

Simulation of a fuzzy logic controller for DC/DC converter and its equivalent gains for a PI controller

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Abstract

Equivalence between conventional linear PI controller and fuzzy logic controller is established in this paper. A 3 zone fuzzy controller for regulating dc/dc converter is described in detail. The proposed fuzzy controller and linear PI controller are evaluated by computer simulation. A 5 zone fuzzy logic controller is also evaluated by computer simulation.

Keywords: Fuzzy controller, DC/DC converter.

Indexing: Fuzzy controller, buck-boost converter, DC/DC converter, equivalent PI gains.

Major Discipline: Power electronics circuits.

1. Introduction

For industrial system the problem of controlling a plant involves the determination of the mathematical model and the choice of an appropriate controller. Even if the model of the plant is not known or known inaccurately, the Fuzzy Logic Controller (FLC) can control the plant. The basis of Fuzzy Logic Controller is the use of linguistic statements¹. The PID controller is a very common configuration for industrial control. However, the complexity of the plant makes the tuning of the controller difficult and often trial and error is used. The Fuzzy Logic Controller, on the other hand is easy to implement and tune as it is based on the commonsense understanding of the nature of the plant to be controlled^{2,3}. Therefore it is worth-while to examine the equivalence between the two types of the controllers⁴. The 3 zone Fuzzy Logic Controller can be analyzed in different zones to obtain expressions for equivalent gains. This demonstrates that the FLC is like a gain scheduled PI controller. Further, converters are often studied using their small signal models, especially for controller design. In such cases, the equivalent gains at the d.c. operating point may be used to model the FLC controller. The values for such gains can be calculated from the expressions derived in this paper.

In this paper a Fuzzy Logic Controller with triangular membership function and defuzzification by 'centre of gravity' method is analysed⁵. The block diagram of the Fuzzy Logic Controller is shown in Fig. 1(a). The plant is a Buck-Boost dc/dc converter shown in Fig. 1(b) which has been modeled with differential equations for the purpose of simulation. It is shown that this Fuzzy Logic Controller is equivalent to a non-linear PI controller with adjustable gains. The triangular membership function is commonly used in Fuzzy

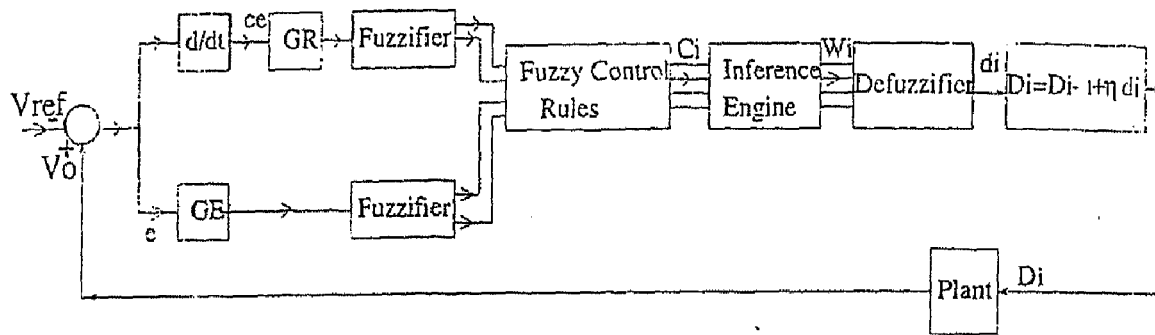


FIG. 1a. Block Diagram.

Logic Controllers as it is easy to implement. Other non-linear membership functions e.g. Gaussian function, have been studied and are not widely used as they are difficult to implement. Therefore they are not considered in this paper. The simulation results of a dc/dc converter (Buck-Boost) with 3 zone and 5 zone Fuzzy Logic Controller are presented⁶.

2. Description of the DC/DC converter and its equations

The converter operates in 2 or 3 modes in one cycle during continuous and discontinuous operation of the converter respectively. In Fig. 1(b) R_L is the resistance of the inductor winding and R_c is the equivalent series resistance of the capacitor and $R_1 = R + R_c$. The switch S is operated periodically with duty cycle D . Thus S is on for duration DT and off for duration $(1-D)T$. If I_l is continuous, the circuit operates in two modes called Mode 1 and Mode 2 below. If I_l is discontinuous during a cycle an additional mode called Mode 3 below occurs. The following equations are obtained for operation of converter in each of these modes.

2.1. Mode 1

In this mode S is on and $0 \leq t \leq DT$ and following equations are obtained.

$$\dot{V}_c = \frac{-V_c}{R_1 C}$$

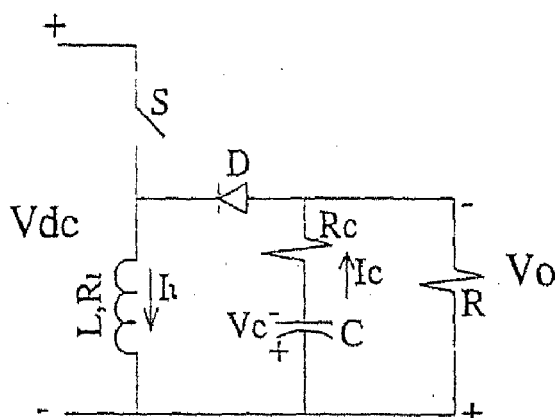


FIG. 1b. Buck-boost converter.

$$\dot{I}_l = \frac{V_{dc}}{L} - \frac{R_l I_l}{L}$$

$$V_c = V_{ci} e^{-\frac{t}{RC}} \quad (1)$$

$$I_l = \frac{V_{dc}}{R_l} + \left(I_{li} - \frac{V_{dc}}{R_l} \right) e^{-\frac{R_l t}{L}} \quad (2)$$

where V_{ci} and I_{li} are initial capacitor voltage and inductor current respectively.

2.2. Mode 2

In this mode S is off and diode D conducts and t is given as $DT \leq t \leq T$. It is assumed that the LC circuit is underdamped and $R_l \approx R$. For this mode, the equations are as follows.

$$\dot{V}_c = \frac{V_c}{R_1 C} + \frac{R_l I_l}{R_1 C}$$

$$\dot{I}_l = \frac{-V_c}{L} - \frac{R_l I_l}{L}$$

$$V_c = e^{\sigma t'} (c_1 \cos \omega t' + c_2 \sin \omega t') \quad (3)$$

$$c_1 = V_{ci}$$

$$c_2 = \frac{\frac{R_l I_{li}}{RC} - V_{ci} \left(\sigma + \frac{1}{RC} \right)}{\omega}$$

$$I_l = e^{\sigma t'} (c_3 \cos \omega t' + c_4 \sin \omega t') \quad (4)$$

$$c_3 = I_{li}$$

$$c_4 = -\frac{\frac{V_{ci}}{L} + I_{li} \left(\sigma + \frac{R_l}{L} \right)}{\omega}$$

$$\sigma = -\frac{\frac{1}{RC} + \frac{R_l}{L}}{2}$$

$$\omega = \frac{\sqrt{\frac{4(R+R_c)}{RLC} - \left(\frac{1}{RC} + \frac{R_l}{L} \right)^2}}{2}$$

where V_{ci} and I_{li} are initial capacitor voltage and inductor current respectively for Mode 2. Time t' is given by $t' = (t-DT)$ and therefore $0 \leq t' \leq (1-D)T$.

2.3. Mode 3

For discontinuous conduction there is an additional mode apart from mode 1 and mode 2. This mode occurs if the inductor current given by equation (4) reduces to zero for a value

of $t' < (1-D)T$. Due to the presence of the diode the inductor current cannot reverse. Capacitor C discharges into the load and diode D is off. Therefore the inductor current remains zero throughout this mode. The time for mode 2, given above then becomes $DT \leq t \leq T_1$ where T_1 is time when the inductor current becomes zero. The value of t' for which the inductor current becomes zero can be obtained from equation (4) by equating the left hand side to zero. Adding DT to this value of t' gives T_1 . The following equations are valid in this mode.

$$\dot{V}_c = \frac{-V_c}{R_l} \cdot \frac{1}{C}$$

$$\dot{I}_l = 0$$

$$V_c = V_{ci} e^{-\frac{t''}{RC}} \quad (5)$$

$$I_l = 0 \quad (6)$$

where V_{ci} is the initial capacitor voltage for mode 3 and t'' is given at $t'' = t - T_1$ and therefore $0 \leq t'' \leq (T - T_1)$. For simulation purposes the analytical solution for converter given in equation (1) to (6) is used.

3. Description of fuzzy logic controller

The block diagram of Fuzzy Logic Controller is shown in Fig. 1(a). Different blocks of controller are described below.

3.1. Fuzzifier

The inputs to the Fuzzy Logic Controller are the error e and rate of change of error ce . These are defined as

$$e = V_o - V_{ref} \quad (4)$$

$$ce = e_i - e_{i-1}$$

Where V_o = output voltage of the converter

V_{ref} = reference voltage of the converter

The symbol i is the suffix denoting the present sampling instant. The values of e and ce are scaled by gains GE and GR respectively to match the range of inputs used by the fuzzifier. Both fuzzifiers are described by the input-output characteristics given in Fig. 2(a).

The input-output characteristics of the fuzzifier consists of three piecewise linear curves called "Membership Functions". These are shown as N, Z and P in Fig. 2(a). These correspond to the classification of input as Negative, Zero and Positive respectively. The fuzzifier assigns two values of the output $\mu_A(x)$ and $\mu_B(x)$ i.e., membership functions to each value of the input x . If input is greater than or equal to L or, less than or equal to $-L$ or, zero these values go to 0 and 1. If the input is between $-L$ and L , the membership

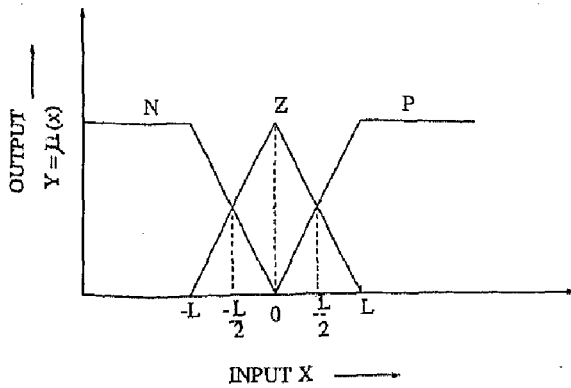


FIG. 2a. Membership function for 3 zone fuzzy controller.

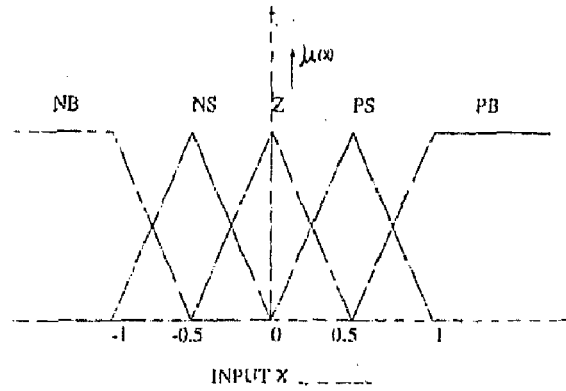


FIG. 2b. Membership function for 5 zone fuzzy controller.

functions have values between 0 and 1. Due to linear nature of curves the sum of the two values of membership functions is always unity i.e., $\mu_A(x) + \mu_B(x) = 1$, for all x . The notion of fuzzification implies that each input may belong to more than one category with a specific value of the membership function. In the fuzzifier shown in Fig. 2(a) an input value is classified as Negative, Zero or Positive with membership function values equal to those given by the respective N and Z or P and Z curves. For large values i.e., magnitude greater than L, the membership function for Zero, curve Z is zero.

3.2. Fuzzy control rules

A control rule relates the category of inputs to the output C_i . It is of the form:

If e_i is A_i and ce_i is B_i then output is C_i .

The above A_i and B_i may each be P, Z or N. Since there are 3 categories of e and 3 categories of ce , therefore total of $3 \times 3 = 9$ rules must be enumerated. These are usually expressed in the form of a control rule table as shown in Table I. L_o is a constant which determines the amount of correction given in the output for a combination of e and ce . A specific case is shown in Table II. Table I gives the values of C_i for each of the nine possible combinations of e_i and ce_i . The zero entries in the table indicate that the converter output voltage is approaching the set point and therefore no control action is required. The positive entries indicate that the converter output voltage is below the set point. So a positive control action is required in order to increase the output. The entries are negative in the table when the output is above the set point so a negative control action is required. For any given value of e and ce there are two categories to which it belongs with two val-

Table I
RuleTable

	P	Z	N
P	-Lo	-Lo	0
Z	-Lo	0	Lo
N	0	Lo	Lo

Table II
Combinations of e&ce

e	ce	μ_{ik}	output (C_{ik})
Z	Z	$\mu_z(e)$	0
P	P	$\mu_p(ce)$	-Lo
Z	P	$\mu_p(ce)$	-Lo
P	Z	$\mu_p(e)$	-Lo

ues of μ i.e., $\mu_A(e_i)$ and $\mu_B(ce_i)$. Similarly for ce . Therefore there are 4 control rules operative for any combination of e_i and ce_i out of possible 9 shown in Table I.

3.3. Fuzzy inference

The fuzzy inference method used in this paper is Mamdani's minimum fuzzy implication which selects a weight factor for C_i and gives a weighted output W_i given by

$$W_i = \min(\mu_A(e), \mu_B(ce))C_i = \mu_i C_i \quad (9)$$

3.3. Fuzzy inference

A nonlinear defuzzification algorithm called 'centre of gravity' method is used. The defuzzifier produces a single crisp value of the output from the multiple weighted values produced by the inference engine. The control action d_i is inferred by

$$d_i = \frac{\sum_{k=1}^4 \mu_{ik} C_{ik}}{\sum_{k=1}^4 \mu_{ik}} \quad (10)$$

3.5. Calculation of D_i

The duty cycle of converter is defined as $D_i = D_{i-1} + \eta d_i$ where η is the gain factor of the fuzzy controller.

4. Equivalent gains of the fuzzy logic controller

The membership function for the curves P, Z and N in X-Y plane are given as

$$\text{Curve P: } \mu_p(x) = \frac{|X|}{L} \quad (11)$$

$$\text{Curve Z: } \mu_z(x) = 1 - \frac{|X|}{L} \quad (12)$$

$$\text{Curve N: } \mu_N(x) = \frac{|X|}{L} \quad (13)$$

in the range $-L \leq X \leq L$. The curves P and N saturate at 1 and curve Z falls to zero outside the above range. For every inference corresponding to a pair of e_i and ce_i the weighting factors $\mu_i (i = 1-4)$ are chosen on the basis of relative values of μ_A and μ_B . But these depend upon the relative values of scaled error and scaled change of error. Therefore e-ce plane is divided into zones as shown in Fig. 3. In the first quadrant each variable can be divided into two ranges $0 - L/2$ and $L/2 - L$. This gives 4 large regions. These are subdivided further into 2 zones each by straight lines of slope -1 and $+1$ passing through the point $(L/2, L/2)$. Each quadrant of e-ce plane can be similarly divided into 8 zones. These are shown in Fig. 3. The division allows us to predict μ_2^* the minimum between μ_A and μ_B stated in equation (9) for W_i . Therefore analytical expressions for μ_2^* are calculated. These are used to calculate d_i . An example for region 1 is given below.

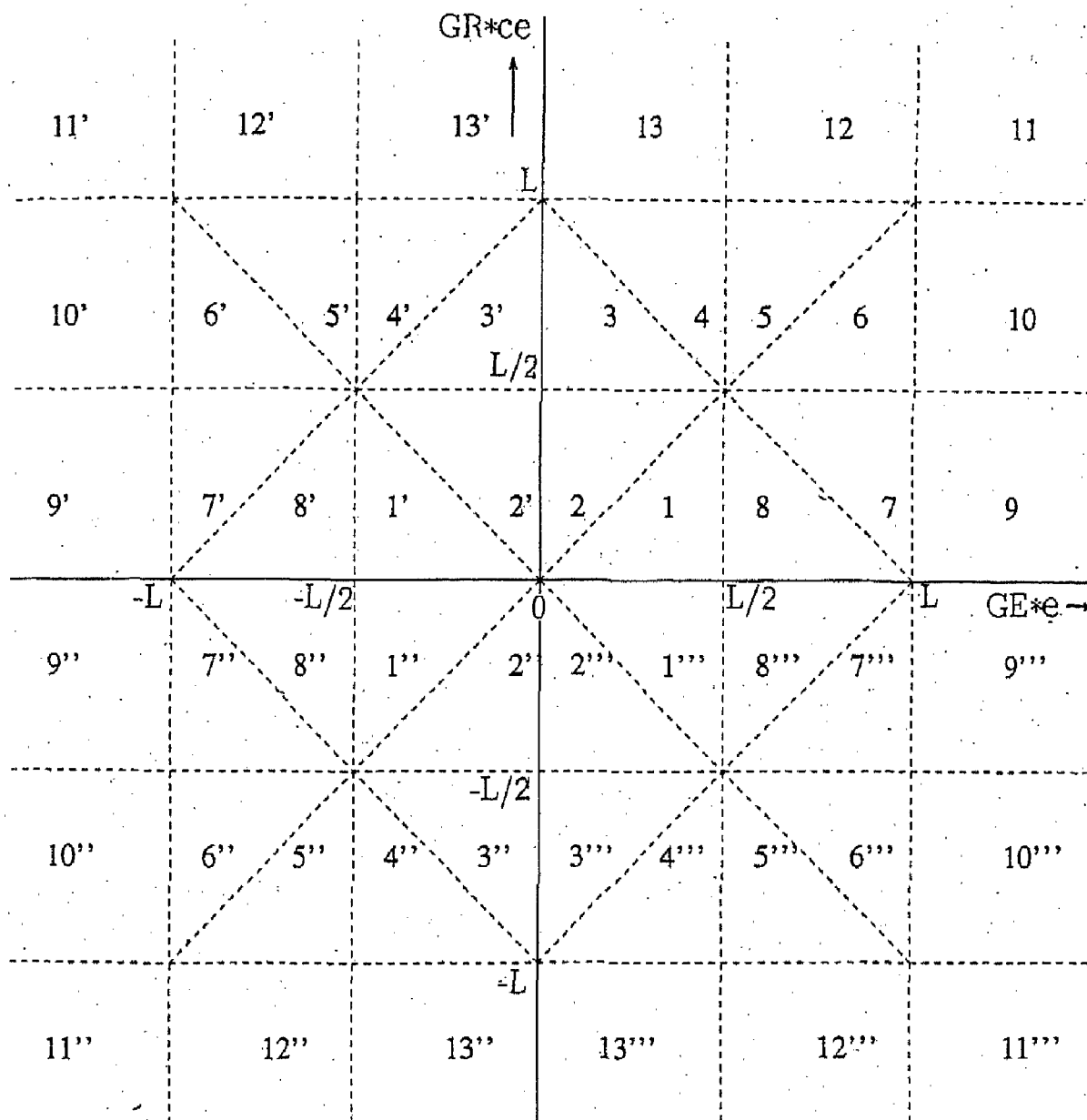


FIG. 3. Division of e-ce plane into zones for calculation of K_i and K_p .

4.1. Region 1

In this region $e > ce$ and both the inputs are in the category P and Z (equation (11) and (12)). The possible combinations of e and ce are given in Table II.

Therefore according to equation (10) d_i is given by

$$d_i = \frac{(-GE|e| - 2GR|ce|)Lo}{(2GR|ce|) + L}$$

In this region $e > 0$, $ce > 0$, therefore $|e| = e$, $|ce| = ce$. Comparing this equation with conventional PI controller.

$$d_i = -(K_i e + K_p ce)$$

$$K_i = \frac{LoGE}{(2|ce|GR) + L}$$

$$K_p = \frac{2LoGR}{(2|ce|GR) + L}$$

The equivalent constants for other regions are given in Table III.

For ranges greater than or equal to L and less than or equal to $-L$ the membership function becomes 0 and 1 and effectively the number of control rules are two. The control action d_i for this range is given in Table IV.

where

$$di1 = \frac{Lo(L - GR|ce|)}{L}$$

$$di2 = \frac{Lo(L - GE|e|)}{L}$$

The above given gains are equal for 1 and 3 quadrant and 2 and 4 quadrant if e and ce both lie in the range $0 < |e|(|ce|) < L$. There are 4 gains for each quadrants of 8 regions. We have taken 2 ranges of e and ce and the number of gains are 8. For n ranges the number of gains will be $2n^2$. Moreover the number of gains are independent of the levels of output. The change in output level only affects the value of gains. Fig 4 shows the characteristics of a Fuzzy Logic Controller on d - e - ce plane. It is seen that d_i is continuous over the e - ce plane.

Table III
Equivalent gains for fuzzy controller

Region	K_i	K_p
1, 1'', 8, 8''	$\frac{LoGE}{(2GR ce)+L}$	$\frac{2LoGR}{(2GR ce)+L}$
2, 2'', 3, 3''	$\frac{2LoGE}{(2GE e)+L}$	$\frac{LoGR}{(2GE e)+L}$
4, 4'', 5, 5''	0	$\frac{Lo(GR - \frac{2L}{e})}{(2GR ce)-3L}$
6, 6'', 7, 7''	$\frac{Lo(GE - \frac{2L}{e})}{(2GE e)-3L}$	0
1', 1''', 8', 8'''	$\frac{LoGE}{(2GR ce)+L}$	$\frac{2LoGR}{(2GR ce)+L}$
2', 2''', 3', 3'''	$\frac{2LoGE}{(2GE e)+L}$	$\frac{LoGR}{(2GE e)+L}$
4', 4''', 5', 5'''	$\frac{LoGE}{3L - (2GR ce)}$	$\frac{LoGR}{3L - (2GR ce)}$
6', 6''', 7', 7'''	$\frac{LoGE}{3L - (2GE e)}$	$\frac{LoGR}{3L - (2GE e)}$

Table IV
Control action for boundary regions

Region	Control action (d_i)
9, 10, 11, 12, 13	-Lo
9'	di1
10'	di1
11'	0
12'	-di2
13'	-di2
9'', 10'', 11'', 12'', 13''	Lo
9'''	-di1
10'''	-di1
11'''	0
12'''	-di2
13'''	-di2

3-D Control Surface for 3 Zone FLC

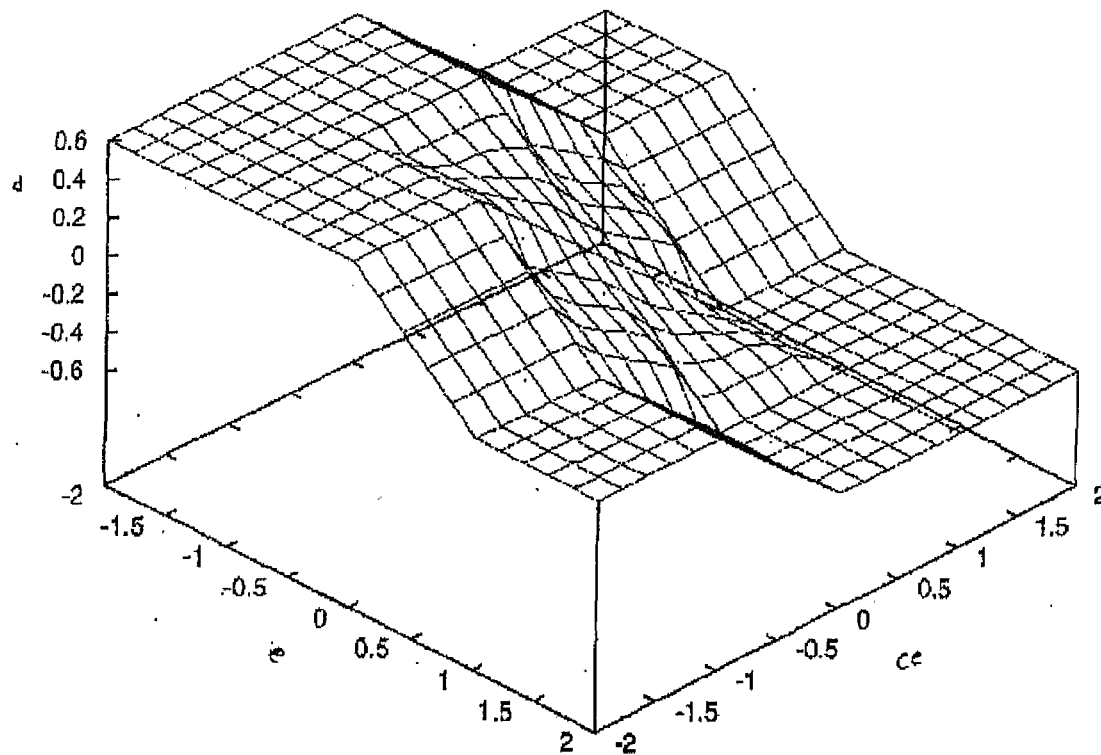


FIG. 4. Variation of control action d_i with e and ce for the 3 zone Fuzzy Controller with $L_o = 0.6$.

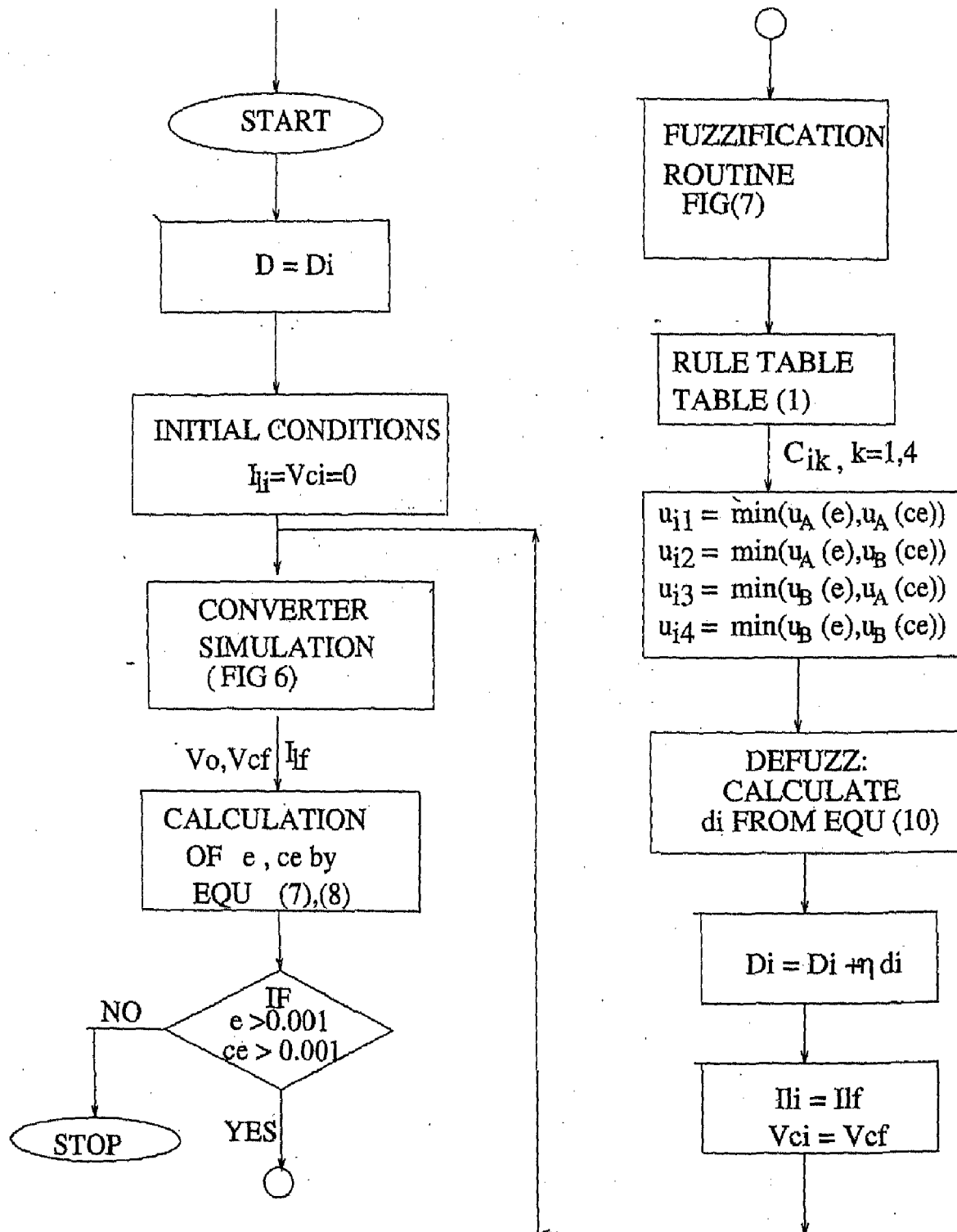
As seen from the Table III, the gains are functions of e and ce . Therefore the fuzzy controller is a nonlinear controller.

5. Simulation results

The above mentioned Fuzzy Logic Control algorithm is now verified by simulation in 'c' language. The converter is simulated by its analytical solution from equations (1)–(6). These equations have been used to simulate the converter using a programme whose flow chart is given in Fig 6. The Fuzzy Logic Controller is simulated according to the block diagram of Fig. 1(a). A flow chart of programme is given in Fig. 5. This flow chart uses the converter simulation shown in Fig. 6 and fuzzification routine shown in Fig. 7. Constant L_o in the rule table (Table I) is chosen as $L_o = 0.6$ throughout the simulation. The constant L shown in Fig. 2(a) has been chosen as $L = 1$. Fig. 8 shows the transient response of the converter with PI controller and 3 zone FLC. Fig. 9 shows the load regulation of the converter with a step change in load resistance from 10 ohms to 5 ohms with linear Pi and 3 zone FLC. Fig 10 shows the line regulation of the converter with a step change in input voltage from 15 V to 10 V with PI and 3 zone FLC. The undershoot and overshoot are much reduced in case of Fuzzy Logic Controller for both types of disturbances. However in the response of the Fuzzy Logic Controller the oscillation frequency is much higher than that in the PI controller. The converter parameters are given as below:

Inductance $L = 100 \mu H$

Output capacitance $C = 100 \mu F$
 Inductance series resistance $R_l = 0.1$ ohms
 Capacitor series resistance $R_c = 0.1$ ohms
 Load Resistance $R = 10$ ohms



5. Main flowchart.

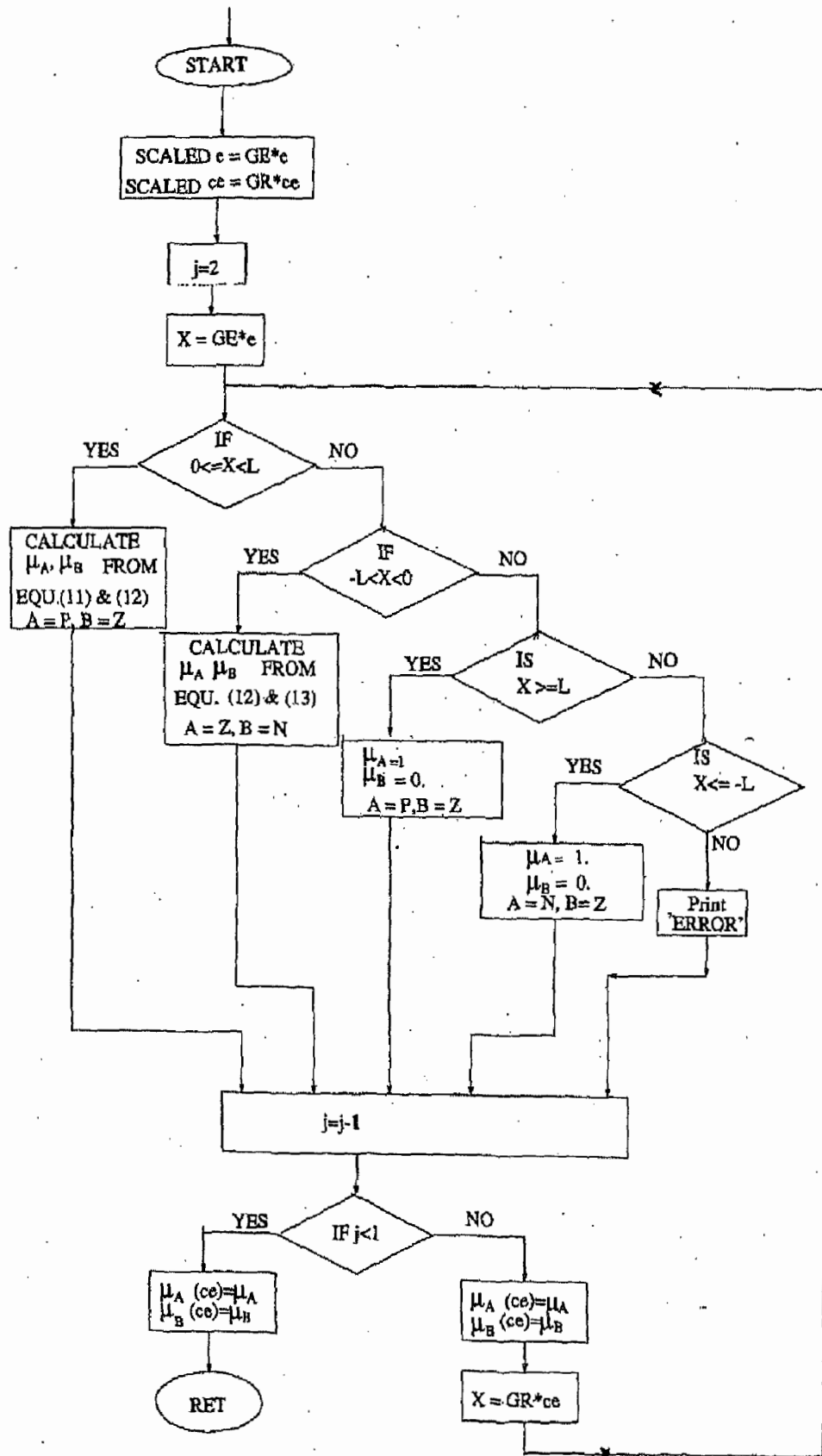


FIG. 7. Fuzzification routine.

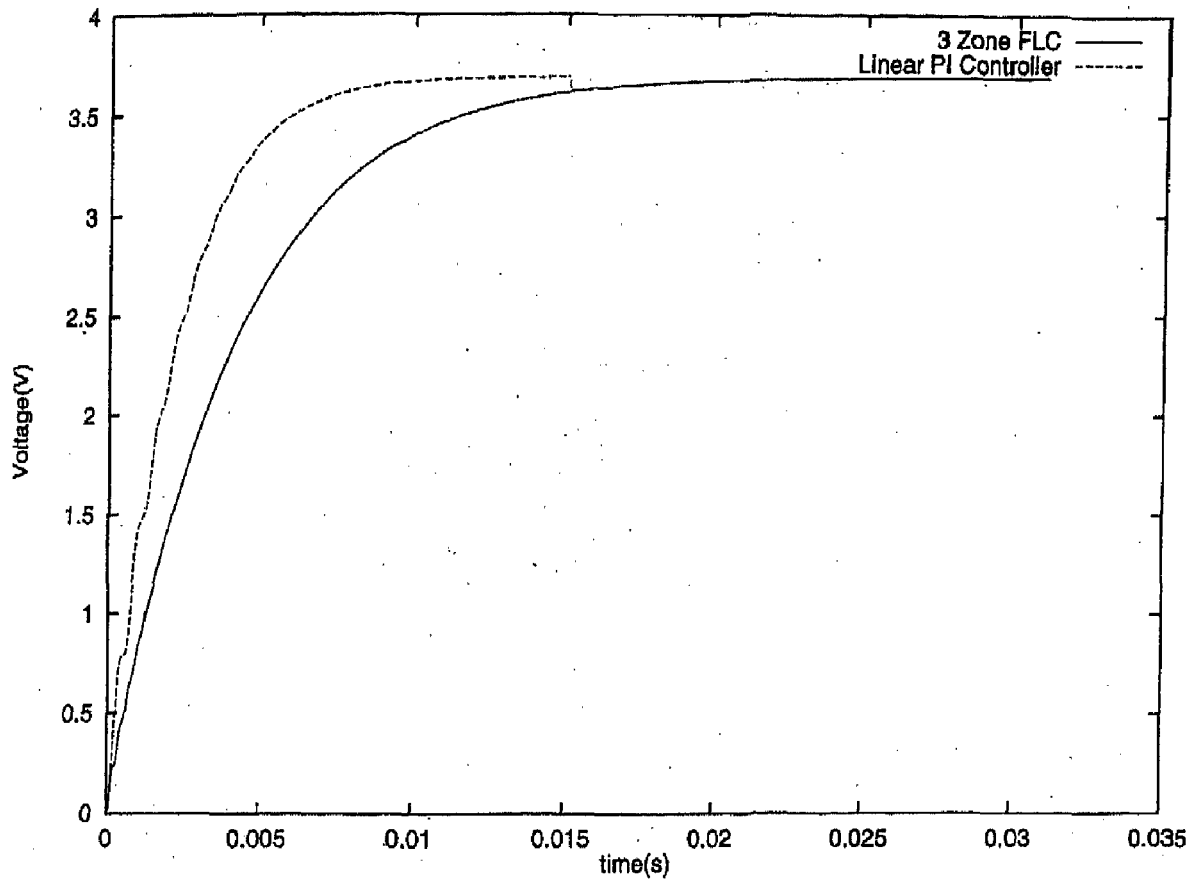


FIG. 8. Transient response of 3 zone fuzzy and linear PI controller.

Gain factor for Fuzzy Logic Controller $\eta = 0.01$

Scaling factor for error, $GR = 0.2$

Scaling factor for change in error, $GR = 66.67$

For Linear PI controller $K_p = 0.012$, $K_i = 0.003$.

5.1. Five zone fuzzy logic controller

A more complex controller can be achieved by the classification of e and ce in 5 zones — Positive Big, Positive Small, Zero, Negative Small, Negative Big called as PB, PS, Z, NS, NB respectively. The membership functions and control rule table for 5 zone FLC is shown in Fig. 2(b) and Table V respectively. The number of constants in the case

Table V
Rule Table for 5 Zone Fuzzy Controller

	NB	NS	Z	PS	PB
PB	-0.3	-0.35	-0.45	0.65	1
PS	0	-0.1	-0.2	-0.35	-0.5
Z	0.2	0.1	0	-0.1	0.2
NS	0.6	0.35	0.2	0.1	0
NB	1	0.65	0.45	0.35	0.3

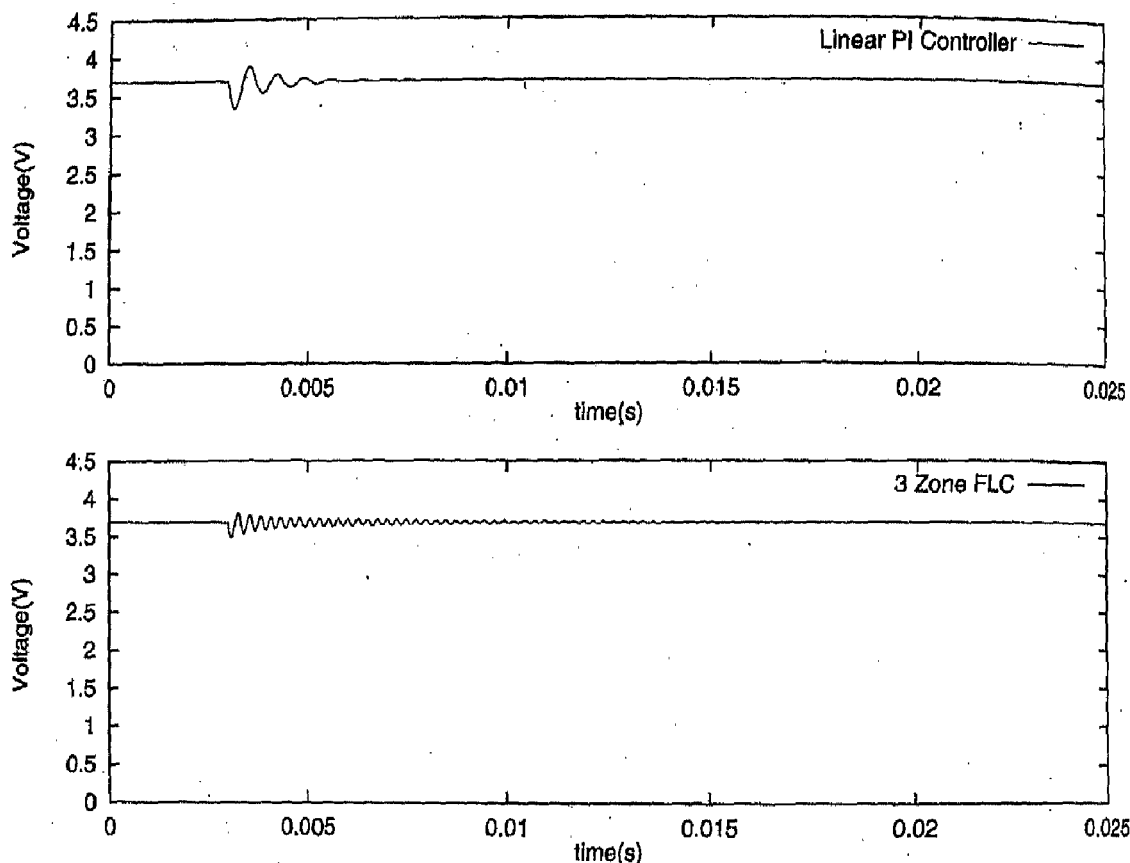


FIG. 9. Load regulation of linear PI and 3 zone fuzzy controller load resistance has step change from 10 ohms to 5 ohms.

of 5 zone FLC is 32 and instead of calculating such large number of constants explicitly, one can directly obtain the control surface for a given set of parameters. The control surface is a plot of function d_i with respect to variables e and ce . This function completely characterises the Fuzzy Logic Controller. The plot of the control surface for a 5 zone FLC whose parameters are given in Fig. 2(b) and Table V is shown in Fig. 11. The equivalent gains at a particular operating point can be calculated numerically as the partial derivatives of the this function for a particular design study. However in this paper the transient response of a 5 zone FLC has been obtained by simulation which does not require the explicit calculation of equivalent gains.

The transient response of 5 zone FLC is shown in Fig. 12. For comparison the response of a 3 zone FLC has also been plotted on the same figure. The two controllers used membership functions varying between same limits as shown in Fig. 2 with $L = 1$. The values of control outputs shown in Table 1 with $L_o = 0.6$ for 3 zone FLC are comparable to the values given in Table V for 5 zone FLC. It is seen that the 5 zone FLC has large number of output levels, while the 3 zone FLC uses only one level. The average of output levels used in 5 zone FLC is 0.45, while the 3 zone FLC uses an output level of 0.6. The same value of $\eta = 0.01$ is used for both controllers. It is seen from Fig. 12 that the transient response of 5 zone FLC is faster than that of the 3 zone FLC for comparable parameters. This shows that

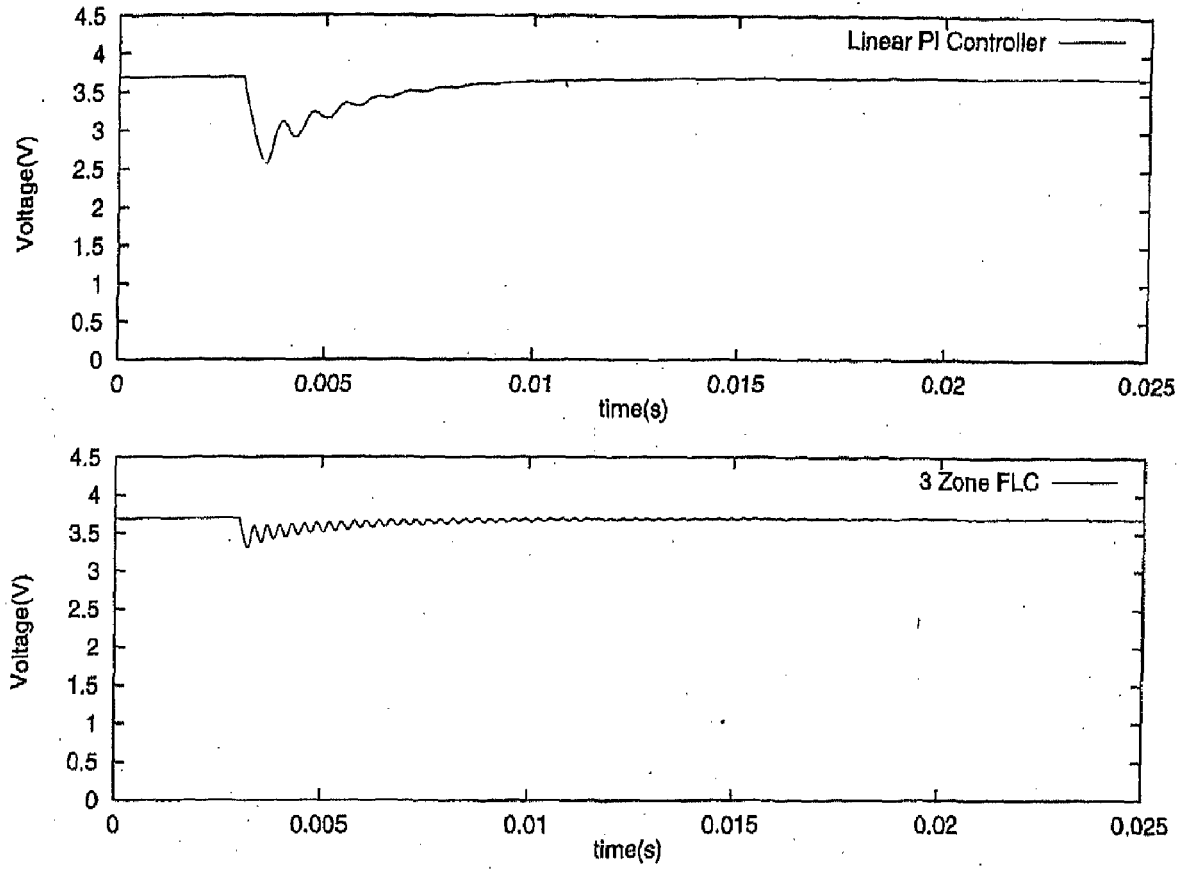


FIG. 10. Line regulation of Linear PI and 3 zone fuzzy controller input has step change from 15 V to 10 V.

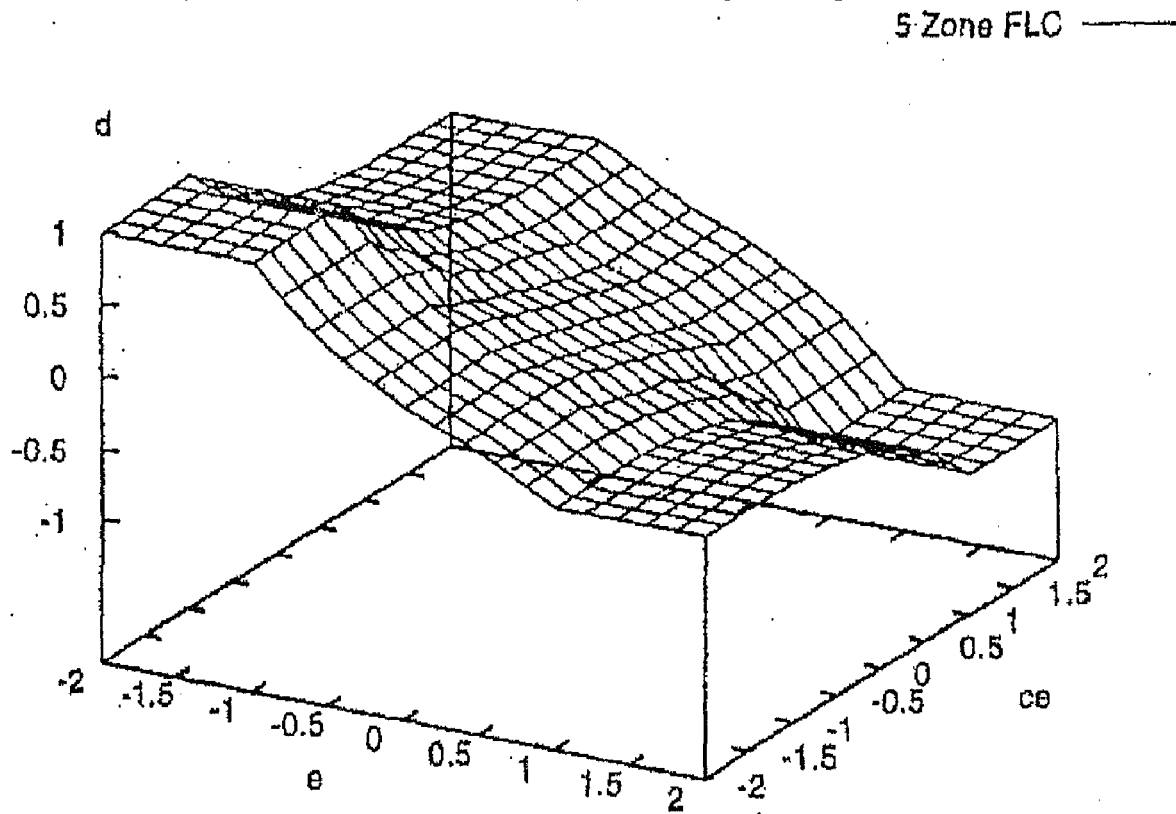


FIG. 11. Variation of control action d_i with e and ce for the 5 zone fuzzy controller.

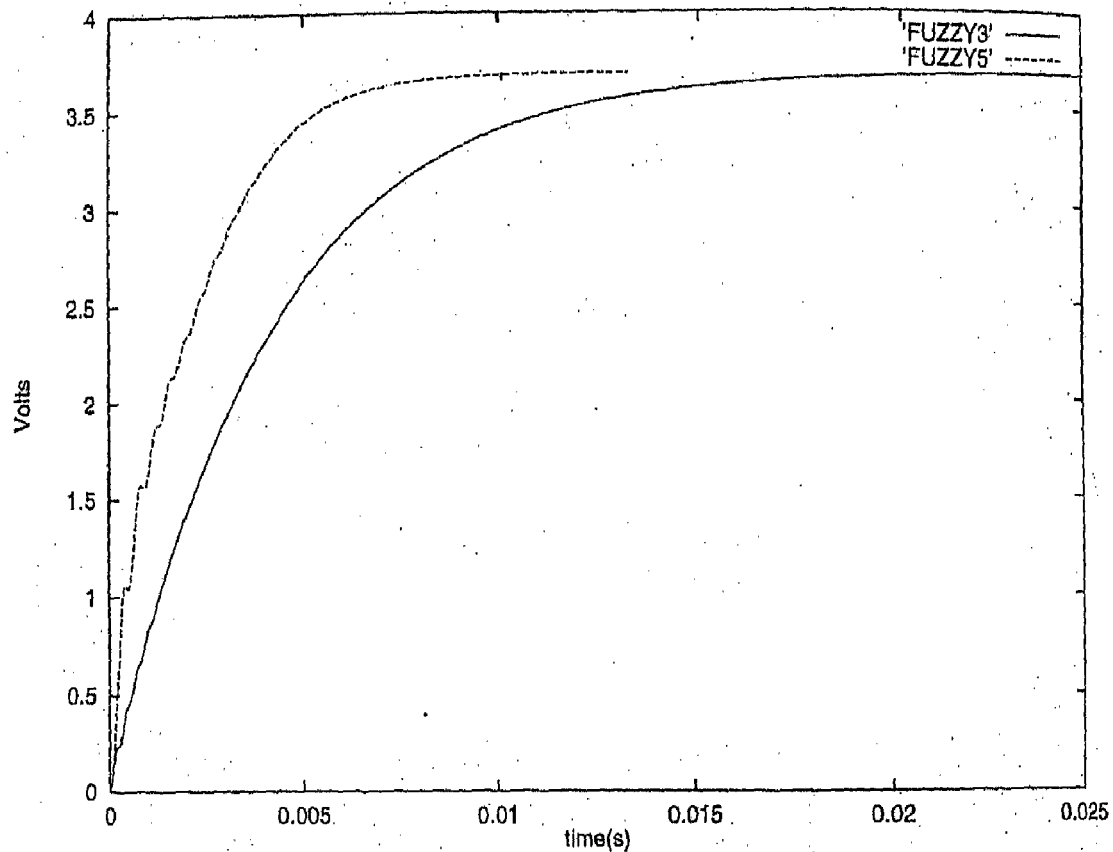


FIG. 12. Transient response of a 5 zone and 3 zone fuzzy controller

increasing the number of zones improves the transient response but increases the complexity of the controller.

6. Conclusion

The fuzzy controller has been seen to be equivalent to a nonlinear PI controller. The expressions for these equivalent gains have been given in Table III and Table IV. Simulation results show that the transient response of a linear PI controller is similar to that of a 3 zone fuzzy controller. However the line and load regulation is better with 3 zone fuzzy controller as this is a nonlinear controller. The transient response of a 5 zone fuzzy controller is better than that of a 3 zone fuzzy controller but more computation is required.

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