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BOOK REVIEWS

The asymptotic behaviour of semigroups of linear operators by Jan van Neerven, Birkhauser Verlag AG, Klosterberg 23, CH-4010, Basel, Switzerland, 1996, pp. 256, sFr. 128.

As the title says, this self-contained book analysis whether a semigroup $T(t)$, $t \geq 0$ of linear operators on a Banach space converges to zero as $t \to \infty$. This is evidently an issue in Dynamical Systems to test the stability of an equilibrium point which, in this case, is the trivial solution. As we know, literature in Dissipative Dynamical Systems concerning the stability / bifurcation of equilibrium points, periodic orbits and the resulting strange attractors is exploding. One may wonder therefore at the relevance of the present book. It must be pointed out that most of the study on Dynamical Systems is concentrated in finite dimensions whereas the present volume develops tools of analysis in infinite dimensions. This is a non-trivial generalization and many new phenomena appear even in the study of approach to equilibrium. This issue is further compounded by the fact that the state space under consideration need not have Hilbert space structure.

The standard method to study a Dynamical System near an equilibrium point is to linearize around it and see whether linear oscillations die out as $t \to \infty$. This issue is explored in great detail in this book. In infinite dimensions, the linearized operator A is usually unbounded (in many applications, A is a Partial Differential Operator) and this is the root cause of all difficulties.

On the side of dynamical equation, a crucial parameter governing the stability of equilibrium is the so-called Lyapunor exponent, denoted here as $\omega_0(T)$. Along with this, another parameter $\omega_1(T)$ governing the stability of orbits emanating from the domain of A is also considered. Because A is unbounded, $\omega_0(T)$, and $\omega_1(T)$ need not coincide. The central question is to know the circumstances under which $\omega_0(T) < 0$ or atleast $\omega_1(T) < 0$. In such a case, it is known that oscillations are killed at exponential rate and so there is exponential stability.

There is a classical connection between semigroup defining dynamics and the spectral equation of A. This can be easily seen by taking Laplace transform of the equation w.r.t time variable. Several questions concerning the behaviour of dynamics can be tackled by performing spectral analysis of A. The analysis in this book is based on this observation.

On the spectral side, two parameters enter the scene naturally: spectral bound $s(A)$ and the abscissa $s_0(A)$ of uniform boundedness of the resolvent of A. Examination of inter-relationships which exist between these four parameters is one of the high points of this book. Several landmark results in this area are presented. A general result states that $\omega_1(T) \leq s_0(A)$. Many other theorems established here are not general but are proved by putting some restrictions on the nature of dynamics (eg: compactness, holomorphy, positivity) (or) on the state space (Hilbert space, L^p space etc.)

Apart from above results, one can also seek sufficient conditions for the uniform stability of the semigroup without using parameters $s(A)$ and $s_0(A)$. An array of Theorems has been given which establish the relationship between stability and the behaviour of the resolvent near the imaginary axis.

Finally, let me point out other potential applications of the results of the book apart form the ones arising in Dynamical Systems which were pointed out earlier. Being abstract, this work assumes that the infinitesimal generator A is given and formulates the end results in terms of A. In practice, this is not the case, however. In many cases, A is defined by a partial differential operator with suitable boundary conditions. The task then is to formulate stability conditions in terms of boundary data, coefficients of the operator etc. One can also think of inverse problems namely to construct suitable boundary condition (or) coefficients so that we have the desired stability. These concrete situations are the ones which first motivated stability questions in infinite dimensions. These include scattering phenomena, exact controllability, stabilization etc. Another area where asymptotic properties of linearized equations have great influence is the study of the corresponding nonlinear equations and global existence for such equations. Since the main thrust of the monograph is on Banach spaces, the results and analysis ought to be useful in PDE posed on L^p spaces. Without going into further details, let us be content with citing some references which may also be read with great benefits along with the present volume under review.

1. LAX, P. D. AND PHILLIPS, R. S., Scattering Theory, Academic Press, New York, Revised Edition 1989.

2. LIONS, J. L., Exact Controllability, Stabilization and perturbations for distributed systems, SIAM Rev. Vol 30(1), 1988, p. 1–68.

3. KOMORNIK, V. Exact Controllability and Stabilization, Masson/J. Wiley copublication, Paris, 1994.

4. CHRISTODOULOU, D. AND KLAINERMAN, S. The Global Nonlinear Stability of the Minkowski space, Princeton University Press, Princeton, 1993.

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The life of Stefan Banach by Ann Kostant and Woyczynski,

This book is a biography of a great, well-known polish functional analyst S. Banach. This is an edited and translated version of an original polish book written by a reporter. Total mathematical content being small, this volume of about 130 pages is easily accessible to a wide educated audience.

It is now more than 100 years since the birth of Banach. There is practically no full account of his life available for English-knowing public. Indeed, it is generally felt that there are more opportunities for people to come to know of life and achievements of mathematicians from Western Europe and USA than from East Europe. It is therefore really not surprising to learn that one edition of Encyclopedia Britannica has Banach listed as a Russian. This biography, I believe, will go a long way to bridge this gap and fulfill the need by showing glimpses of life and work of S. Banach.

Banach is widely acknowledged to be a founding father of functional analysis. The algebraic-topological structures introduced by him (namely Banach Spaces) play a fundamental role in modern functional analysis and without them, theory of numerical approximations cannot exist. Uniform Boundedness Principle, Hahn-Banach Theorem and the Closed Graph Theorem form three cornerstones of the subject of linear analysis. On the side of nonlinear analysis, he established another fundamental result: Principle of contraction mapping. This is the technique by which one first attempts to prove existence of solutions to nonlinear equations. All these materials now form part of M.Sc. curriculum in India and all over the world. Such is the forceful impact of contributions made by Banach to mathematics. His book "Theorie des operations lineaires" describing above developments is a world classic.

A person knowing these achievements of Banach will naturally be anxious and eager to know about his life. What sort of person he was? How he was discovered? What was the atmosphere like when he carried out his work? What is the significance of his work? How it compared with mathematics of that period? The present book seeks to answer these questions. To this end, the author has conducted interviews with people who knew and remember Banach and conversed with mathematicians familiar with Banach's work. It is by no means an easy task. Nevertheless, the author has done a wonderful job of telling a fascinating story. Particular mention must be made of the lucid description of mathematical, political and social discussions that took place at the famous Scottish Cafe at Lvov and the consequent appearance of the Scottish Book. The reader can find also an account of the extent of the French influence on the Polish school at that time.

It is really amazing to read about a man who, even at times of war, aggression and occupation, dedicated himself to mathematics. Banach wrote elementary books for school children because he was in need of money. Such a man, when offered unlimited amount to migrate to USA, declined the offer saying that it was too small to leave his native land. True patriot indeed!

In conclusion, let me state that apart from the usual pleasure of reading, one can also derive some inspiration by knowing Banach's life and achievements in mathematics. Several photographs connected with his life which decorate this volume is an added piece of attraction

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Essential Wavelets for Statistical Applications and Data Analysis by R. Todd Ogden, Birkhauser, Verlag AG, Klosterberg 23, CH-4010 Basel, Switzerland, 1997, pp. 224, sFr. 68.

The mathematical theory of wavelets is very well developed by now, and there are many good books available in this area. However, for those interested in learning the applications of wavelets, the choices are few. The present book can certainly be recommended to those interested in statistical applications of wavelets. It is not a great book even for this readership since many would need a book like E. Hernández and G. Weiss (*A First Course on Wavelets*, CRC Press, 1996) at hand for further mathematical clarifications, but the approach needed for statistical applications is provided here quite well requiring only basics of statistical theory and familiarity with calculus and linear algebra from the readers.

This book consists of a Prologue, nine chapters and an appendix. Prologue briefly explains where and when wavelets can be profitably used. A very brief introduction to wavelet analysis (including multiresolution analysis) follows in Chapter 1. Most readers will need to consult other books on wavelets to gain insights into much of what is discussed here before proceeding further. A short chapter on kernel smoothing and another short chapter showing how wavelets can instead be used follow. These brief discussions, perhaps, give too narrow a view on the uses of wavelets in statistics. Chapter 4 continues wavelet analysis from where it was left off in Chapter 1, and contains a lot of material which is very important to understand applications of wavelets. Rest of the book concerns applications of wavelets in statistics, except the last chapter which explains some generalizations of one-dimensional wavelets to higher dimensions.

This reader finds Chapters 7 and 8 especially useful for statisticians. What are statisticians doing with wavelets currently is discussed here. Further, some of the recent work on applications of wavelets in the area of Bayesian nonparametric regression is also discussed Chapter 8 making this book useful to those interested in Bayesian inference as well.

A couple of minor errors: Preface refers to a non-existent Chapter 10 (perhaps Appendix and at one place on page 10, 2^{-j0} appears as 2^{j0}.

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