Mollenauer at AT & T Laboratories in the US This data rate is an important difference. Current systems operate at 5 Gbit s^{-1} and do not connect easily with land-based networks which have standard data rates of 2 5 and 10 Gbit s^{-1} NRZ data at 10 Gbit s^{-1} has a limited range and does not allow significant wavelength multiplexing, but solitons can be multiplexed at this data rate. Soliton systems can be improved further by using dispersion management techniques, and Masatoshi Suzuki and colleagues at KDD Laboratories, Japan, have shown that a system with a data rate of 20 Gbit s^{-1} can work over 10,000 km, even without components to control jitter.

However, established NRZ format can achieve high data rates by multiplexing many channels at lower data rates (2 5 or 5 Gbit s^{-1}). Indeed, a new link, Sea-Me-We3, will be the first NRZ system to use WDM Soliton systems must therefore provide extra, unique advantages to increase their chances of being adopted In fact, solitons do have some rather subtle properties that could make them the preferred choice for future systems. Two of these are particularly noteworthy

First, solitons could be unaffected by an effect called polarization mode dispersion. This is caused by the unavoidable imperfection in the circular symmetry of any fiber which leads to a small and variable difference between the propagation constants of orthogonally polarized modes This creates a form of modal dispersion which is similar to the well-known problem in multimode fibers In the case of single-mode fiber there are nearly two identical modes that propagate with orthogonal polarization The difference in propagation constants is extremely small, but this dispersion becomes a major problem over long distances and at high data rates Current NRZ systems use the so-called polarization scrambling which rapidly varies the input polarization of the signal to avoid this problem, but it may not arise in soliton systems

Second, solitons are compatible with all optical processing techniques. A long-term goal is to create networks in which all of the key high-speed functions, including routing, clock recovery, demultiplexing and switching are performed in the optical domain so that signals do not have to be converted to electrical form along the way. Most of the devices and techniques proposed for these tasks work only with well-separated optical pulses, so are particularly effective with solitons.

It also appears that solitons, when controlled properly, can be more robust than NRZ pulses. Schemes have been devised that can not only provide control over the temporal positions of the solitons, but also remove noise added by amplifiers. Such schemes would allow the separation between amplifiers to be extended. Recent experiments have demonstrated amplifier separations up to 140 km, well above those that can be used with NRZ pulses. This is probably the strongest economic argument for using solitons, since fewer amplifiers would be needed for the same data capacity.

9. Summary and conclusions

We have reviewed the concept of optical solitons in nonlinear fiber optics with a brief explanation of the various important physical concepts related to nonlinear fiber optics The possibility of optical soliton in the presence of higher-order effects like third-order dispersion, self-steepening and stimulated inelastic scattering was discussed as also the possibility of wavelength division multiplexing using solitons

In erbium-doped optical fiber, effects like GVD, SPM and SIT are fundamental, and the soliton solution of the NLS–MB equation applies The parametric condition for the same has been explicitly shown Including higher-order effects like higher-order dispersion, self-steepening and stimulated Raman scattering, we have studied two interesting systems, namely, the H–MB system and HNLS–MB fiber system, and shown for the first time, the possibility of soliton propagation in HNLS–MB fiber system for two parametric choices

Now we are investigating the effect on damping and dissipation on solitons and also are analysing the possibility of solitons with the inclusion of inhomogeneous effect like variable dispersion nonlinearity and amplification

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