J. Indian Inst. Sci., July–August 1997, 77, 375–393.

Book Reviews

Analysis of thin concrete shells by K. Chandrashekhra, New Age International (P) Limited, Publishers, 4835/24, Ansari Road, Darya Ganj, New Delhi 110 002, 1996, pp. 462, Rs. 650/-.

This book "Analysis of Thin Concrete Shells" comprising nine chapters deals with the problem of shells in a systematic and comprehensive manner. This book is primarily addressed to graduate students and practicing engineers. The style and writing reflect the approach of a gifted and experienced teacher of several years of standing.

The author has presented the fundamentals for differential geometry, the assumptions made in theory of thin shells in Chapter 1 and these are required for a student who begins the graduate course in Theory of Thin Shells. Membrane theory of shells of Revolution has been discussed in the second chapter. This includes the examples of circular cylindrical shell filled with liquid, cylindrical shell subjected to wind loading. Conical and spherical shell subjected to various loading are also discussed. The author has presented the results for membrane forces and displacements for various shell geometry and loading in tabular form in chapter 2 to enable the designer to arrive at quick design. The membrane theory of shells of translation has been developed in chapter 3. This includes cylindrical shells, synclastic shells such as elliptic paraboloid and anticlastic shells such as hyperbolic paraboloids and conoids. Though in these two chapters the author has discussed the membrane theory neglecting bending resistance he has highlighted serious shortcomings of this at appropriate places in these chapters. Bending theory of cylindrical shells with and without edge beams is illustrated in chapter 5. The author also shows that short shells may be approximately analysed using membrane theory, while long shells may be analysed using simple beam theory. A brief description for designing and detailing the reinforcement for cylindrical shell with edge beam is also presented. Bending analysis of shells of revolution such as cylindrical, conical and spherical shells subjected to axisymmetric edge loads has been discussed in chapter 5. In this chapter the author has also presented finite element method to solve the problem of shells of variable thickness including multishell structures. Again the results are given in a tabular form for various shells.

The bending analysis to cylindrical and conical shells of revolution subjected to arbitrary loads is discussed in chapter 6 and the applications have been illustrated through examples. The concept of equivalent cylindrical shell is interesting. In chapter 7 the bending analysis to shallow doubly curved shell is presented along with a simple method to determining the boundary effect which the designers may find very useful. By using his method with membrane theory of shells several practical shell structures could be designed. The author has developed the analysis of long and short folded plate structures in chapter 8 and analysis of long folded plate has been illustrated through an example. These days model tests also are carried out to verify theoretical solution but also for design. In chapter 9 experimental analysis of shell structures has been discussed.

In this book the author has presented both mathematical analysis and design aspects of shell structures in a balanced way. He has also presented the results in the form of tables to enable the designer to arrive at quick design. The author could have included chapters one on Buckling of shells and the other on Finite Elements Analysis of Shells. Even though the title of the book is Thin Concrete Shells, the theory developed in this book could also be applied to shell of any other material. I am sure that the book "Analysis of Thin Concrete Shells" will serve its intended purpose.

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Handbook of Brownian Motion-Facts and Formulae, by A. N. Borodin and P. Salmien, Birkhauser Verlag AG, Klosterberg 23, CH-4010 Basel, Switzerland, 1996, pp. 462, SFr. 138

Brownian motion has come a long way since the day botanist Robert Brown saw, in early last century, what he considered as a 'peculiar' movement of pollen suspended in liquid. Mathematically, however, it took off only in this century, beginning with Bachelier (1900) who studied it in order to model stock prices, followed soon by Einstein (1905) who predicted and analysed such a motion without apparently being aware of Brown's findings. As a mathematical object, its rigorous foundations were laid by Norbert Wiener in 1923. He constructed the 'Wiener measure', the probability law of Brownian motion, on the space of continuous functions. Coming as it did in quick succession after the theory of Lebesgue measure on Euclidean spaces which had not quite sunk into the collective consciousness of mathematicians, it took a while before its importance was realized. But the man who really took up Brownian motion as a mission was the Frenchman Paul Levy, exploring its fine properties, many of which have revealed their true value only in recent times. With Levy, Brownian motion had 'arrived'.

The physicists were, of course, not content with pure Brownian motion as a model for phenomena they were looking at and more realistic models were sought, incorporating, for example, 'inertia'. This led to objects like the Smoluchowski process, the Ornstein-Uhlenbeck process, which were early examples of what has come to be known as diffusion processes.

There are a few well-defined landmarks in the general theory of Brownian motion and diffusion processes. An early contribution was by Kolmogorov, who, using the Markov property cast as the Chapman-Kolmogorov equation, obtained the 'infinitesimal laws of motion' for the probability densities of such processes in forward and reverse time. This led to the forward and backward equations of Kolmogorov, the former generalizing the Fokker-Planck equation for Brownian motion. These, however, required strong regularity conditions on the 'drift' and 'diffusion' coefficients associated with the process, which specify incremental conditional mean and variance. The next step, led by Feller and Dynkin, was to abstract out the Markov property as a semigroup of positive linear operators on a function space, giving an analytic theory based on the semigroup theory developed by Hille, Yosida and others. Then came Ito's stochastic differential equations, which gave a valuable probabilistic framework for these processes, putting at one's disposal the powerful machinery of martingale theory developed by Doob, Meyer and others. The next big jump combined elements of both analytic and probabilistic formulations. This was the martingale formulation of Stroock and Varadhan, with which things fell into place. The next major step jumped categories of abstraction – it dealt directly with functionals of Brownian motion in terms of an infinite dimensional 'calculus' on Wiener space. This is the 'Malliavin Calculus', the last major Brownian adventure to date. Undoubtedly there will be more.

Applications have not been lagging behind. A big customer is of course physics. (A celebrated early survey on this aspect is by S. Chandrasekhar). Quantum mechanics in particular has had a deep relationship with Brownian motion, particularly through the Feynman-Kac formula for path integrals. To mention another strand, Nelson's stochastic mechanics tries to base quantum mechanics in a more fundamental way on Brownian motion and diffusions. The other notable application area has been control and communication engineering, where Brownian motion served as a rigorous (but not uncontroversial) model for integrated 'white nose' in continuous time. More recently, reflected Brownian motion as a model of queues in heavy traffic limit has gained much currency as a computational tool. Finally, a very very recent hype has been in its application to finance, particularly option pricing, bringing us back in a full circle to where Bachelier began.

Given this, a 'handbook' couldn't have come at a more opportune time. And a handbook it is, indeed! A two part book, its first part is devoted to 'everything you wanted to know about Brownian motion, but were afraid to ask' compressed into hundred odd pages. This includes the stochastic process prerequisities, linear diffusions, stochastic calculus, Brownian motion curios, Feynman-Kac formula,....., and lots and lots of 'local time', a process even more peculiar than the Brownian motion, measuring the net time it spends at any given level.

This part naturally is a terse, rapid overview of facts without details and cannot replace a good textbook for the same. (One that comes to mind is 'Brownian Motion and Stochastic Calculus', I. Karatzas and S. Shreve, 2nd Ed., Springer Verlag, New York 1991). There are some ommissions. The p.d.e connection could have been explored further, leading perhaps to Malliavin Calculus. But then there has to be a choice and the authors' clearly did not include this.

The second part deals with explicit formulae for quantities pertaining to Brownian motion and related processes (such as the Bessel processes, the best known of which measures distance of the Brownian motion from the origin). This constitutes the bulk of the book and is just a long list of categorized formulae for the aforementioned.

This is perhaps the only book that does this, providing thereby a valuable source for people who need to explicity calculate such quantities, be it to analyse a communication network or a stock market. The book does a great job of compiling such facts and is unlikely to be outdone in this respect for a while to come.

To summarize, a great 'handbook', but don't expect to learn or teach from it.

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Robust nonlinear control design, (State-Space and Lyapunov Techniques) by R. A. Freeman & P. V. Kokotovic, Birkhauser Verlag AG, Klosterberg 23, CH-4010 Basel, Switzerland, 1996, pp. 270, SFr. 98.

The subject matter of this text is dynamical systems defined by a system of nonlinear ordinary differential equations in Euclidean spaces, their properties and their controls. Let us start with a system {EMBED Equation} where $x = x(t)$ is the variable characterizing the state of the model under study. t is, of course, the time variable and the dot denotes the derivative w.r.t time. We are aware of the rich, bewildering diversity in the asymptotic behaviour of such nonlinear systems such as instability, bifurcation, chaos, strange attractors etc. There is a huge literature on these topics. Viewing the above system as the state equation, the next practical step is to consider the following system with control input.

{EMBED Equation} $= f(x, u, t)$ (1)

where $u = u(y, t)$ is the variable representing the control exerted on the system. Note that u utilizes only $y = y(x, t)$ (referred to as measurement variable) which may be a part of the full state variable x. Since u depends on y (hence on the solution x), it is known as partial feedback control. The idea of introducing u is to control the (asymptotic) behaviour of the trajectories $x(t)$ as $t \to \infty$. For instance, can one choose u such that trajectories are asymptotically stable? (More generally, we would like to design controls such that the dynamical system has an attractor which is prescribed apriori). On this problem, see, for instance, the paper E.D. SONTAG, A Lyapunov - like characterization of asymptotic controllability, SIAM J. Contr. Optimiz., 21 (1983), pp. 462–471.

In the book under review, the systems considered are of the following form:

{EMBED Equation} $= f(x, u, w, t)$ (2)

Here, we see the introduction of a new variable $w = w(x, u, t)$ which represents disturbances and uncertainties in the model. No model is perfect and there are always uncertainties. We wish to design the system in such a way its essential functions are maintained even in adverse conditions in which the model no longer accurately reflects reality. Such a task motivates the following question: does there exist a control u such that all solutions x of (2) are asymptotically stable for all admissible disturbances w?

What are the necessary and sufficient conditions for an affirmative answer? If the answer is yes, how one goes about actually constructing such a control? This is a particular case of the class of questions treated by the authors of this volume. Controls of the above type are called robust controls for they provide robustness in the above sense. This explains partly the title of the book. Now, we will try to explain the remaining part of the title. Since nonlinear models reflect reality better, the system(2) has to be nonlinear. If it is linear w.r.t. x, then the uncertainties have to incorporate nonlinear effects. This kind of approach ignores the good effects of non-linearities present and leads to excessive control effort. This effort can be greatly reduced if the base model is nonlinear. However, there are well-known difficulties to deal with non-linearities. For instance frequency - space and Fourier transform techniques which are so dominant in linear theory are not easily generalize to the nonlinear case. On the other hand, techniques based on Lyapunov functions are available; indeed, this seems to be the only route (general enough) of attack of the stability questions of nonlinear systems.

To implement this circle of ideas, the authors introduce, what they call, robust control Lyapunov functional (rclf), for the system (2). Just as the existence of control Lyapunov functionals (clf) is equivalent to the nonlinear stabilizability of systems (1) without uncertainties, the existence of rclf is equivalent to the robust stabilizability of systems with uncertainties (see chapter 3). The task of constructing an rclf there by becomes a crucial step and this is undertaken in Chapter 5-8 of the book in a systematic fashion. In Chapter 4, the authors assume that such an rclf has been found and construct a robust controller for (2).

In carrying out these tasks, the authors extend certain existing novel ideas to the present case. For instance, let me cite the idea exploited in Chapter 4. To avoid excessive efforts, one has to choose controls judiciously; for example as an optional control of a suitably chosen cost functional. How to choose it? Given a cost functional J, one can form the associated Hamilton-Jacobi-Isaacs equation (HJI) for the value function V(x). The new idea is to take V(x) to be the rclf and consider the inverse problem of determining J. It is well-known that there are no practical ways to solve the (HJI) equation in higher dimensions. Hence it is not clear how to go about solving this inverse problem. It comes as a surprise to see that the authors are able to calculate the optional control directly from rclf without recourse to the HJI equation. Before developing the theory, the authors illustrate the above procedure in an illuminating example.

Let me briefly mention yet another place (Chapter 5) where we see once again exploitation of simple novel ideas. An clf V need not be in general an rclf. However, the requirement that a clf V be an rclf for (2) imposes severe restrictions on the structure of (2). These, known as matching conditions, had been a stumbling block for many years. The idea here is to turn the tables around and view the matching condition as a guide for choosing rclf V rather than a constraint on (2). Thus rclf V has to satisfy additional properties apart from the usual ones. How to achieve this for a system of the form (2)? The idea followed in the book is to consider only one state variable say x_1 and consider the rest as control variables and proceed recursively. Surprisingly, this latter problem can be solved in a number of interesting cases and the solution leads to a form of rclf for the original system (2). Above idea is proved to be successful in systems which are in strict feedback form (or) lower triangular form. The net progress is that such a structural hypothesis on the system is much weaker than the classical matching conditions.

Having described the main contents of the book. let us briefly discuss its structure and usefulness. I find it very well written. The authors illustrate new ideas in some simple motivating examples before proceeding to develop a theory. Introductions at the beginning of each chapter and summaries at the end make a pleasant reading. This method of writing updates even a lazy and forgetful reader on the progress made and takes him along till the end. Even though there are many books dealing with linear systems, this is the first book on nonlinear systems with uncertainities. In the course of the book the authors clearly highlight their contributions vis-a-vis existing literature: Questions to be tackled in future include the analysis of distributed systems with uncertainities. In this context, let me cite

J. L. Lions, Controlabilité exacte, perturbations estabilization de systèms distributés, Vol 1, 2, Masson, Paris (1988).

V. Komornik, Exact controllability and Stabilization, The Multiplier Method, Masson - J. Wiley, Paris (1994).

Finally, I am of the opinion that this is an advanced text suitable for readers with some maturity and understanding of the field. The well-compiled bibliography at the end of the book will, no doubt, be useful to a wide audience.

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Yakov Ilich Frenkel - His work, life and letters, by Victor Ya-Frenkel, Birkhauser Verlag Basel - Boston, translated by Alexander S. Silbergleit, 1996, pp. 332, SFr. 168.

Yakov Ilich (J. I.) Frenkel was one of the leading theoretical physicists of the erstwhile Soviet Union during the second quarter of the present century. Along with his teacher Abram Fyodorovich Ioffe he helped establish physics in the modern sense in the far corners of his country. In turn he was senior and mentor to a host of talented Soviet physicists, the best known among them being Vladimir Fock, Lev Davidovich Landau and Igor Tamm. Pyotr Kapitza was a contemporary of J. I.

This book, written by J. I.'s son Victor Yakov, is an account of J.I.'s life, work and personality, seen through his many letters to parents and wife, and from personal family reminiscences. It provides a fascinating glimpse into life and science in the turbulent times when the USSR was born and established itself, and brings out the crucial points of contact with the more advanced schools of physics in the West, and what they mean for Soviet physics.

J. I. was born in 1894 in Rostov-on-Don to Jewish parents of modest means. (Parenthetically one must say that to most of us names like Rostov-on-Don, Azov, Kazan, Simferopol and Sevastopol, Yalta and the Crimea do conjure up visions of romantic and strange lands and cultures, some with an oriental tinge!) Those were Tsarist times, and J.I.'s father, a member of a revolutionary group, had been twice imprisoned. To escape from an anti-Jewish wave in the early 1900's the entire family moved from Kazan to Switzerland for refuge for a while. From a very young age J. I. displayed high talent in music and painting; by his mid teens his flair for mathematics and physics - thanks to self-study and good books - began to show. By 1909 the family was in St. Petersburg (or Petrograd or Leningard!), and J. I. entrolled at the Gymnasium there. At eighteen he had composed a memoir of over a hundred pages on terrestrial magnetism and atmospheric electricity. Soon after completing his course at the Gymnasium, J. I. travelled to the U.S.A. to examine the prospects of further education at a University in that country. However, as he was admitted to the University at St. Petersburg he came back and began his studies there.

The teachers in physics were excellent, though a research atmosphere was lacking. In mathematics, on the other hand, stalwarts like Markov, Steklov and Smirnov were around. J. I. and Kapitza were both members of Ioffe's seminar - which turned out to be the cradle of Russian physics. J. I.'s thesis was on an application of the Rutherford atom model to study the surface electrical properties of solids and liquids. Following this period with Ioffe, J. I. (and his brothers) spend several years in the Crimea, including the period of the 1917 October revolution. These were really hard times - very unlike what any contemporary physicist is likely to have ever witnessed. While J.I. was elated by the Revolution, politics itself seems to have been rather distant from his thoughts. After a few years of teaching at a fledgling University in Simferopol, and following his marriage to Sarra Isaakovna in December 1920, J. I. came back to St. Petersburg to rejoin the department under Ioffe.

Victor Frenkel gives detailed accounts of political events and episodes to create a vivid picture of those times, and quotes from many then secret and confidential documents. One sees the arm of the police state intruding into private and family affairs all the time, and one realises the many difficulties involved in the setting up of new scientific institutions in a young state. All decisions depended on the state which however, after the Revolution gave very high priority to science.

J. I.'s key ideas and style in physics were also being formed around this time - surface phenomena in solids and liquids, vapour-solid collisions at surfaces, theory of adsorption, dissociation and diffusion of ions in crystals, ideas linking the solid and liquid states of matter. All these were developed in a direct and physical manner, staying close always to experiment. J. I. also felt a deep obligation to teach and teach well, and to write advanced level text books in all the major areas of physics to meet the needs of the times. Indeed he became the first physicist to compose a comprehensive course on theoretical physics in the Soviet Union. Many of his specialised books were first in their genre, and ultimately he ended up writing as many as twenty five monographs spanning the whole range of physics at an advanced level. Around this time, J. I. came into close contact with the legendary Paul Ehrenfest. It was the latter who took J. I. under his wing and helped him get a Rockefeller Scholarship to spend a year in Europe, on condition he would then go back home!

The year from November 1925 to November 1926 spend by J. I. in Europe proved crucial to his development, and established his reputation on the world scene. He spend time in Germany, France and England, and made a very good impression everywhere. Close contacts with Einstein, Pauli, Born, Dirac and Langevin were forged. One learns that in those times state support for science was quite weak in both France and Italy. J. I. spend extended periods with Pauli at Hamburg and Born at Gottingen. The former became a lonely time, while the latter turned out more enjoyable in every way. Often we read of J. I. having to cut himself off socially and be alone due to difficult finances, non arrival of scholarship amounts and the like.

The events of this year are conveyed largely through his letters to his parents and to Sarra. He set himself a punishing pace and was extremely productive. This was the period of the blossoming of quantum and wave mechanics. It also essentially coincided with the period that S. N. Bose from India spend in Europe; but it is evident that the European community really took in J. I., and he got far more out of these opportunities than Bose ever did. J. I.'s well known work on the classical theory of electron spin, created with critical inputs from Pauli, belongs to this period. He also completed his landmark monograph on classical electrodynamics. By this time, J. I. was acknowledged as the leading theoretical physicist of the U. S. S. R.

Returning to St. Petersburg, J. I. built up a famous group at the Physico-Technical Institute (the PTI) in association with Ioffe. Invitations to the prestigious Como Conference in 1927, an address there, and contacts with giants like Sommerfeld followed. By temperament, J. I. was always willing to help his students, and even take the not-so-gifted under his wing. In this respect he was very different from the much more stern and demanding Tamm and landau. Among his physics accomplishments in this phase, we recall the first ever text book treatment of wave guides, one of the earliest texts worldwide on wave mechanics, the wave equation for spin one particles well before Proca, and new ideas on para and ferro magnetism. He continued his work on the electron theory of metals, and a multitude of applications of the then young quantum mechanics. His arguments were always simple and direct, showing a fine intuitive understanding of phenomena, and his many derivations became standard lore. Among other things he worked hard to propagate quantum mechanics among chemists.

As crucial for J. I. as the year spent in Europe was a year spent in the U. S. A. starting September 1930. He was invited to teach a course on wave mechanics at the University of Minnesota in Minneapolis, which he did outstandingly well. All the events and impressions of this year are described through his many letters to the family. Along with his course, he kept working on his books, on attending and speaking at numerous meetings of the American Physical Society, giving talks to public audiences about the USSR, and making professional visits to a host of universities including Harvard, Columbia, Princeton, Chicago, Yale, Michigan and Purdue. In his letters we see how much sympathy there was in the USA at that time for "the Russian experiment", and we read of the great contrasts in U. S. society that struck J. I. very sharply. He also remarks on how difficult it was (and is!) for women to make careers for themselves in the U. S. The role and importance of music and painting, the need for interests beyond science to make a full life, find eloquent expression in his letters. The year was a triumph, and his standing was widely acknowledged.

Back in the USSR, J. I.'s interests and activities continued to flourish in high gear. He did pioneering work on a microscopic theory of plastic deformations of solids, electrical properties of dielectrics and semiconductors, orientational lattice melting, liquid crystals and polymer physics. In the 1940's he completed his master work on a kinetic theory of liquids. J. I. also ventured into the then new field of nuclear physics, and his model for fission was even ahead of the better known Bohr-Wheeler theory. As mentioned earlier, out of a sense of duty he completed the first ever course on theoretical physics written in the USSR, and helped launch Landau, Fock, Tamm, Pomeranchuk and Berestetsky on their careers. The PTI became a kind of mother institution in the Soviet Union. Many world leaders of science including Bohr and Dirac were close to J. I. and deeply sympathetic to Soviet science.

In the last phase of his life, however, J. I. ran into political problems. Coinciding with the McCarthy era in the U.S.A., J. I. was accused of violating the tenets of dialectical materialism, and of having employed too anthropomorphic images and analogies in his expositions of wave mechanics. Here we recall Fock's appeal to Bohr to express himself more cautiously on the interpretation of quantum mechanics, to avoid hardships to his followers in the Soviet Union. Few of us have had to live through experiences comparable to these. By this time his health began to give way and he died in early 1952, having reached the position of "top theoretical physicist of the USSR" in the words of the U. S. media.

J. I.'s life was extraordinarily productive, and was permeated with the constant desire to help others. In his physics he was quick and intuitive, preferring simple analogies and pictures to formal mathematics. He published his ideas, even preliminary ones, very rapidly to inspire others to follow him. He rubbed shoulders with the likes to Einstein, Ehrenfest, Pauli, Born, Dirac and Langevin, and his achievement was phenomenal. He had an encyclopaedic knowledge of physics and at the same time music and painting were vital to his life. He set himself a gruelling pace, and in his personal affairs he was simple and direct, even naive and gullible.

This biography by his son brings out J. I.'s personality very well. The final chapter written from within the family circle is really well done. Sometimes the fact that this translation from Russian into English is by a Russian is noticeable, and sometimes one wishes the historic photographs had been less restored and touched up by the artist's pencil. These are minor blemishes in an otherwise absorbing and rewarding account of a tremendously full and inspiring life.

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Clifford algebras with numeric and symbolic computations, by R. Ablamowicz, P. Lounesto, J. M., Parra, Birkhauser, AG, Averlag, Klesterberg. 23, CH-4010, Basel, Switzerland, 1996, pp. 332, SFr. 98.

Works of Hamilton, Grossmann and Clifford saw fruitation in that of Pauli and Dirac and have had a wide sway in, e.g. advances of Witten-Donaldson—Atiyah, Penrose, and in quantum fields. The present book under review is part of spurt in cross-subject books and conference proceedings around the world with added dimension of computer help; e.g. 'Clifford Algebra and Spinor-valued Functions' (Kluwer, 1992) by Delange et al. even encloses a software packet. At least one recent book - 'Spinors and Calibrations' (Academic, 1990) by H. F. Rees (Math. Revs. 91e, 53056), finds no mention in these twenty articles, of which fourteen involve computer use. My own interest, via Uncertainty Principle, was to discover Shift (Phase) Operator as infinite dimensional limit of a Circulant matrix - at the base of order two Generalized Clifford Algebra (GCA) forming a nonsingular matrix basis. At MATSCIENCE in late sixties Alladi Ramakrishnan and students were deep in Dirac matrices, Gell-mann -Nishijima relations and generalisations identical to GCA's - reflected in the diverse works of Morinaga-Nono, Yamazaki, and Morris to which I drew their attention. There followed the first International Conference of Ooty, 1971. Though my own review was late, in obscure Proc. T. N. Science Academy (Vol. 2/2 (1979) 107–132), the ideas gained fair currency through Shanthanam and collaborators from MATSCIENCE. Here articles 6 and 15 (numbers refer to order of occurrence of articles) refer to Ramakrishnan and group; first gives resume of early GCA literature with application to SU(2), and the second lists other applications. Article five stresses Octohedral group in complex Pauli algebra with CLICAL programme to obtain Gell-mann-

Nishijima relations with plenty of mystery and galimatias; reference to Ramakrishnan was deserved. The first article by Lounesto reports a search, using CLICAL programme, of counter examples, in Clifford Algebra (CA), to the works of living mathematicians to afford public debate, p. 20 mentions for $n > 3$ that conformal mappings are just restrictions of $x \to (ax + b)/(1 + c \cdot x)$. The latter generates $SL(n + 1; R)$ and shares with conformal maps only the subgroup of Homotheties (=Similitudes). Indeed on $R^{p,q}$, homotheties do not generate an $SO(p + 1, q + 1)$, when extended by $x \to x/(1 + c \cdot x)$; but the latter (or pure affines) extend the conformal algebra to infinite dimensional one of quasi-conformal maps! What involves here (and Knop's article p. 156) is to identify a vector in $R^{p+1, q+1}$, with a matrix {EMBED Equation} of invariant pseudo-determinant $-x$ $u^2 + v^2$. As nonsingular basis of a matrix algebra CA's, modulo p, q relations, do give $SO(p + 1, q + 1)$ representations, but precise identification with non-linear conformal action warrants care. Here follows summary of the rest of the grouped articles.

2-Use of REDUCE for CA in euclidean/Lorentz spaces, and forms in gravity/gauge theory.

3-MATHEMATICA software for Clifford calculus- Nabla, Forms.

4-MAPLE V to Pauli algebra applications.

18-Matrix and real CA isomorphisms explored by enumerating CA orthonormal basis sets for n = 4 with FORTRAN.

19-Visual geometry of complex plane is extended to higher dimensions with CA and C_{++} language.

20-Focuses library of data types and algorithms for numerical Clifford computations employing C_{++} programme.

7-discusses Clifford algebraic vector continued fractions/Pade approximants.

8-illustrates MAPLE package LUCY for Clifford Spinor Calculus.

9-illustrates MPALE for Pauli spin algebra in central force problems.

11-illustrates MAPLE V package CLIFFORD to discuss bilinear forms with antisymmetric part.

12-Hyperbolic version of complex plane using unipotent element $u^2 = 1$, $u \neq \pm 1$, is illustrated with a cubic and MATHEMATICA package.

13-Classifies Octonian multiplication to select an X-product, discovered 1993, and applied to String theory!

14-Complex variable concept extended formally (using 2 i's and 2×2 complex matrices) to four dimensions; no applications are indicated.

16. Follows Hestenes to develop projective geometry with examples of CLICAL use.

17. Explores applications to quasi-crystals in projective space using MATHEMATICA.

In computer methods, first six stress elements and development of techniques, and the rest use it as a partial tools. A thought: it is a wonder of the free market that despite effectively dwindling library funds with conflicting demands of books, journals and now conference proceedings the publication explosion is unabated.

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Solar Energy: Principles of Thermal Collection and Storage, by S. P. Sukhatme, Tata-McGraw Hill, Publishing Company Ltd., 4/21, Asaf Ali Road, New Delhi 110 001, 1996, pp. 426, Rs. 120/-

The first edition of this book appeared in 1984 and was reviewed in this journal. The second edition of this book is much larger than the first and contains two new chapters. After the spurt in oil prices in 1973, there has been a renewed interest in the use of renewable energy. Most colleges around the world started courses in Solar Energy. Hence a number of books appeared in the late 1970's and early 1980's. None of these books were useful to teachers and students in India because these books did not discuss the solar energy availability in India or the use of solar energy devices in India. The book on Solar Energy by S. P. Sukhatme appeared in 1984 and filled the void. The book was suitable for an elective course in Solar Energy at the undergraduate or postgraduate level. The second edition of the book contains two new chapters which deals with economic analysis and other methods of utilization of solar energy.

The first chapter of the book provides an overview of the various sources of energy in India and the world. This chapter contains useful data on the energy production in India during the past 40 years. In the second chapter there is a detailed enumeration of the various methods of collection and storage of solar energy for thermal applications. In chapter 3, the nature of solar radiation and its availability is discussed. The average solar radiation available in different parts of India in different seasons has been presented. Empirical methods for the estimation of solar energy in different cities in India has been provided. In Chapter 4, the use of flat plate solar collectors with liquid as the heat transfer medium is discussed. The analysis of the steady-state and transient performance of flat plate solar collectors is elaborated in great detail. In Chapter 5, the analysis of solar air heaters is presented. Chapter 6 deals with concentrating collectors. The analysis of the performance of parabolic trough concentrators, compound parabolic collectors, and central receiver systems has been presented.

Solar energy is unavailable at nights and on cloudy days. Hence the storage of solar energy in some form becomes essential. The topic of thermal energy is covered in Chapter 7. The use of sensible, latent and chemical energy storage is highlighted. Chapter 8 deals with Solar ponds. Solar ponds collect and store solar energy. This chapter is much larger in this edition than in the earlier edition. This is because major advances in Solar Pond technology occurred in the last ten years. This chapter contains a lot of useful information on Solar Ponds in India. In Chapter 9, the economic analysis of solar energy systems is discussed. In chapter 10, the use of solar energy converters such as photovoltaic cells, wind energy, biomass, biogas, and ocean thermal energy conversion is discussed. Appendix 3 contains solar radiation data for different cities in India and Appendix 5 provides wind data for some cities in India. The book contains a large number of numerical examples and many problems at the end of each chapter. This book will therefore be ideal as a text book for an elective course in Solar energy. This book will also be found to be useful by engineers in the industry who want information on alternative energy technologies

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Analytic Number Theory – Volumes I & II, by Bruce C. Berndt, *et al.*, Birkhauser Verlag AG, Klosterberg 23, CH-4010, Basel, Switzerland, 1996, pp. 449, SFr. 128.

The two volume work on "Analytic Number Theory" is a collection of 50 research papers (24 in Volume I and 26 in Volume II) by eminent number theorists all over the world brought out as a proceedings in honour of Professor Heini Halberstam, University of Illinois at Urbana – Champaign, USA, who has served the mathematics community as a dedicated researcher and teacher of Number theory for over four decades. In short, a Festschrift volume dedicated to Professor Heini Halberstam, held during May 16–20, 1995.

The papers cover a wide range of distinct topics mainly in analytical number theory and provide a glimpse into the modern viewpoints, the several themes and other approaches in this classical topic. Papers included are of high quality and the reading is a challenge to any budding mathematician intending to digest and assimilate the factuals.

The book is strongly recommended for every institutional library. The editors, Professors B. C. Berndt, H. G. Diamond and A. J. Hildebrand have to be wholeheartedly commended for their excellent job of bringing out the proceedings and full credit should also be paid to Birkhauser for publishing the volumes in its Progress in Mathematics Series (138 & 139).

A short summary of the fifty papers appearing in alphabetical order is given below:

Volume I

1. K. Alladi – *Weighted partition identities and applications*, pp. 1–15.
   In this paper, new identities for the partition functions of Euler, Gauss, Rogers-Ramanujan are obtained a new interpretation for Jacobi's triple identity is given.
2. G. E. Andrews – *Rogers-Ramanujan polynomials for modulus 6*, pp. 17–30.
   In this paper, polynomial generating functions related to partitions of n into parts $\not\equiv 0, \pm$ (mod 6) are employed to obtain interesting representations involving Gaussian polynomials and q-trinomial coefficients.
3. J. Bae – *On subset-sum-distinct sequences*, pp. 31–37.
   The author shows that $2 \leq \lambda \leq 3.6906742...$, where λ is the supremum of all s that satisfy $\sum_1^{m_s} a_i \geq \left(1-2^{ms}\right)/\left(1-2^s\right)$, for all integers $m \geq 1$, wherein $\{a_n\}_1^\infty$ satisfies: $a_1 + ... + a_n \geq 2^n - 1$.
4. R. C. Baker & G. Harman – *The Brun-Titchmarsh theorem on average*, pp. 39–103.
   In this paper, bounds for the number of primes satisfying a certain linear congruence are obtained in a precise manner. As a consequence it is found that, for an arbitrary a, the greatest prime factor of $p + a$ exceeds $p^{0.676}$ for infinitely many primes p, improving upon the work of several mathematicians.
5. A. Balog, H. Darmon & K. Ono – *Congruences for Fourier coefficients*, pp. 105–128.
   Sufficient conditions for the existence of Fourier coefficients of a half integer weight modular form divisible by a given integer almost always are obtained and illustrated with nice examples explaining the congruences through Galois representation of odd rank elliptic curves.
6. P. T. Bateman – *The asymptotic formula for the number of representations of an integer as a sum of five squares*, pp. 129–139.
   It is interestingly shown that the formula in the title could be derived using Jacobi's formula for the number of representations of an integer as a sum of four squares.
7. P. Bleher & J. Bourgain – *Distribution of the error term for the number of lattice points inside a shifted ball*, pp. 141–153.
   This paper deals with the distribution of the normalized difference between the number of lattice points and the volume of a ball, as a function of its radius w.r.t. its centre
8. N. Boston – *A probabilistic generalization of the Riemann zeta function*, pp. 155–162.
   This is a nicely written article, containing expressions for the probability function defined via Haar measure over certain topological groups.

9. D. Bowman – *A general Heine transformation and symmetric polynomials of Rogers*, pp. 163–171.
A generalization of Heine transformation in multivariables is obtained in such a way that it yields the discrete analogues of basic hypergeometric functions expressed in terms of Rogers symmetric polynomials.

10. D. Bradley – *A sieve auxiliary function*, pp. 173–210.
This paper deals with a new sieve function which helps in the determination of the sieving limit of the combinatorial sieve and includes a table of values with high accuracy.

11. T. Cochrane – *Bounds on complete exponential sums*, pp. 211–224.
Results on complete exponential sums involving polynomials in several variables over finite fields are obtained, with emphasis on the bounds for weights of characteristic values.

12. J. B. Conrey – *A note on the fourth power moment of the Riemann zeta-function,* pp. 225–230.
Explicit formulae for the terms in the asymptotic expansion of the mean fourth power of the Riemann zeta-function on the critical line are obtained.

13. H. Daboussi – *Effective estimates of exponential sums over primes*, pp. 231–244.
The problem in the title is nicely analysed leading to results concerning asymptotic behaviour of certain exponential sums and the associated bounds.

14. H. Delange – *On products of multiplicative functions of absolute value at most 1 which are composed with linear functions*, pp. 245–263.
An asymptotic formula with a small error is obtained for products involving multiplicative functions of absolute value at most unity.

15. H. G. Diamond, H. Halberstam & H. –E. Richert – *Combinatorial sieves of dimension exceeding one II,* pp. 265–308.
This classic paper presents a full solution for all the dimensional values of constructing sieves for a boundary value problem involving a pair of simultaneous linear differential delay equations, thereby completing the earlier article of the authors, viz, *J. Number theory*, **28** (1988), 306–346.

16. D. Eichhorn & K. Ono – *Congruences of partition functions*, pp. 309–321.
Several interesting congruence properties are obtained in this nice article dealing with the partition function of Ramanujan and a generalized version for the number of partitions into a distinct number of colours.

17. P.D.T.A. Elliott – *Fractional power large sieves*, pp. 323–332.
This paper deals with certain inequalities concerning large sieves in which powers of 2 may be replaced by α, such that: $1 < \alpha < 2$.

18. P. Erdos – *Some problems I presented or planned to present in my short talk,* pp. 333–335.
This talk contains a large number of unsolved problems in number theory and combinatorics – No doubt, by one of the greatest number theorists of this century who passed away on 20–09–1996.

19. P. Erdos, S. W. Graham, A. Ivic & C. Pomerance – *On the number of divisors of n!*, pp. 337–335.
This paper includes several results on the number of divisors of n!, various estimates and differences concerning highly composite numbers.

20. R. Evans – *Generalized Lambert series*, pp. 357–370.

This paper contains results of certain sums expressed in terms of Eisenstein series together with modular properties and contains systematic proofs of q-series identities of Ramanujan.

21. M. Filaseta – *A generalization of an irreducibility theorem of I. Schur*, pp. 371–396.
This paper contains several results on the irreducibility of polynomials over rationals, including extensions of the theorem of Schur.

22. M. E. Flahive & A. C. Woods – *Small values of indefinite binary quadratic forms*, pp. 397–409.
The notion of convergents to a full lattice in two dimensions is developed in this paper and numerically small values are investigated in the case of indefinite binary quadratic forms.

23. Friedlander & H. Iwaniec – *Bombieri's sieve*, pp. 411–430.
This article contains numerous results, estimates and applications associated with Bombieri's sieve

24. S. W. Graham – *B_h sequences*, pp. 431–463.
This paper deals with upper bounds of certain sequences of finite sums of positive integers which are required to be distinct.

Volume II

25. D. R. Heath-Brown – *An estimate for Heilbronn's exponential sum*, pp. 451–463.
In this paper, various results concerning Heilbronn's exponential sum are obtained. It is observed that the numbers n^p are uniformly distributed modulo p^2, for a large prime p.

26. M. Helm – *On B_3-sequence*, pp. 465–469.
A conjecture of Erdos in the even case concerning B_3-sequence is proved in this paper (see also S. W. Graham, pp. 431–449; article no. 24 above).

27. C. Hooley – *On an elementary inequality in the theory of Diophantine approximation*, pp. 471–486.
The following is one of the interesting results proved in this paper: "There exists infinitely many coprime pairs of integers r and m, for a given irrational number θ and $\in > 0$, such that $\log_n |m^k\theta\text{-r}| < \in - 1/(2^k - 1)$".

28. M. N. Huxley – *The integer points close to a curve II*, pp. 487–516.
In this article, the author proves several interesting results on the existence of integer values close to real functions satisfying various constraints. This paper is a continuation of his earlier article on the same topic, viz, *Mathematika*, **36**, (1989), 198–215.

29. M. Jutila – *The fourth moment of Riemann's zeta function and the additive divisor problem*, pp. 517–536.
In a native sense, the fourth moment of zeta-function is a mean value problem for exponential sums involving the divisor function. This paper explores the connections and includes results in a generalized setting of the theory of L-functions attached to Mass wave forms.

30. N. M. Katz & Z. Zheng – *On the uniform distribution of Gauss sums and Jacobi sums*, pp. 537–558.
Certain equidistribution estimates with relatively smaller error terms for Gauss and Jacobi sums involving additive and multiplicative characters over finite fields are obtained in this paper, together with moment estimates.

31. M. N. Kolountzakis – *A problem of Steinhaus: Can all placements of a planar set contain exactly one lattice point?*, pp. 559–565.

The author obtains a measurable set in the plane which is very small outside large strips parallel to a fixed direction and which is found to violate the following property of Steinhaus: "A subset of the plane, no matter how translated and rotated, contains a unique point with integer coordinates".

32. X. -J. Li – *A note on the Riemann-Roch theorem for function fields*, pp. 567–570.
The author provides a proof of Riemann-Roch theorem for function fields using Fourier transforms over adele rings and the adelic Poisson summation formula.

33. J. Liu & T. Zhan – *Estimation of exponential sums over primes in short intervals II*, pp. 571–606.
This is a nicely written article containing numerous results on exponential sums over primes in short intervals, forming a continuation of an earlier article by the authors on the same topic (yet to be published).

34. L. Lucht & K. Reifenrath – *Weighted Wiener-Levy theorems*, pp. 607–619.
The authors prove general weighted Wiener-Levy theorems which lead to the existence of certain multidimensional series expansions making use of Gelfand's theory of commutative Banach algebras.

35. W. Luo – *Spectral mean-values of automorphic L-functions at special points*, pp. 621–632.
This paper confirms the validity of Lindelof conjecture for a class of automorphic L-functions associated with Maass cusp forms.

36. H. Maier – *The size of the coefficients of cyclotomic polynomials*, pp. 633–639.
Various upper bounds for the coefficients of n-th cyclotomic polynomial are incorporated in this paper.

37. Y. Motohashi – *On Kuznetsov's trace formulae*, pp. 641–667.
This is a paper dealing with connections between several trace formulae and spectral sums and Kloosterman sums written in an attractive style, obtaining a relation between Kuznetsov's sum and Fourier coefficients of cusp-forms over the full modular group.

38. M. R. Murty & C. S. Rajan – *Stronger multiplicity one theorems for forms of general type of* GL_2, pp. 669–683.
This is a very nicely written article on cuspidal automorphic representations and their associated L-series. Numerous results are obtained. One such is the explicit formula technique employed to estimate exponential sums.

39. M. Nair & A. Perelli – *A sieve fundamental lemma for polynomials in two variables*, pp. 685–702.
This paper deals with a sieve fundamental theorem of a polynomial in two variables with integer coefficients assumed to be irreducible and with no fixed prime divisors. A general form of the prime ideal theorem valid in algebraic number fields is proved in the appendix which assumes the convergence of certain infinite products.

40. C. Pomerance – *Multiplicative independence for random integers*, pp. 703–711.
An estimate is obtained for a sequence of random integers uniformly distributed to be multiplicatively dependent. It is surprising that the same estimate also yields when a subsequence has its product as a square.

41. R. A. Rankin – *On certain meromorphic modular forms*, pp. 713–721.
This paper presents a classic construction of modular forms of even weight represented by Poincare zeros of a special type. The results are highlighted by a number of illustrative examples.

42. E. Scourfield – *Comparison of two dissimilar sums involving the largest prime factor of an integer*, pp. 723–735.
This paper mainly examines conditions under which two dissimilar sums may be asymptotically equal, involving strictly increasing positive continuous functions with stipulations.

43. H. M. Stark – *On the determination of an L-function from one value*, pp. 737–743.
It is shown that two elliptic curves in Weierstrass normal form are isogenous over a number field whenever their periods are linearly dependent or are proportional.

44. K. B. Stolarsky – *An approximation to the q-analogue of n involving the n-analogue of a golden number*, pp. 745–753.
This paper contains a number of interesting results and approximations of q-analogues of integers.

45. R. C. Vaughan – *Small values of Dirichlet L-functions at 1*, pp. 755–766.
This paper deals with the behaviour of Dirichlet L-functions associated with real non-principal characters. It is shown that two specific values are always attained asymptotically quite frequently by these L-functions at 1.

46. D. Wan – *Global zeta functions over number fields and function fields*, pp. 767–775.
This paper clearly explains the complex global zeta function over a finitely generated commutative algebra with identity along with many illustrative examples.

47. S. Wong – *Consequences from the study of concentration functions on shifted twin primes*, pp. 777–801.
This paper contains a number of results from probabilistic number theory. One such interesting result is the criterion of weak convergence to a limiting distribution of a set of frequencies.

48. T. D. Wooley – *An affine slicing approach to certain paucity problems*, pp. 803–815.
This paper deals with the estimates for the number of solutions of a pair of symmetric diagonal equations, one cubic and one quadratic. An asymptotic formula for the solutions is also obtained.

49. L. -C. Zhang – *Ramanujan's class invariants, Kronecker's limit formula and modular equations (II)*, pp. 817–838.
This is a continuation of the paper of the same title ((D. C. Berndt, H. H. Chan & L. -C. Zhang - Part I - Trans. Amer. Math. Soc. (to appear)) detailing the Weber-Ramanujan class invariants which could be viewed as generators of the Hilbert class field of $Q(\sqrt{-n})$ or the ring class field of the order $Z[\sqrt{-n}\,]$, written in an attractive style.

50. W. -B. Zhang – *Probabilistic number theory in additive arithmetic semigroups 1*, pp. 839–885.
This paper provides a lucid account of probabilistic properties of additive functions over a free commutative monoid having a countable generating set of primes and admitting a degree function. Various analogues of classical results are obtained in algebraic function fields. It is found that the number of distinct irreducible polynomial divisors of polynomials of degree n are normally distributed.

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The simple science of flight: From insects to jumbo jets, by Henk Tennekes, The MIT Press, 55, Hayward Street, Cambridge, Mass 02142, USA, 1996, pp. 137, $20.00

This is a delightful title book by a well-known fluid dynamicist who taught aeronautics at Penn State, headed the Royal Dutch Meteorological Institute and has been an enthusiastic private pilot. In his introduction, he says the book is an act of revenge; when as a young faculty member he talked about sparrows and butterflies in his course on aircraft performance, the head of his department chided him, saying that "animals that flap wings are none of our business" in an engineering department. Tennekes, of course, things and knows otherwise, and effortlessly talks about birds and planes in the same breath throughout this book, to show how they can both be understood on the basis of the same few principles of flight.

He achieves his purpose through a series of what I see as back-of-the envelope calculations, based on the classical formulae of aircraft performance. It is fascinating to see how ducks and dragonflies, the Boeing 747 and the Fokker 100, and Paul MacCready's human-powered *Gossamer Condor*, all fit into the general overall scheme. Indeed the broad characteristics of the man-made fliers are often derived or designed in a few pages of arguments and calculation, giving the reader an extra-ordinary perspective on why birds and planes are the way they are. The book may in fact be seen as a sustained (and very pleasant) argument about the consequences of the simple scaling laws of aerodynamic performance.

The second great strength of the book is the author's ability to make numbers come alive; and he does this by casting and recasting them in terms that the lay reader can easily relate to. For example, the load sustained by each square meter of a Boeing wing is compared with figures for a sparrow (which are much lower) and for a woman wearing high heels (much higher), with explanations for the differences. Similarly the fuel consumption of aircraft, automobiles and humming birds. This is done so well that by the time the author reaches the end of the book, the reader is mentally prepared to consider with equanimity such questions as how the Beoing 747 would have to be redesigned if it had to run on peanut butter (costs 15c per megajoule) rather than gasoline (only 1c per megajoule), if not steak (which is a very expensive source of energy, at $5 per MJ).

The author is a great admirer of the Boeing 747, which he sees as "the commuter train of the global village". On the one hand, his Great Flight Diagram shows that the 747 is "a perfectly ordinary 'bird', with ordinary wings and a middle-of-the-road wing loading", following the same trend as the gnat and the hawk and the goose. On the other hand it is the "only one [aircraft] that obeys ruthless engineering logic" - (the others are all shown to be compromises in some way), and is "one of the great engineering wonders of the world", like the pyramids or the Eiffel Tower. Correspondingly, the supersonic Concorde gets some unfavourable attention. It is of course a commercial failure, but it is a technological success and, at least to this reviewer, a beautiful machine, out of the ordinary, and standing out so conspicuously amid all those other drab aircraft looking more or less the same in Heathrow or JFK.

At the end of a thoroughly enjoyable book, one begins to notice that something is missing. It is somehow implied in the book that the aerodynamics of birds, insects and aircraft - whether human-powered or supersonic - are all the same. This is of course not true. The new aerodynamic principles on which the Concorde was built were forcefully and eloquently described by Kuchemann; Weis-Fogh and Lighthill elucidated the totally different aerodynamics of the moth as compared to a bird. What is common to all of them is the arithmetic of performance. The book is therefore really about the fundamentals of flight performance; you will not find out *why* or *how* a wing generates lift, but you will find out why, given its lift, the body it holds up delivers a certain performance. From this point of view the word 'Science' in the title may be misleading to some.

But it is a very interesting book. It is beautifully printed (with very few misprints; fortunately the one on p.11, where the weight of a Boeing 747 is quoted as 3.4×10^{-6} Newton's, will mislead nobody). It has nice sketches of all kinds of fliers scattered throughout, the handiwork of both God and man, and is a great introduction to flight in particular and to a kind of engineering logic in general. It is strongly recommended to all people interested in aircraft or birds, lay or professional; and will be a perfect gift to you inquisitive nephew or niece.

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Partial differential equations and functional analysis (In memory of Pierre Grisvard), by J. Cea, *et al.* Birkhauser Verlag AG, Klosterberg 23, CH-4010, Basel, Switzerland, 1996, pp. 352, SFr. 128.

This is a collection of articles presented at a conference held in 1994 in memory of one of the distinguished French mathematicians P. GRISVARD who has disappeared prematurely. His specialty was Functional Analysis and its applications to Partial Differential Equations. More precisely, he has made deep and fundamental contributions in the following areas: Boundary value problems for Elliptic equations and Initial value problems for Parabolic equations in L^p spaces, Interpolation Theory and Applications to regularity of solutions, Elliptic boundary value problems in irregular domains, non-smooth data and non-smooth coefficients. He has created his own legacy in these areas. Following the tradition of Grisvard, his friends, colleagues and collaborators have presented interesting articles which are collected in this volume. They are not surveys on a single topic. They are quite diverse and advanced and meant for specialists working in the field. Apart from these articles, this volume contains one paper by Grisvard himself which is published posthumously along with a complete list of his publications. Among these, the monograph 'Elliptic problems in non-smooth domains" which appeared in Pitman Series in 1985 is well-known among mathematicians working in the general area of Partial Differential Equations.

Having said several general things about this volume, let us briefly scan its contents. The contributions from Grisvard, Apel & Nicaise, Dauge, Geymonat & Tcha-Kondor, Kozlov & Maz'ya, Zerner concern the regularities I singularities of solutions of elliptic equations in non-smooth domains with cusps, corners, edges etc. What is the impact of these singularities of the domain on the solutions? How to describe it explicity/ These are the goals of the papers mentioned above.

There are two articles on elastic shells. One of them deals with a new class of unstable problems which, when perturbed slightly, may fail to admit solutions. Such problems are referred to as "sensitive". Exact controllability is the subject of two papers included in this book. While one of them analyzes partial controllability, the other one examines the effects of singularities of the domain on the propagation of waves. The influence of boundary points on the unique continuation of harmonic functions is discussed in a paper by Baouendi & Rothschild. The articles by Da prato, Farini, Sincestran present interesting generalizations of a method of solving evolution equations originally proposed by Grisvard.

Nowadays, these is a tremendous interest in boundary value problems on non-smooth domains because of the discovery of fractal structures in applications and in nature. Grisvard's works can be considered as a first step taken in the direction of studying such structures. The articles included in this volume present interesting generalizations and update Grisvard's ideas. Thus, the book under review can prove to be a very valuable asset for scientists interested in qualitative behaviour of solutions of Partial Differential Equations arising in applications.

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Beyond the quartic equation, by R. Bruce King, Birkhauser, Boston, USA, 1996, pp. 150, SFr. 68.

The theory of equations has had a fascinating history. All the ancient civilizations had successfully solved the linear equation and they had varying degree of success with regard to the quadratic equation. The cubic and the quartic had to wait till the 16th century when Cardano and Tartaglia gave formulae for their solutions. That is, by the end of the 16th century one had formulas for the roots of linear, quadratic, cubic and quartic polynomials in terms of the coefficients of the polynomials. The formulas involved the four basic operations and extracting square and cube roots. Then, naturally four basic operations and extracting square and cube roots. Then, naturally enough, mathematicians started looking for the solution of a general quintic along the same lines. It is a fairly well known fact that in the early 19th century N. H. Abel and E. Galois independently, and by different methods proved the impossibility of finding a formula for the roots of a general quintic involving only extraction of roots (of complex numbers). In other words, the quintic cannot be solved by *radicals*. (We say that a polynomial $p(x)$ over a field F *is solvable by radicals over* F if we can find a sequence of fields $F_1 = F(\omega_1)$, $F_2 = F_1(\omega_2), \ldots,$ $F_k = F_{k-1}(\omega_k)$ with $\omega_1^{r_1} \in F, \omega_2^{r_2} \in F_1$, etc., for some integers $r_1, r_2, \ldots,$ such that the roots of $f(x)$ all lie in F_k). It is unfortunate that this beautiful theorem had a very negative pedagogical influence in that the theory of equations proper stopped at this point! No mention is made of the work of C. Jordan, L. Kiepert, F. Klein, and others in the late 19th century relating to the solution of quintics and higher degree polynomials. The book under review strives to remedy the situation by unearthing some of these gems and bringing them to light.

While, usually, the formula for the roots of polynomials of degree 2, 3, or 4 involves extraction of square and cube roots, in some cases, notably the solution of the general cubic with three real roots, it is more convenient to make use of trigonometric substitutions. The radicals $\sqrt[n]{a}$ (a, a complex number) and the trigonometric functions sin, cos are examples of *transcendental functions* that can be expressed in the general form

$$f(x) = \int \frac{dx}{\sqrt{p(x)}}$$

where $p(x)$ is a quadratic polynomial. However, logarithms and trigonometric functions are no longer sufficient for the solution of the general quintic equation. We need transcendental functions of a more general type than these defined by (*). It turns out that *elliptic functions*, which are transcendental functions that can be derived from integrals of the type (*) where $p(x)$ is of degree 3 or 4, are sufficient to describe the solutions of the general quintic. (The discovery of the elliptic functions had immediately followed the proof of insolubility of the general quintic). The initial work in this area was done by Hermite and developed by Gordon which culminated in the classic paper of L. Kiepert in 1878 describing a quintic equation algorithm in terms of elliptic functions. But this algorithm appeared intractable before the era of computing machines.

The author of the book under review, R. B. King, tried to implement Kiepert's algorithm on a computer and the results of his efforts from the major portion of the book. As the author himself says "many of the key ideas appear to have been forgotten by the subsequent generations of mathematicians during the past century so that some of the underlying mathematics has the status of a lost art"! With the appearance of this book there is now a strong case for this topic to be included in a course on elliptic functions.

The book starts off with the necessary background material from group theory and the theory of elliptic functions, and then goes on to describe the Kiepert's algorithm. The last chapter briefly describes general results on the solution of higher degree polynomials.

The book has been written in admirable style but I should confess that the layout looks a bit jarring to those used to TEX-outputs!

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