J Indian Inst Sci., Mar.–Apr. 1996, 76, 201–211.

Coupled mode analysis of tapered optical waveguides

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Received on October 25, 1994

Abstract

In integrated optics, tapered optical waveguide-based devices are in need, especially as end-to-end couplers between different devices and optical fibres. Light propagation in tapered waveguides is characterized by mode perturbation and loss arising due to coupling of guided mode power into radiation modes. In this paper, coupled mode theory of dielectric optical waveguides is developed specially to include coupling between guided and radiation modes. A two-dimensional tapered structure with a quadratic taper function along propagation axis and a parabolic refractive index profile for transverse cross-section is solved analytically. Numerical calculations in the case of $LiNbO_3$-based device, for the power variation of the guided mode, reveal a loss in the range 1–5 dB within a propagation distance of 100 μm.

1. Introduction

Optical integrated circuits based on light propagation in dielectric and semiconductor waveguides have gained importance in recent times[1]. Applications of integrated optics include optical communications, optical signal processing and sensing, and optical computing.

Tapered optical waveguides[2] structurally consist of optical fibre components and a dielectric or semiconductor material, possibly fabricated on the same substrate as other integrated optic circuits, and gradually taper according to input–output and efficiency criteria. They find many applications in fibre and integrated optics. Coupling a channel waveguide having a rectangular beam cross-section to an optical fibre supporting a circular shape is a typical example. Further, the dimensions of semiconductor integrated optic circuit waveguides are typically 1–3 μm compared to standard single-mode optical fibres used for communications that are typically 6–10 μm, necessitating tapered optical waveguides for input and output coupling.

Analysis of light propagation in tapered optical waveguides is complicated due to the varying nature of waveguide cross-section along the direction of light propagation. Hence, the analysis of tapered waveguides without resorting to numerical methods is difficult[3–6]. Recently, numerical methods like beam propagation method[4] and approximate analytical methods like conformal mapping[6] are attempted to solve the problem of light propagation in tapered optical waveguides. Simpler and more general methods are still necessary to investigate the problem.

Coupled mode analysis has long been used for many applications in fibre and integrated optics[7]. Recently, it is being improved to include light propagation in nonparallel waveguides[8]. In coupled mode theory, deviation from a standard waveguide is supposed to bring about a coupling among its normal modes. The idea presented in this paper is that, locally along the propagation axis, the tapered waveguide may be supposed to be a deviation to straight waveguide. This helps in investigating light propagation in tapered waveguides. Coupled mode theory appears to provide a simpler and approximately analytical solution to tapered waveguide problem. In this paper, coupled local mode relations are derived between forward propagating guided and radiation modes. This method is applicable to any waveguide cross-section and to any taper function.

2. Coupled mode theory

In this section, a brief outline of the coupled mode theory as applicable to dielectric optical waveguide problems is given[7].

Coupled mode theory is based on eigenmode expansion principle. The problem is to determine the field associated with light propagation along a given waveguide structure. In coupled mode approach, the given waveguide structure in terms of its refractive index profile $n^2(x,y,z)$ is decomposed into a standard known structure like a straight waveguide $n_0^2(x, y)$ and its deviation $\Delta n^2(x,y,z)$.

We suppose $\Psi_p\ (x,y)$ to be the eigenmodes for straight waveguide with the refractive index profile $n_0^2(x,y)$. Then the total field in the given structure, $\Psi(x,y,z)$ is considered as a superposition of the orthonormal modes $\Psi_p\ (x,y)\ \exp(-i\gamma_p z)$ of the standard structure.

$$\Psi(x,y,z)=\sum_{p=1}^{N} U_p(z)\Psi_p(x,y). \tag{1}$$

The functions $U_p(z)$ are the normal mode field weightages and consitute the coupled mode spectrum. The relations between them are the coupled mode equations. They can be derived by various methods like wave equation or reciprocity theorem[7]. In the case of uniform waveguide they result in $U_p = U_p^\circ\ \exp(-i\gamma_p z)$ where U_p° is the mode spectrum at $z = 0$. The general form of the coupled mode equations is as follows:

$$\frac{dU_p}{dz}=-i\gamma_p U_p-\sum_{q=1}^{N} K_{pq}\, U_q, \tag{2}$$

where K_{pq} are the coupling coefficients and contain perturbation terms, and hence the various parameters of the given structure. They are given by

$$K_{pq}=\frac{k_0^2}{2\gamma_p}\iint \Psi_q^* \Delta n_q^2 \Psi_p dS. \tag{3}$$

Here, γ_p are the propagation constants of the normal modes and $k_0 = 2\pi/\lambda$. By knowing the initial mode spectrum U_p°, the set of coupled mode equations (2) can be solved for $U_p(z)$ giving the interaction among the modes due to perturbation. Power calculations can be done once the field patterns are determined.

The existing coupled mode theory of parallel waveguides is not applicable to tapered waveguides as it is because it uses an unperturbed straight waveguide as standard structure throughout the propagation distance. In the case of tapered waveguide the local normal mode patterns and their number vary. In addition, radiation modes may play an important role. In the next section, a set of coupled mode equations are derived as applicable to tapered type of optical waveguides.

3. Tapered waveguide analysis

A tapered waveguide is described in terms of the taper function $S(z)$ denoting its cross-sectional geometry along the propagation axis, z. A typical tapered waveguide structure is shown in Fig. 1. Of special interest are the initial and final cross-sections. $n^2(x, y; \xi)$ describes the transverse refractive index profile at $z = \xi$. The local normal modes at any ξ are those of a uniform waveguide with the same cross-sectional features. The mode fields $E(x,y,\xi)\exp(-i\gamma\xi)$ satisfy the corresponding wave equation

$$\nabla_T^2 E + (k_0^2 n^2 - \gamma^2)E = 0, \tag{4}$$

where ∇_T^2 is the transverse Laplacian and $\gamma(\xi)$ represents the local propagation constant. The domain of γ consists of a discrete set of values for guided modes and a continuous set for radiation modes.

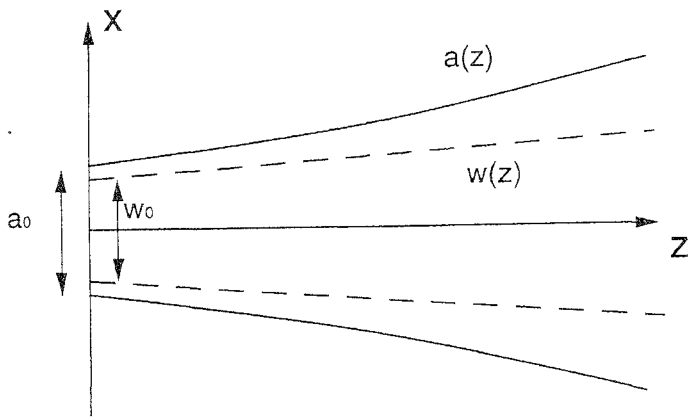

FIG. 1. Tapered waveguide structure.

Here, assume that the fields vary slowly with taper, *i.e.*,

$$\left|\frac{\partial E}{\partial \xi}\right| << \left|\gamma E\right|, \tag{5}$$

as can be noted from the following equation for z-variation of E,

$$\frac{\partial^2 E \exp(-\mathrm{i}\gamma\xi)}{\partial \xi^2} = (\frac{\partial^2 E}{\partial \xi} - 2\mathrm{i}\gamma \frac{\partial \mathrm{E}}{\partial \xi} - \gamma^2 E) \exp(-\mathrm{i}\gamma\zeta). \tag{6}$$

If the local guided and radiation modes E_g and E_r at any point on propagation axis $z = \xi$ are known, the total field E_t can be expressed as their sum owing to their orthogonality. Considering a single guided mode and a spectrum of radiation modes, the total field is given by

$$E_t(\xi) = u_g(\xi)E_g(\xi) + \int_\Gamma u_r(\xi, \gamma_t)E_t(\xi, \gamma_t)d\gamma_t\,, \tag{7}$$

where Γ, when loss also is included, is a complex contour for radiation mode propagation constants γ_r, and the dependence of E fields on x,y coordinates is implied.

The variables $u_g(\xi)$ and u_t (ξ,γ_t) represent the interaction between modes and the relation between them is to be determined now. As the wave propagates to another point $z = \xi' = \xi + \Delta\xi$, the new functions u_g (ξ') and $u_t(\xi,\ \gamma_t)$ can be determined by decomposing the total field $E_t(\xi')$ as

$$u_g(\xi') = \int\int_A E_t(\xi')E_g^*(\xi')\mathrm{d}S; \tag{8}$$

$$u_r(\xi', \gamma_r') = \int\int_A E_t(\xi')E_t^*(\xi', \gamma_t')\mathrm{d}S. \tag{9}$$

Here the integrals extend over the entire x–y space A, and * indicates complex conjugate.

The local normal modes satisfy the following orthornormal equations

$$\int\int_A E_g E_{g'}^*\ \mathrm{d}S = \delta_{gg'}; \tag{10}$$

$$\int\int_A E_t E_{r'}^*\ \mathrm{d}S = \delta(r - r'), \tag{11}$$

where $\delta_{gg'}$ is the Kronecker delta function and $\delta(r–r')$ is the Dirac delta function. In the limit as $\Delta\xi \to 0$, the functions u_g, u_r, E_g and E_t at ξ and ξ' can be related as follows:

$$f(\xi + \Delta\xi) = f(\xi) + \Delta\xi \frac{\partial f}{\partial \xi}. \tag{12}$$

The total field $E_t(\xi')$ at $z = \xi$ can be approximated by the field obtained by propagating the individual modes from ξ to ξ'

$$E_t(\xi+\Delta\xi) = u_g(\xi)E_g(\xi)\exp(-\mathrm{i}\gamma_g\Delta\xi)\int_\Gamma u_r(\xi,\gamma_r)E_r(\xi,\gamma_r)\exp(-\mathrm{i}\gamma_r\Delta\xi)d\gamma_r . \quad (13)$$

Substituting (13) in (8) and using (12); we get

$$u_g + \Delta\xi\frac{\mathrm{d}u_g}{\mathrm{d}\xi} = \int\int_A [u_g E_g + \int_\Gamma u_r E_r \mathrm{d}\gamma_r][E_g^* + \Delta\xi\frac{\partial E_g^*}{\partial\xi}]\mathrm{d}S. \quad (14)$$

Simplifying this and making $\Delta\xi \to 0$, we get

$$\frac{\mathrm{d}u_g(\xi)}{\mathrm{d}\xi} = [\kappa_{gg}(\xi,\gamma_g) - \mathrm{i}\gamma_g]u_g(\xi) + \int_\Gamma \kappa_{gr}(\xi,\gamma_r)u_r(\xi,\gamma_r)\mathrm{d}\gamma_r . \quad (15)$$

For radiation modes, substituting (13) in (9) and using (12), we get

$$u_r + \Delta\xi\frac{du_r}{d\xi} = \int\int_A [u_g E_g + \int_\Gamma u_r E_r d\gamma_r][E_r^* + \Delta\xi\frac{\partial E_r^*}{\partial\xi}]\mathrm{d}S. \quad (16)$$

Simplifying and making $\Delta\xi \to 0$, we get

$$\frac{\partial u_r(\xi,\gamma_r)}{\partial\xi} = \kappa_{rg}(\xi,\gamma_r)\,u_g(\xi) + \int_\Gamma [\kappa_{rr}(\xi,\gamma_r,\gamma_r') - \mathrm{i}\gamma_r]u_r(\xi,\gamma_r')\mathrm{d}\gamma_r' . \quad (17)$$

Equations (15) and (17) constitute the coupled mode equations, where the coupling coefficients are given by

$$\kappa_{gg}(\xi) = \int\int_A E_g(\xi)\,.\,\frac{\partial E_g^*(\xi)}{\partial\xi}\mathrm{d}S; \quad (18)$$

$$\kappa_{gr}(\xi) = \int\int_A E_g(\xi)\,.\,\frac{\partial E_r^*(\xi)}{\partial\xi}\mathrm{d}S; \quad (19)$$

$$\kappa_{rg}(\xi) = \int\int_A E_r(\xi)\,.\,\frac{\partial E_g^*(\xi)}{\partial\xi}\mathrm{d}S; \quad (20)$$

$$\kappa_{rr'}(\xi) = \int\int_A E_r(\xi)\,.\,\frac{\partial E_{r'}^*(\xi)}{\partial\xi}\mathrm{d}S. \quad (21)$$

4. Parabolic index taper

As an illustrative example consider the two-dimensional taper waveguide shown in Fig. 1 and 2. The waveguide supports a single-guided mode and a continuum of radiation modes. In this paper, our aim is to choose an appropriate structure such that the local normal modes have Gaussian and sinusoidal profiles for guided and radiation modes, respectively. Consider the refractive index profile given by

$$n^2 = \begin{cases} n_1^2\left\{1 - 2\Delta\left(\dfrac{x^2}{a^2}\right)\right\} & |x| < a/2 \\ n_2^2 & |x| \geq a/2 \end{cases} \quad (22)$$

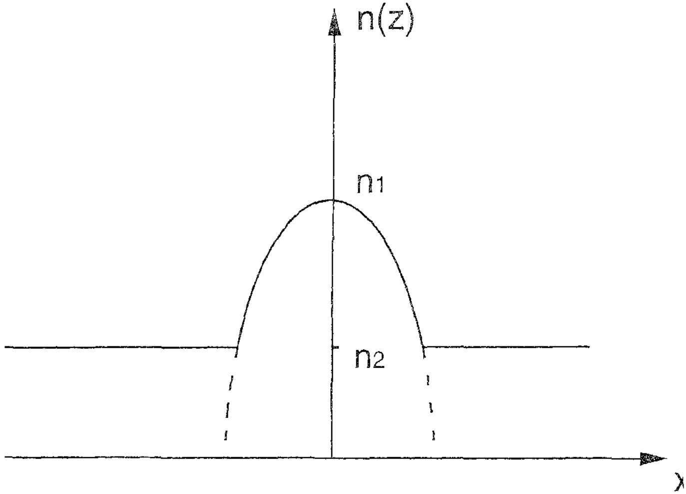

FIG. 2. Refractive index profile.

where $\Delta = (n_2 - n_1)/n_1$, n_1 and n_2 are the refractive indices at the centre of core and in the cladding, respectively, and $a(z)$ is the width of the waveguide.

For such a profile the guided modes can be approximated by Hermite Gauss functions inside the core and exponentially decaying functions in the cladding[10]

$$E_g(x) = \begin{cases} AH_m(x)\exp(-x^2/w^2) & |x| < a/2 \\ A'\exp(-\alpha_2|x/a|) & |x| \geq a/2 \end{cases} \tag{23}$$

where $\alpha_2 = \sqrt{\gamma_g^2 - k_0^2 n_2^2}$ and $A' = A\ \exp(-\alpha_1)H_m(a)\exp(-a^2/4w^2)$. For the dominant mode $H_0(x) = 1$ and so the guided mode has a Gaussian field distribution. The propagation constant γ_g and the normalization factor A may be given as follows:

$$\gamma_g = n_1 k_0 \left(1 - \frac{\sqrt{2\Delta}}{ak_0 n_1}\right)^{1/2}; \tag{24}$$

$$A = (2/\pi)^{1/4}/\sqrt{w}. \tag{25}$$

The beam width function is related to the actual waveguide width as

$$w^2(z) = \frac{2a(z)}{\sqrt{2\Delta}n_1 k_0}. \quad (26)$$

In contrast to guided modes, the radiation modes are oscillatory in nature both in core and cladding. They may be represented in the following form:

$$E_r \begin{cases} A_1 \exp(-i\gamma_1 x) + A_2 \exp(+\gamma_1 x) & |x| < a/2 \\ B \exp(\pm \gamma x) & |x| \geq a/2 \end{cases}. \quad 27)$$

Here, $\gamma_1 = \sqrt{k_0^2 n_1^2 - \gamma_r^2}$, $\gamma = \sqrt{k_0^2 n_2^2 - \gamma_r^2}$, and the radiation mode propagation constant varies from 0 to $k_0 n_2$. The constants A_1 and A_2 are related by matching the fields at the boundaries

$$A_1 = (\frac{\gamma_1 + \gamma}{2\gamma_1}) B \exp\{i(\gamma - \gamma_1)a/2\}; \quad (28)$$

$$A_2 = \left(\frac{\gamma_1 - \gamma}{2\gamma_1}\right) B \exp\{i(\gamma - \gamma_1)a/2\}. \quad (29)$$

B is obtained by normalizing the radiation mode field, while noting that the fields extend to infinity[10]

$$B = (\gamma/2\pi)^{1/2}. \quad (30)$$

Now, for the selection of a suitable taper function, we consider the relation between the guided mode width (beam width $w(z)$) and the actual waveguide width $a(z)$ (26). So a quadratic taper profile would result in a linear beam width profile. Thus, the taper profile is chosen as

$$a(z) = a_0 (1 + s.\frac{z}{w_0})^2, \quad (31)$$

so that the beam width is linear

$$w(z) = w_0 + s.\ z. \quad (32)$$

Here, a_0 and w_0 are the initial waveguide width and the initial beam width, respectively, and s is the slope of the beam width taper function.

To evaluate the coupling coefficients for the tapered structure we need to substitute the expressions for the mode fields (23, 27) into (18–21) and simplify. Thus, the evaluation is simplified as outlined below

$$k_{gg}(z) = \int_{-\infty}^{+\infty} E_g \frac{\partial E_g^*}{\partial z} dx$$

$$= \int_{-\infty}^{+\infty} A \exp(-x^2/w^2) \exp(-i\gamma_g z) \frac{\partial}{\partial z} [A \exp(-x^2/w^2)\ \exp(i\gamma_g z)] dx$$

$$= i\gamma_g - \frac{s}{w} \tag{33}$$

$$k_{rr'}(z) = \int_{-\infty}^{+\infty} E_r \frac{\partial E_{r'}}{\partial z} \mathrm{d}x$$

$$= \int_{-\infty}^{+\infty} B\exp(\mathrm{i}\gamma x)\exp(-\mathrm{i}\gamma_r z)\frac{\partial}{\partial z}[B\exp(-\mathrm{i}\gamma x)\exp(\mathrm{i}\gamma_{r'z})]\mathrm{d}x$$

$$= \mathrm{i}\gamma_r \delta(\gamma_r, \gamma_{r'}) \tag{34}$$

$$k_{gr}(z) = \int_{-\infty}^{+\infty} E_g \frac{\partial E_r}{\partial z} \mathrm{d}x$$

$$= \int_{-\infty}^{+\infty} A\exp(-x^2/w^2)\exp(-\mathrm{i}\gamma_g z)\frac{\partial}{\partial}[\exp(-\mathrm{i}\gamma x)[\exp(\mathrm{i}\gamma_r z)]\mathrm{d}x$$

$$= (\mathrm{i}\gamma_r)\left(\frac{1}{2\pi}\right)^{1/4}\sqrt{w\gamma}\exp(-\frac{w^2\gamma^2}{4})\exp\{-\mathrm{i}(\gamma_g - \gamma_r)z\}. \tag{35}$$

$$k_{rg}(z) = \int_{-\infty}^{+\infty} E_r \frac{\partial E_g}{\partial z} \mathrm{d}x$$

$$= \int_{-\infty}^{+\infty} B\exp(\mathrm{i}\gamma x)\exp(-\mathrm{i}\gamma_r z)\frac{\partial}{\partial z}[A\exp(-x^2/w^2)\exp(\mathrm{i}\gamma_g z)]\mathrm{d}x$$

$$= \left(\tfrac{1}{2\pi}\right)^{1/4}\sqrt{w\gamma}\left(\mathrm{i}\gamma_g - \frac{3s}{2w} + \frac{sw\gamma^2}{2}\right)\exp\left(-\frac{w^2\gamma^2}{4}\right)\exp\left\{-\mathrm{i}(\gamma_r - \gamma_g)z\right\} \tag{36}$$

where $\delta(\gamma_r, \gamma_r')$ is the Dirac delta function which vanishes when the radiation mode propagation constants are different.

To obtain the variation of guided and radiation mode fields by solving (15,17) using (33–36) we note the phase variation of u_g and u_r and seek the solution in the form

$$u_g(z) = G(z)\exp(\mathrm{i}\gamma_g z) \tag{37}$$

$$u_r(z) = R(z)\exp(\mathrm{i}\gamma_r z). \tag{38}$$

Substituting (15,17) and changing the independent variable to $w = w_0 + sz$, we get

$$\frac{\mathrm{d}G}{\mathrm{d}w} = -\frac{s}{w}G + \int_0^{k_o n_2} K(w)R d\gamma_r \tag{39}$$

$$\frac{\mathrm{d}R}{\mathrm{d}w} = (\mathrm{i}\gamma_r)K'(w)G \tag{40}$$

where

$$K(w) = (1/2\pi)^{1/4}\sqrt{w\gamma}\{\mathrm{i}\gamma_g - 3s/2w + sw\gamma^2/2\}\exp(-w^2\gamma^2/4) \tag{41}$$

$$K'(w) = (1/2\pi)^{1/4}\sqrt{w\gamma}\exp(-w^2\gamma^2/4). \tag{42}$$

From these we can get a second-order differential equation for the variation of guided mode field by differentiating (39) with respect to w and using (40). Thus, we get

$$\frac{d^2G}{dw^2} + \frac{1}{w}.\frac{dG}{dw} - \frac{1}{w^2}G = p(w)G, \tag{43}$$

where the factor $p(w)$ is as follows:

$$p(w) = \int_{-k_0n_2}^{k_0n_2} (i\gamma_r/s^2)\,(w\gamma/\sqrt{2\pi})$$

$$(i\gamma_g - 3s/2w + sw^2\gamma^2/2)\ \exp(-w^2\gamma^2/2)Gd\gamma_r. \tag{44}$$

We note that this contains factors of the form $\exp(-w^2\gamma^2/4)$ and so represents a small perturbation to the guided mode fields due to coupling to radiation modes. So this may be approximated by a constant average perturbation defined as follows:

$$\tilde{p} = (1/L)\int_0^L p(w)dw, \tag{45}$$

where L is the length of the taper.

Equation (43) may be solved for $G(z)$ either directly or numerically or through a perturbation method subject to the initial condition that $G(0) = 1$. Thus, we get

$$G(z) = \left\{\frac{w_0}{w_0 + sz}\right\}^{1+\tilde{p}}. \tag{46}$$

The solution of coupled mode equation for the radiation mode fields $R(z)$ can be readily obtained using the solution for guided mode fields,

$$R(z, \gamma) = \int K'(w)(i\gamma_r)G(z)dz. \tag{47}$$

5. Results and discussion

The derivation is general except that the waveguide supports only one guided mode. Many modes can be easily included resulting in their coupled mode equations. The nature of the taper function is indirectly involved in the expression for the coupling coefficients. In solving for a particular taper geometry the coupling coefficients may be simplified to include the derivatives of refractive index profile along the propagation direction. The results of coupled mode theory for tapered optical waveguides derived here may be contrasted with that of straight waveguides[7,10].

The coupled mode equations (15,17) relate u_g and u_r, the guided and normal mode spectra. In the analysis while only one guided mode is considered, all the radiation modes are taken into account. The radiation mode spectrum is represented by $u_r(\xi, \gamma_r)$.

Hence, eqn (15) relates a single component u_g to many radiation modes u, In contrast, eqn (17) relates each radiation mode to the guided mode and all radiation modes. So, the coupled mode equations are not symmetric in this sense. This is also apparent when the coupled mode equations are rewritten with the radiation mode spectrum discretized.

Similar arguments apply to the expressions for coupling coefficients (18–21). For example, k_{gr} represents the coupling coefficient between the guided mode γ_g and each of the radiation modes γ_r, and is a function of γ_r. In contrast, K_{rr} represents coupling between two radiation modes, say γ_{r1} and γ_{r2}, and is a function of both.

The coupling coefficients and the coupled mode equations are z-dependent, showing that they are inhomogeneous. Thus, it can be inferred that the coupling is not a periodic process in the case of tapered waveguides.

The theory may be extended and improved in many ways; for example, by including abrupt tapers. One important inclusion that may prove necessary is that of reflected modes, both guided as well as reflected. Comparisons may be made with other methods, notably beam propagation method which is well known to attack nonparallel waveguide problems.

In the example, only a single-moded taper is considered to give importance to radiation modes. To an approximation we note that the guided mode power decays reciprocally along the propagation axis. Calculations have been made using the parameters corresponding to $LiNbO_3$ waveguide devices: $\lambda = 1.3\ \mu m$, $n_2 = 2.2$, $\Delta = 1\%$ and $a_0 = 3\ \mu m$. We assume a radiation perturbation of $\tilde{p} = 1\%$. In the graph of Fig. 3, $\theta = 1°$, $0.5°$, $0.1°$ correspond to slope of the beam width variation ($w_0 = 2\ \mu m$) ($s = \tan(\theta)$). We find that the power falls to over -5dB within a propagation distance of $100\ \mu m$ for $\theta = 1°$, but less than 1 dB for $\theta = 1°$.

6. Conclusion

In this paper, a new simple and general method to analyse tapered optical waveguide is suggested. An important feature of it is that the parameters κ_{pq} are functions of z. The coupled mode equations can be solved for simple problems like tapered fibres and tapered slab waveguides. In addition, the analysis becomes even simpler in such problems where only the dimensions change and not cross-sectional shape. The method can be easily extended to tapers with many guided modes, including reflected modes. As an example, a two-dimensional graded index tapered waveguide is solved analytically. In future studies, we hope to compare the method using the results from a standard numerical method like the beam propagation method.

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