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# Mode propagation in planar segmented waveguides with parabolic index segments*

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**Abstract**

Propagation characteristics of planar periodic segmented waveguides consisting of infinitely extended parabolic refractive index segments in the transverse direction have been analysed using the matrix formulation of Gaussian beam propagation. Modal propagation and conditions for the existence of guided mode in such a segmented waveguide have also been obtained.

**Keywords:** Mode propagation, segmented waveguides, waveguide stability.

## 1. Introduction

Segmented waveguides consisting of a periodic linear array of high index regions embedded in a lower refractive index region (Fig. 1) have recently attracted considerable attention for applications to nonlinear interactions, particularly for second harmonic generation (SHG)[1]. Periodic segmented waveguides have been used to achieve efficient phase-matched SHG in KTP and $LiTaO_3$[1-5]. In these structures, domain inversion and waveguides can be achieved simultaneously, making them very attractive for use in nonlinear guided wave interactions employing periodic structures. Segmented waveguides by themselves are also interesting since the effective index and mode spot size of the propagating mode can be controlled by simply varying the duty cycle of segmentation which can be used in the efficient design of $z$-variant linear waveguide devices such as mode expanders, polarisation converters, wavelength filters, etc., and in periodic structures such as Bragg reflectors[6], mode expanders, wavelength filters and polarisation converters. Some studies on the linear characteristics of segmented waveguides have been reported recently by Li and Burke[7], and Weissman and Hardy[8]. However, there is no simple analytical method to study propagation in these structures. There have also been some experimental studies on the linear characteristics of segmented waveguides, employing the concept of equivalent nonsegmented (uniform) waveguides[9, 10].

In a recent study, we have considered a planar periodic segmented waveguide formed by segments characterised by an infinitely extended parabolic refractive index variation in the transverse direction[11]. Since the transverse distribution of the fundamental mode of a waveguide characterised by an infinitely extended parabolic index profile is a

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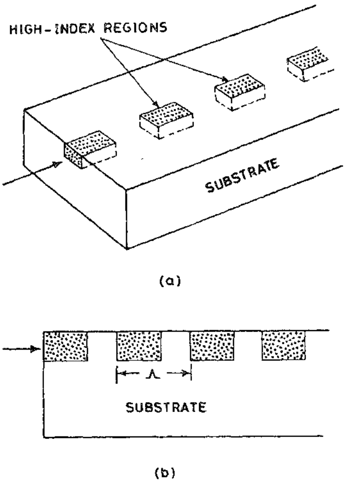



FIG. 1. a. A periodic segmented waveguide comprising a linear array of high index regions. b. Schematic of the longitudinal cross-section of a segmented waveguide; Λ is the period of segmentation.

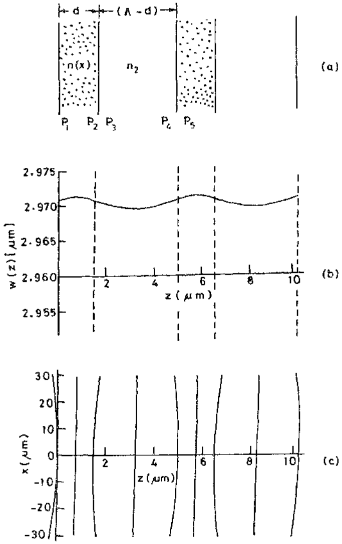



FIG. 2. a. A periodic segmented waveguide with period Λ and duty cycle 0.3. Variation of beam spot size (b) and phase front (c) of the Gaussian mode along *z*.

Gaussian (see, Thyagarajan *et al.*[12]), and since a Gaussian beam remains Gaussian on diffraction in a homogeneous medium, we expect the modes of the above segmented waveguide to be 'Gaussian-like'. Using the matrix formulation of Gaussian beam propagation, we have obtained simple analytic expressions for determining the fundamental mode parameters[11]. The analysis shows that the effective index of the mode varies almost linearly with the duty cycle of segmentation, and is independent of the period of segmentation; the spot size of the mode varies along the waveguide periodically, repeating after every period of segmentation and the average value of the spot size of the mode increases with decreasing duty cycle. These results are consistent with the reported results on segmented waveguides[7–10]. In this communication, we discuss the stability of a mode in such a segmented waveguide, and obtain the condition for the existence of guided modes in such waveguides. We show that, depending on the waveguide parameters, there exists regions in which the field distribution becomes unstable and the beam diverges rapidly as it propagates through a distance of a few periods. Using typical values for the waveguide parameters, we have illustrated the propagation behaviour corresponding to different regimes of waveguide stability. For the sake of completeness, we have outlined the analysis in the following section.

## 2. Analysis

Figure 2a shows a periodic segmented waveguide with a period $\Lambda$ and duty cycle $\gamma\,(= d/\Lambda)$ consisting of alternate regions of high- and low-index media. We assume the refractive index profile of the segmented waveguide to be given by

$$\begin{aligned} n^2(x) &= n_1^2(1-\alpha^2x^2) \qquad && 0<z<d \\ &= n_2^2 && d<z<\Lambda \\ n^2(x, z+\Lambda) &= n^2(x,z), && \end{aligned} \tag{1}$$

where $n_1$ is the axial refractive index in the high index segment and $\alpha$ is a measure of gradation of the refractive index profile.

Since a segmented waveguide is not uniform along the propagation direction, the conventional definition of a mode may not be applied. We may, however, define the mode of a segmented waveguide as that transverse field distribution which repeats after every period of segmentation; the reduction in amplitude of the field will correspond to the propagation loss, while the phase change with propagation determines the effective index of the mode. Such a definition is very similar to the definition of a mode in a lens waveguide[13,14] or an oscillating mode in an open optical resonator[12]. Since the fundamental mode of a waveguide is characterised by an infinitely extended parabolic index variation, one expects the fundamental mode of such a segmented waveguide also to be a Gaussian under the paraxial approximation. We thus assume the field distribution of the fundamental mode of this segmented waveguide to be Gaussian and of the form

$$f(x,z) = \exp\left[\frac{-ikx^2}{2q(z)}\right], \tag{2}$$

where $q$ is the complex beam parameter, and is defined as (see Yariv[14])

$$\frac{1}{q(z)} = \frac{1}{R(z)} - \frac{i\lambda}{\pi w^2(z)}, \tag{3}$$

$\lambda$ is the wavelength in the medium, $k = 2\pi/\lambda$, $R$, the radius of curvature of the wavefront and $w$, the beam spot size

Under the paraxial approximation, the transformation of a Gaussian beam through an optical system is described by[14]

$$q_o = \frac{Aq_i + B}{Cq_i + D}, \tag{4}$$

where $A$,$B$,$C$ and $D$ are elements of the ray matrix of the optical system which relate the ray parameters at the input plane to those at the output; $q_i$ and $q_o$ are the complex beam parameters at the input and the output planes, respectively. Thus, knowing the values of the beam radius $R(z_i)$ and spot size $w(z_i)$ at any given plane with $z = z_i$, the corresponding values at any other plane with $z = z_0$, can be obtained using eqns (2)–(4).

Propagation in a segmented waveguide consists of propagation through a homogeneous medium of length $(\Lambda - d)$, through a parabolic index medium of length $d$ and refraction at the interfaces between the two regions. The $ABC$ and $D$ matrices corresponding to these are given by

*Propagation through a homogeneous medium*

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 1 & \Lambda - d \\ 0 & 1 \end{pmatrix}. \tag{5}$$

*Propagation through a parabolic index medium*

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} \cos \alpha d & \dfrac{\sin \alpha d}{\alpha} \\ -\alpha \sin \alpha d & \cos \alpha d \end{pmatrix}. \tag{6}$$

*Refraction at planar interface between media of indices of $n_1$ and $n_2$*

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & \dfrac{n_2}{n_1} \end{pmatrix}. \tag{7}$$

For propagation in the segmented waveguide shown in Fig. 2a, the $A$, $B$, $C$ and $D$ matrix elements corresponding to one period of segmentation are given by the product of the four ray-transfer matrices corresponding to two refractions, propagation through a medium of index $n_2$ of length $(\Lambda - d)$ and through a parabolic index medium of length $d$:

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} \cos \alpha d - \dfrac{n_1}{n_2} \alpha (\Lambda - d) \sin \alpha d & \dfrac{\sin \alpha d}{\alpha} + \dfrac{n_1}{n_2} (\Lambda - d) \cos \alpha d \\ -\alpha \sin \alpha d & \cos \alpha d \end{pmatrix}. \tag{8}$$

In writing the ray-transfer matrices for the interface we assume the refractive index in the region with parabolic refractive index as $n_1$, which is the axial refractive index.

As mentioned earlier, to determine the modes of the segmented waveguide, we look for those field distributions which repeat themselves after one period except for a change in amplitude and phase. Thus for an initial field $f(x, z)$, if $u(x, z + \Lambda)$ is the field distribution after one period, then for a mode we must have

$$u(x, z + \Lambda) = |\sigma| f(x, z) e^{-i\phi}, \tag{9}$$

where $|\sigma|$ is a measure of the loss and $\phi$ gives the phase change in one period. In the present case since we are considering infinitely extended segments, the segmented waveguide will be lossless, *i.e.*, $|\sigma| = 1$. If $q_j$, $R_j$ and $w_j$ are the Gaussian beam parameters in the plane $P_j$, $j = 1, 5$ (see Fig. 2a), then for a mode we have $q_5 = q_1$, or

$$w_5 = w_1, \text{ and } R_5 = R_1. \tag{10}$$

Using eqns (4), (8) and (10), we get the parameters of the mode as

$$w_1^2 = \frac{\lambda}{\pi}\left[\frac{1}{|B|}\sqrt{1-\frac{(A+D)^2}{4}}\right]^{-1}, \tag{11}$$

$$\frac{1}{R} = \frac{-(A-D)}{2B}. \tag{12}$$

In the above analysis we have neglected the reflection losses at the interface which are significant only when the condition for Bragg diffraction is satisfied. The total phase change in one period is given as

$$\phi = k_1 d + k_2(\Lambda - d) + \frac{\phi_1+\phi_2}{2} - \frac{\pi}{2}, \tag{13}$$

where $k_i = (2\pi/\lambda_0)n_i$; $i = 0, 1$; $\delta = \sqrt{k_1\alpha}$.

$$\phi_1 = \tan^{-1}\left[w_1^2\left(\frac{\delta^2}{2}\cot\ \alpha d + \frac{k_1}{2R_1}\right)\right]$$

$$\phi_2 = \tan^{-1}\left[\frac{w_3^2 k_2}{2}\left(\frac{1}{R_3} + \frac{1}{(\Lambda - d)}\right)\right].$$

The effective index of the mode is then given by

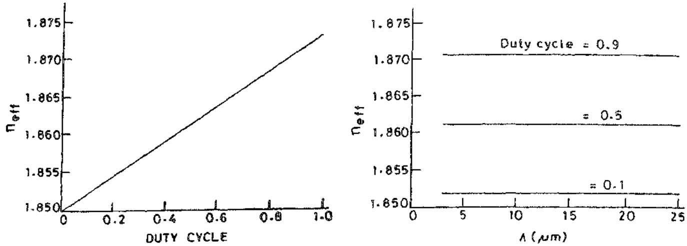
$$n_{\text{eff}} = \frac{\phi}{k_0\Lambda}. \tag{14}$$

From eqn (11), we see that if $\left|\frac{A+D}{2}\right| > 1$, then $w_1$ and hence $n_{\text{eff}}$ become complex and hence under such conditions the segmented waveguide does not support any guided mode. This condition is similar to the stability criterion for a lens waveguide[14]. Thus for stable modes in a segmented waveguide, we must have

$$-1 < \frac{A+D}{2} < 1, \text{ or}$$

$$-1 < \cos\ \tilde{\alpha}\gamma - \frac{n_1}{2n_2}\tilde{\alpha}(1-\gamma)\sin\ \tilde{\alpha}\gamma < 1, \tag{15}$$

where $\tilde{\alpha} = \alpha\Lambda$ is the normalised period and $\gamma = d/\Lambda$ is the duty cycle. It is interesting to note that the stability condition given by eqn (15) is independent of the wavelength of operation and depends only on the waveguide parameters.




a b

FIG. 3. Variation of $n_{eff}$ of the fundamental mode with a. duty cycle (for $\Lambda = 5$ μm) and b. period of segmentation (for $\gamma = 0.1, 0.5, 0.9$) for a segmented waveguide with parameters given by eqn (16).

## 3. Numerical results and discussion

Consider a segmented waveguide with the following values of various parameters:

$$\lambda_0 = 0.85\ \mu\text{m},\ \Lambda = 5\ \mu\text{m}$$

$$n_1 = 1.875,\ n_2 = 1.85,\ \alpha = 3 \times 10^{-2}\ \mu\text{m}^{-1}. \quad (16)$$

Figures 3a and b show the variation of the effective index of the fundamental (Gaussian) mode with the duty cycle (for a given period) and the period of segmentation (for three different duty cycles). These results are consistent with the recent experimental results of Thyagarajan *et al.*[9] and Baldi *et al.*[10] and also with the theoretical results of Li and Burke[7] and Weissman and Hardy[8]. Figures 2b and c show the variation of the beam spot size and the wavefront of the fundamental mode along the length of the waveguide. It is obvious that the mode of the segmented waveguide periodically diffracts and refocusses as it propagates through the waveguide. This is very similar to propagation in a lens waveguide. Figure 4 shows the variation of the maximum value of the spot size of the mode (for a given duty cycle) with duty cycle. For smaller duty cycles, since the beam has to propagate over a longer distance in the homogeneous medium, the spot-size of the mode automatically increases to minimise the diffraction effects. In an actual segmented waveguide with finite transverse boundaries, this would also result in lower losses.

The stability diagram for the segmented waveguide is shown in Fig. 5. The shaded area corresponds to the region where $|\frac{1}{2}(A+D)| > 1$ (*i.e.*, where eqn (15) is not satisfied). Hence, the shaded area indicates the region where the segmented waveguide does not support any guided mode. The stability predicted by our analysis is due to the fact that the diffraction effects are either under- or overcompensated by the convergence provided by the parabolic index segments, depending upon the waveguide parameters. Figures 6a and b show the variation of spot size of the Gaussian field along the length of the waveguide corresponding to the stable and unstable regions of segmented waveguide. The waveguide parameters are chosen such that in the first case the diffraction effects

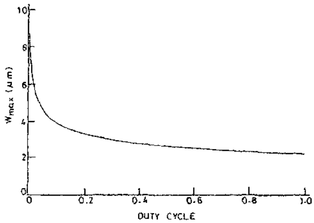



FIG. 4. Variation of maximum spot size as a function of duty cycle for $\Lambda = 5$ μm.

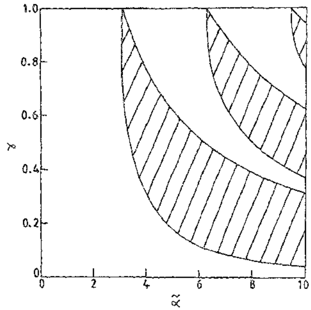



FIG. 5. Stability diagram of a parabolic index segmented waveguide. The shaded area corresponds to regions of unstable operation.

are overcompensated by the convergence (Fig. 6a), and are undercompensated in the second case by the convergence provided by the parabolic index segments (Fig. 6b); thus in the unstable region any input field grows in transverse extent with propagation and eventually leaks out of the waveguide. This aspect is very important for practical realisation of low loss segmented waveguides. From Fig. 5 one can see that for $\tilde{\alpha} \geq 3$ the waveguide is stable for all duty cycles, but for $\tilde{\alpha} \geq 3$ there are regions of nonguidance. Practical parabolic index waveguides have $\alpha \simeq 0.03$. Thus, for stable operation $\tilde{\alpha} < 3$ implies, $\Lambda \leq 100$ μm. Hence, most practical segmented waveguides would correspond to the stable region of operation.

Although our analysis is presented for a segmented waveguide with infinitely extended segments (in the transverse direction), it brings out many of the salient aspects of

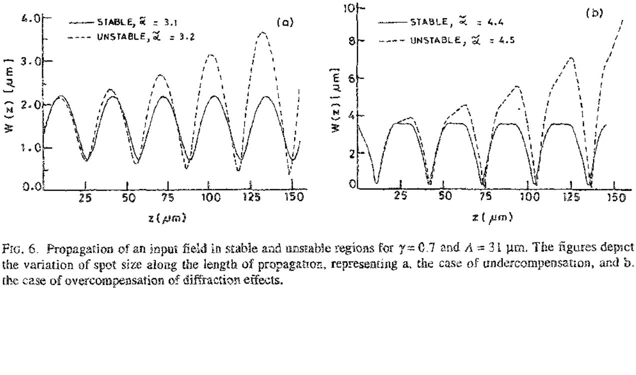



FIG. 6. Propagation of an input field in stable and unstable regions for $\gamma = 0.7$ and $\Lambda = 31$ μm. The figures depict the variation of spot size along the length of propagation, representing a. the case of undercompensation, and b. the case of overcompensation of diffraction effects.

propagation in segmented waveguides. The present analysis can also be easily extended to segmented waveguides with segments consisting of infinitely extended parabolic index variation in both transverse directions. Our analysis may be applicable with little modification to study buried graded index segmented waveguides.

In conclusion, we have modelled segmented waveguides with infinitely extended parabolic index segments in terms of propagation of the mode, and dependence of the effective index on the duty cycle and the period of segmentation using the matrix formulation of Gaussian beam propagation. We have also obtained conditions for the existence of guided modes in such a segmented waveguide.

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