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Book Reviews

Recent developments in operator theory and its applications by I. Gohberg. *et al.*, Birkhauser Verlag AG, Klosterberg 23, CH-4010, Basel, Switzerland, 1996, pp. 448, sFr. 148.

The present volume contains the proceedings of an International Conference held in Winnipeg, Canada during 1994. Out of 64 papers which were presented at the conference, 21 articles have been chosen and published in this book. As the title indicates, these articles are devoted to various aspects of Operator Theory. Much space is devoted to the study of operators on indefinite scalar product spaces. Other main topics covered in this volume include spectral theory, system theory from engineering point of view etc. Of course, it is difficult to specify the targeted audience for such books. The best one can do is to scan through the contents of some of the articles published here. Though it is not possible to go into the details, we can definitely indicate the kind of questions tackled by various contributors. This will, hopefully, convey to the reader an idea of the contents of the book.

First few presentations deal with topics in Operator Theory à la Gohberg. The first one, for instance, considers the inverse problem of obtaining the coefficients of an operator from certain functions of solutions associated with the operator. This is done for a special class of first order systems of linear ordinary differential equations. Associated with scattering solutions is reflection coefficient function. Formulas are obtained to determine the coefficients of the operator in terms of the reflection coefficient function.

Linear operators and their structures on a Hilbert space is a classical topic. An important property used quite often is the positive definiteness of the inner product. What happens when we drop this assumption? One typical way to generate such scalar products is to start with a Hilbert space and a self-adjoint, invertible operator acting on it. The bilinear form associated with this operator defines a scalar product which need not be definite. In a more practical situation, IR^4 equipped with Minkowski metric provides an important example of indefinite inner product space. Though there are some books devoted to the study of such spaces and operators acting on them, it is by no means complete. It is an active area of current research.

For instance, one article here considers polar decomposition of a matrix X in space of the type described above. This means that X is factorized as UA with U unitary and A self-adjoint with additional restrictions. The article in question examines this issue under various hypotheses on X.

A couple of subsequent papers deal with eigenvalue problems of elliptic operators involving some weight functions taking both positive and negative values. A prototype example is the following:

$$\Delta u = \lambda\,(sgn\ x_n)u \text{ in } \mathrm{IR}^n.$$

We see the need to work in indefinite inner product spaces. It is a theorem that the operator $(sgn\ x_n)\Delta$ is similar to a self-adjoint operator in the Hilbert space $L^2(\mathrm{IR}^n)$. Thus the study of the spectrum is reduced to classical cases.

There are couple of other articles devoted to the study of operators on spaces with indefinite scalar product. One studies the effect on the spectrum when the operator under consideration as well as the scalar product structure are both perturbed. There perturbations may be small or compact. Another work is concerned with minimization of quadratic forms where the matrices involved need not be positive definite. This is one of the fundamental problems frequently encountered in Mechanics, Calculus of Variations etc. In such situations, solutions are not always guaranteed to exist. The idea of the paper by SAYED, HASSIBI & KAILATH is to link the above problem with an equivalent estimation problem for which Kalman-type algorithms apply and yield information on the original problem.

There is one article studying spectral properties of Schrodinger type operators containing complex potentials. Real potentials which occur in many physical models have been treated in many classical texts. Notwithstanding the relevance of the model considered in this paper, the ideas advanced here should prove useful in treating various non-selfadjoint models.

Next appears a somewhat unusual paper. It suggests a novel algebraic approach to multi-parameter spectral problems by introducing what are called coalgebraic structures. According to the authors of the paper, their technique yields new results in spectral theory.

One can find some papers on systems theory and control. One of them is devoted to completely symmetric systems. Another one deals with instabilities in feed-back systems.

Spline approximation of Wiener-Hopf equation is the subject matter of one contribution. There is another paper which too has numerical flavour and that concerns the evaluation of line integrals over smooth curves for which an explicit parametrization is not known. This paper concentrates on the computation of certain singular integrals.

Finally, let us finish our description of the contents by mentioning the contribution from SHKALIKOV. His article deals with some physically interesting spectral problems where eigenvalue parameter appears quadratically. Eigenvalues in the right half plane determine the instability of the system. A rather elegant formula counting the number of such eigenvalues is given in the article.

General feeling one gets after going through this volume is that articles are well written giving necessary background and motivation. They also provide a well-compiled bibliography. However, Operatory Theory being vast discipline, the topics covered are wide spread and there is no focus in any particular direction. This is the defect of many conference proceedings . Since the level of the presentations is quite high, this book could be accessible to the specialists interested in the kind of questions described above.

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Resonance of Ramanujan's Mathematics, Volumes I and II by R. P. Agarwal, Birkhauser Verlag AG, Klosterberg 23, CH-4010 Basel, Switzerland, 1996, pp. 259, Rs. 450.

I begin with some general remarks. SRINIVASA RAMANUJAN'S letters from Madras Port Trust to G. H. HARDY constitute a golden epoch in the history of world mathematics. Under the leadership of G. H. HARDY a number of mathematicians of UK (and later of other parts of the world) who belonged to the Hardy school brought out the best of Ramanujan's results to the limelight of the day. G. H. Hardy in collaboration with Ramanujan developed ideas of circle method which I believe were very very fruitful in solving many serious problems of number theory (like Goldbach Problem and Waring's Problem). One of the things necessary for the additive theory of quadratic forms was the theory of modular functions, which forms part of the rich theory of q-series (Lambert series and basic hypergeometric series) and hypergeometric series and so on developed in his own way by Ramanujan. R. P. Agarwal, a very distinguished student of W. N. Bailey (a great exponent on these topics who belonged to the Hardy school), has set out in these two volumes to expound the tremendous impact of Ramanujan's work in this field. The reader will benefit from the rich research experiences of R. P. Agarwal who is one of the best experts in these topics to-day. (One side remark: Sri W. NARAYANAN is the adopted son of Smt. RAMANUJAN and not her step son). Both these volumes consist of five chapters with many parts in each chapter beautifully arranged. One of these chapters (in volume I) deals with the work of Ramanujan on definite integrals.

To give an idea of the volumes to a general reader it is necessary to introduce some technical definitions.

Let

$$(a)_0 = 1, (a)_k = a(a+1)\dots(a+k-1) = \frac{\Gamma(a+k)}{\Gamma(a)} k > 0.$$

The generalized hypergeometric series is defined by

$$pF_q\begin{bmatrix} a_1,\dots,a_p;z \\ b_1,\dots,b_q \end{bmatrix} = pF_q[(a_p);(b_q);z] = \sum_{r=0}^{\infty}\frac{(a_1)_r\cdots(a_p)_r z^r}{r!(b_1)_r\cdots(b_q)_r}.$$

where the a's and b's and z are in general complex numbers. The series converges for all z when $p \le q$, at least when none of the denominator parameters are zero or a negative integer. It converges for $|z| < 1$ when $p = q + 1$ and it converges for $z = 0$ only if $p > q + 1$. At $z = 1$ the $p + 1F_p$ series converges for $Re[\Sigma b_p - \Sigma a_{p+1}] > 0$, and at $z = -1$ it converges for $Re[\Sigma b_p - \Sigma a_{p+1} + 1] > 0$. The author omits the argument z from $p + 1F_p$ symbol when it is equal to one (as part of the notation). Then $p + 1F_p(z)$ series is called "well poised" if $1 + a_1 = a_2 + b_1 = a_3 + b_2 = \dots = a_{p+1} - b_p$ and "a k-balanced series" if $b_1 + \dots + b_p = k + a_1 + a_2 + \dots + a_{p+1}$ (k– a positive integer). A "1-balanced" series is usually called balanced or Saalschutzian.

The generalized hypergeometric series is a special case of a more general type of series called generalized basic hypergeometric series defined as follows: Let

$$[a]_n = (a; q)_n = (1 - a)(1 - aq^2)\dots(1 - aq^{n-1}),\ n \ge 1,\ [a]_0 = (a\ ; q)_0 = 1,$$

and

$$[a]_\infty = (a;q)_\infty = \pi_{n=0}^{\infty}(1 - aq^n), |q| < 1.$$

Then the generalized basic hypergeometric series is defined as

$r\Phi_s\ [a_1, a_2,\dots, a_r; b_1, b_2,\dots, b_s; q\ ; z]$

$= r\Phi_s\ [(a_r); (b_s); q\ ; z]$

$$= \sum_{n=0}^{\infty}\frac{[a_1]_n\cdots[a_r]_n}{[q]_n[b_1]_n\cdots[b_s]_n}\left[(-1)^n q^{\frac{1}{2}n(n-1)}\right]^{1+s-r_{,n}}$$

where $q \ne 0$ is a complex number. There is a somewhat corresponding classification into "well poised", "nearly well-poised series of the first kind", "nearly well-poised series of the second kind", "k-balanced series", "the 1-balanced series or Saalschutzian series of simply balanced series".

With these definitions we can give a rough idea of the problems of the subject. It consists of evaluations of these functions in terms of well-known functions of analysis (such as Γ-functions in the case of generalized hypergeometric series) inter-relations between these functions and inter-relations between various transforms of these functions (such as definite integrals of these functions). The interplay of arithmetic and q-series is brought out by various identities such as

$$\left(\sum_{n=-\infty}^{\infty} q^{n^2}\right)^2 = 1+4\sum_{n=0}^{\infty}(-1)n\frac{q^{2n+1}}{1-q^{2n+1}} = 1+4\sum_{n=1}^{\infty}\left(d_1(n)-d_3(n)\right)q^n$$

where $d_1(n)$ = the number of divisors d of n such that $d-1$ is divisible by 4 and $d_3(n)$ = number of divisors d of n such that $d+1$ is divisible by 4. These date back to L. Euler and C. G. Jacobi. Ramanujan discovered many more beautiful identities. Some of these are transformations of mock-theta functions of order 3, 5, 7 and Roger-Ramanujan identities (between q-series and q-products) such as

$$\sum_{n=0}^{\infty} \frac{qn^2}{(1-q)\left(1-q^2...\left(1-q^n\right)\right.} = \pi_{n=-\infty}^{\infty}\left(1-q^{|5n+1|}\right)^{-1}$$

and

$$\sum_{n=0}^{\infty} \frac{q^{n(n+1)}}{(1-q)\left(1-q^2...\left(1-q^n\right)\right.} = \pi_{n=-\infty}^{\infty}\left(1-q^{|5n+2|}\right)^{-1}.$$

These and many more identities explained in the book are a feast to the reader. Incidentally the reviewer records that the author is the first to explain what it means to say order of the mock-theta-function. It should be mentioned that the subject though studied for its own sake finds applications in statistical mechanics such as the Hard Hexagonal model and so on. These are very well explained in the book. After Ramanujan there is a great spate of papers by various authors working in the subject and it is almost impossible to mention the references to various authors and the author has done a very good job in this connection also. One finds scores stray papers on each of the topics considered by the author, but it is perhaps the first attempt to present an up-to-date, chronological and critical description of all these works at one place. It is in this respect, particularly, that these volumes will be very useful to the research workers. The reviewer takes this opportunity quote the following result of his from the paper which is cited in volume I chapter-I. Let $H_n = 1+\frac{1}{2}+...+\frac{1}{n}$. Then

$$\sum_{n=1}^{\infty}(-1)^n\frac{H_n^3}{n} = \frac{9}{8}\zeta(3)\log 2+\frac{1}{9}(\log 2)^4-\frac{1}{8}(\pi\log 2)^2-\frac{\pi^4}{144}$$

Here LHS is not a hypergeometric series. He has also results involving $H_n^k(k>0$ any integer). But these later results are not published. But certainly these are inspired by the work of Ramanujan.

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Organising for the Use of Space: Historical Perspectives on a Persistent Issue by Roger D. Launius, Univell–Inc., P. O. Box 28130, San Diego, California 921 98, USA, 1995, pp. 234, $. 60.

In 1961, a young and glamorous president of the United States announced the intention of his Government to put a man on the moon and bring him back. The announcement electrified his nation and the rest of the world, and is often held up as the perfect example of the way a political leader can focus scientific enterprise by setting an ambitious technological goal for the country. As well all know, Kennedy's goal was achieved with a series of spectacular successes during the following decade. But NASA, the agency responsible for these successes, is in serious trouble 35 years later; its budgets have been slashed, its staff retrenched and its contractors - who at the peak of the space programme took 90% of its funding and included the big corporations as well as the small high-tech companies that mushroomed around NASA laboratories (often with expatriate Indian talent) - these contractors have found their business vanish.

As these events occur one tends to praise or blame individual political or business or scientific leaders, but, as this volume shows, there have been deep historical forces at work as well. The eight essays collected in the volume trace how the various decisions were made and executed - by politicians, scientists, bureaucrats and generals - and how various internal battles among them were fought and won. (The odd man out in the collection is a fascinating comparison between the Jet Propulsion Laboratory in Pasadena CA, which did pioneering work on solid rockets in the US, and Peenemuende in Germany, which made the dreaded V2 rockets during the War; there were strong similarities (a heavy reliance on theory, for example) as well as many differences in their scientific philosophy, approach to technology development and style of management.) The essays illuminate how the US has gone full circle from the late President Eisenhower in the 50s to Bill Clinton in the 90s. In the process the book demands a reevaluation of various historical figures, in particular of Eisenhower, who comes out as a sober, rational leader and a man of great integrity who kept his head when so many of his countrymen were losing theirs.

When the Russian announced Sputnik in 1957, many Americans would not believe it till they had seen and heard it; I recollect vividly trudging to the terrace, along with a large number of American colleagues and friends in a Caltech laboratory, to check whether Sputnik was there (- a brief glint off the setting sun showed that it was). The Russian feat set off national hysteria: as Lyndon Johnson said at the time, the "Soviets have beaten us at our own game -daring, scientific advances in the atomic age". The atomic and hydrogen bombs that the Soviets had exploded could be explained away as stolen American technology, but Sputnik could not.

Eisenhower, the hero of World War II who was the US President at the time refused to panic, but his cool only invited derision. His image was that of a conservative, golf-loving, laid-back politician: an opposition leader wrote

You say on fairway and on rough
The Kremlin knows it all,
We hope our golfer knows enough
To get us on the ball,

and others dubbed the Eisenhower White House the "Tomb of the Well-known Soldier". But Eisenhower did not want to spend large sums of money on what he thought of as a "stunt race" in space; he assured the nation that the Soviets had no lead of military value. He was suspicious of a clever and scheming "military-industrial complex", about whose designs he warned his countrymen as his presidency drew to a close. But all of this convinced nobody. There was widespread fear that, just as those who controlled the oceans ruled the world in the 19th century, those who controlled space might do the same in the 20th. A fearful nation that had been fed stories on the "missile gap" by Kennedy elected him as Eisenhower's successor, and was jubilant when the new President announced an ambitious manned space programme.

Two essays in the volume reveal the true position. Eisenhower knew (from the secret flights of the high-altitude U2 plane) that the Russians had nowhere near the missile power of the US. It is remarkable how well-kept this secret was: Eisenhower did not want to reveal it in public (because the U2 flights were illegal) and apparently Kennedy did not know it either. (Would it be possible in India to keep such secrets?) Eisenhower considered that the main military application of satellites would be surveillance, and supported a suitable programme (dressed in scientific finery - as we shall shortly see). His scientific advisers told the nation that bombing from space made no sense - it would be far cheaper to do it from ground (or the oceans). But all of this fell on deaf ears.

Kennedy found out soon after his election that the missile gap was, if anything, the other way round, and this helped him to face down Khrushchev during the Cuban missile crisis. But he still asked his advisers for suggestions on "any other space programme which promised dramatic results in which we could win?" That is how the Apollo project was born. I remember an American Senator who visited Bangalore - perhaps in the late 60s or early 70s - who said that the US space programme had three objectives: (i) to demonstrate to the world that the US could organize itself to achieve complex and ambitious technological goals; (ii) to enhance national security; and (iii) to promote the advancement of science. And, he said, each objective was only about a tenth as important as the preceding one.

Kennedy's decision to go to the moon was followed by intense discussion on how the country should organize itself for the effort. Much of this book is concerned with how the innumerable issues that arose were slowly sorted out. All the moves and countermoves, in the White House, the Pentagon and elsewhere, are recounted. This dynamics is not unfamiliar in India, but there appear to be two differences. First, the number of players in the decision-making seems much larger; secondly, it can be so well documented in the US (because of an extensive system of well-preserved historical archives in all the agencies involved) that very specific lessons for the future can be drawn. The kind of analysis that is presented here would seem virtually impossible in India, if only because the records would in the first place most likely not be carefully preserved, and if kept would be hard to trace and access.

To complete the story, Reagan, when he became President, was horrified to discover that US strategic policy had for decades been based on the principle of Mutual Assured Destruction. He considered this senseless, and asked whether, instead, a policy for Assured Survival was not feasible. That is how the Strategic Defence (or Star Wars) Initiative was born. Whether this initiative was technologically sound or not, it turned out to be an astute political move: It was the last straw on the back of the Russian camel, and convinced Garbachev that his country's economy could not afford this race. So the Cold War ended, the USSR broke up, and, the US having finally achieved all its military and psychological objectives, the Space Programme had to be wound down to Eisenhowerian proportions. Hence (to cut a long story short) all the recent turmoil in NASA.

But amidst the din of political and bureaucratic battle one can begin to see how all American system works. Eisenhower wants to establish that space is free for all, so he *lets* the Russians launch the first satellite, and is keen that American *scientists* (not generals) should launch one of their own; and the National Academy of Science is persuaded to make a suitable proposal as part of the International Geophysical Year - a proposal that is immediately approved. Kennedy wants to remove the sense of weakness that had infected his country, and hardness a team of romantic scientists and ambitious generals to put a man on the moon. And Reagan gives the final push, signalling the Russians that he can take the sting out of their missiles.

As the Cold War ends a commercial technology war begins - in Asia this time; the Missile Gap and SDI give way to Intellectual Property Rights and super-301. I wonder how *that* history will read 40 years from now.

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Mathematical Encounters of the second kind by Philip J. Davis, Birkhauser, Verlag AG, Klosterberg 23, CH-4010 Basel, Switzerland, 1997, pp. 256, sFr. 40.

Professor Philip Davis has written a unique book which contains suggestions to teachers and to mathematicians in particular, in conveying information to the learner, written in an attractive style, more or less like a narration containing numerous anecdotes, stories etc., which carries the reader through and floats him or her in the atmosphere of the imaginations of the author. He richly deserves hearty appreciations for the contents included in this book.

The book mixes a number of topics on mathematics, philosophy, mysticism, literature, statistics, history, biography, cryptography, psychology and fantasy and mathematical genii and catches the reader with its lucid style of narration, partly being, fiction and imagination. Mathematics, of course, with a little niceties on geometry and number theory, is strewn into the theme of the book as encounters with several characteristic characters, real and existing as well as imaginary. Throughout the book, which is divided into four parts and each part into several sections, one finds beautiful and imaginative quotations, phrases, philosophical statements and facts, worth contemplating and memorizing. The book includes accounts of the various styles including life-styles, of mathematicians, real as well as imaginary, living as well as (dead, and conveys the reader in a very effectual way) 'abstraction' as easily as an item of conversation.

In this book, one finds a cute definition of Mathematics as an art, a science and a craft, and as a mother of many other subjects such as philosophy and computer science, being an intellectual creation of the human mind possessing very great depth and used as a language all over, based on strange and wonderful ideas comprising of several techniques, distinct methods and which develops a positive analytic attitude in anyone who practices it.

Some of the statements/results from the book, which could be highlighted are:

1. The crank would not crank and the grinder would not grind. There is no royal road to geometry.
2. Mathematics is created by individuals. It is neither found on tablets carried down from mounts nor in baskets at churches and never is it a product of nameless and faceless committees.
3. Mathematics is a pure form and a pure pattern. It is an abstraction, a generalization, a logical deduction and an unadulterated reason. In mathematics, there is never a final revelation or a resolution.
4. If learning exhibits a phosphorescence, it is a radiation of inner enthusiasm that makes its way out of the scholar.
5. An interesting majic square of order 6 (SUN) with 14 sums, each totaling 216 (comprising of only the odd numbers from 1 to 71), appears on page 122.

6. The largest known prime till date is the Mersenne number, M(859433), containing 258716 digits, where: $M(n) = 2^n-1$ (smallest: M(2) = 3).

7. For $n \geq 5$:

$$1 < \frac{n}{3\log n} < P(2n \setminus n) < \frac{7n}{5\log n}$$

where P(2n\n) is the number of primes between n and 2n. (The distribution of primes exhibits both regularity as well as chaos!)

The reviewer finds this book comprising topics of a diverse nature having a solid literary base, worth reading for times to come and highly enjoyable. The book is recommended for every mathematician/scientist/scholar who intends to convey abstract ideas in a simple manner as possible.

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Intersections of Random Walks by Gregory F. Lawler, Birkhauser, Verlag AG, Klosterberg 23, CH-4010 Basel, Switzerland, 1996, pp. 226, sFr. 58.

Random walk models in statistical physics have a long history. However, using the so called "self-avoiding walks" (that is, random walks conditioned to have no self-intersections) to model polymers is fairly recent. Because of the non Markovian nature of the self-avoiding walks only a few mathematically rigorous results are known. One approach is to study "intersection of paths of random walks". And the book under review is the second printing of a masterly account of the "problems dealing with the non-intersection of paths of simple random walks".

The contents of the book are as follows:

1. Simple random walk
2. Harmonic measure
3. Intersection probabilities
4. Four dimensions
5. Two and three dimensions
6. Self-avoiding walks
7. Loop-erased walk

Though bulk of Chapter 1 (like local central limit theorem, strong Markov property, martingales associated with random walks, etc.) are fairly standard, discrete versions of difference estimates for harmonic functions and Harnack inequality presented in Section 1.7 may not be familiar even to many people working in random walks.

Chapter 2 concerns harmonic measure, viz. the hitting distribution of a set for a random walker starting at infinity; this can also be interpreted as a problem of non intersection of a random walk with a fixed set. Existence of harmonic measure (along with bounds on the rate of convergence) is derived using results of section 1.7. Discrete capacity is developed as a measure of the probability that a random walk will hit a set. Using this, harmonic measure of a line segment is derived. This chapter ends with an introduction to "Diffusion-limited aggregation", a growth model due to Witten and Sander; this is a Markov chain whose state space is the set of all finite connected subsets of the d-dimensional lattice containing zero.

The next three chapters study the probability that the paths of independent random walks intersect. In Chapter 3 the basic framework is explained; expected number of intersections and bounds on the hitting probability are obtained. The expected number of intersections gives rise to a natural conjecture about the order of the probability of "long range" intersections, which is also proved; the critical case when the dimension $d = 4$ requires a careful analysis. The problem of estimating the probability of intersections of two random walks starting at the same point is also considered; the problems turns out to be easier when one of the random walks is two-sided and the other one-sided; in this case, the probability of no intersection is shown to be inversely proportional to the expected number of intersections.

The critical dimension for such intersection problems, viz. $d = 4$, is characterised by logarithmic behaviour of the interesting quantities; for example, the probability of non intersection goes to zero like an inverse power of the logarithm of the number of steps. The exact power, and similar asymptotic expressions are derived in chapter 4 for the critical dimension.

The next chapter concerns similar questions in dimensions two and three. Here the probability of no intersection goes to zero like a power of the number of steps. The result are weaker than in chapter 4, as only upper and lower bounds for the exponent are given. The approach taken here is to relate the problem to a corresponding exponent for intersection of Brownian paths.

The last two chapters are about self-avoiding walks. Heuristic discussion of "connective constant" and "critical exponent" are given in sections 6.2 and 6.3. Other ways to model self-repulsion are also introduced in chapter 6, viz. discrete Edwards model (a configurational model), and Laplacian random walk (a kinetically growing walk).

Chapter 7 is devoted to a rigorous analysis of Laplacian random walk; this model can be obtained by "erasing loops" from a simple random walk. The definition is a bit complicated in two dimensions due to recurrence of simple random walk. For dimensions greater than or equal to four, this model converges to Brownian motion (with, of course, a logarithmic correction when $d = 4$).

An appendix of "recent results" (since the first printing of the book) is also included.

The thrust of the book is probabilistic, though some of the results (especially in chapter 3) have been obtained earlier by physicists (using heuristic arguments, "renormalization group approach", etc.). Much of the material in chapter 3-7 concern the author's own research. The book is very well focused on its central theme, and the story is told clearly and in a readable fashion. The prerequisite is a "standard measure theoretic course in probability theory including martingales and Brownian motion". One you have the prerequisite, wading through this book can be a very rewarding experience.

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S. RAMASUBRAMANIAN

A modern approach to probability theory by Bert Fristedt and Lawrence Gray, Birkhauser, Verlag AG, Klosterberg 23, CH-4010 Basel, Switzerland, 1996; pp. 776, sFr. 98.

The book contains a large amount of material in Probability theory and Stochastic processes and research scholars beginning their career in Probability theory would find it a very useful text. Such a student can begin his voyage in Probability theory with this book and gain enough knowledge from it to be able to start research in a wide choice of topics in Probability theory/Stochastic processes.

The book covers several topics that are usually not covered in a book of this level: stable distributions, a characterization of Brownian motion, conditional *distributions*, Point processes, Levy processes and Interacting particle systems.

The book has a distinct style of presentation: very few proofs are given if full. Most proofs are exercises with enough hints. One learns the subject much better by working out the details of the proofs by himself/herself and the style of this book is likely to be imitated by a few authors at least (or so one hopes!). After all, it is well known that a book that gives very detailed proofs encourages lethargy and the student may miss some crucial points, leading to some gaps in his/her understanding.

The book provides a large number of exercises (apart from the ones intended to complete proofs of the theorems in the book). These exercises are an important contribution of this book. Solutions/hints are available at a web site (whose address is provided in the preface of the book).

The title of the book contains the word 'modern' and the authors spend some time in the preface trying to justify calling the approach 'modern'. While it is true that the book is modern in some respects, it is old fashioned in others and it is difficult to accept some of the arguments put forth by the authors in this regard. For instance, the fact that abstract topology does not play a major role in the book doesn't make it modern! A unified treatment of random variables with values in general measurable spaces makes it modern, but the treatment of infinitely divisible/stable distributions is old fashioned since the modern trend in this area is to treat Banach space valued random variables rather than just real valued random variables. The book gives fairly recent proofs of many results: Strong law of large numbers and Renewal theorem to name some and it treats topics like Interacting particle systems, the martingale problem, coupling methods etc, which make the treatment modern.

The book gives construction of several stochastic processes (e.g., Markov chains and Brownian motion) by special techniques. These constructions are nice and elegant. However, a basic text on Probability theory and stochastic processes must include the general existence theorem of Kolmogorov. In practice, stochastic processes are specified by finite dimensional distributions and so called consistency conditions, which are, of course, necessary for the process to exist). This seems to be notable omission in the book.

Conditional probabilities are defined using Hilbert space projections (cf. Definition 1 on page 404) and Radon-Nikodyn theorem gets an under exposure in the book. The Radon-Nikodym theorem and the Lebesgue decomposition theorem follow easily using Hilbert space projections (cf. Real and Complex Analysis by Walter Rudin) and this proof could have been presented easily in an exercise. In the opinion of this reviewer, a book of this nature should also aim at providing a good training in basic measure theory. In fact, some of its readers may use this as the only source for basic theory, it should have been given enough prominence in the text.

A word of caution to the readers: there is one *glaring* error in the book. Theorem 10 in Appendix C (page 702) is well known to be FALSE and almost any book on point-set topology gives a counter-example to this statement.

Despite the few negative points mentioned here, the book is an excellent source for basic material on Probability theory and Stochastic processes and the book is a must for any department interested in Probability theory (especially those teaching courses on the subject).

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