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Intuitive modeling of inter-causal relationships based on cognitive experiments

VADHRI SRINIVAS*, B. SHEKAR# AND M. R. RAO†
Quantitative Methods and Information Systems Area, Indian Institute of Management Bangalore, Bannerghatta Road, Bangalore 560 076, India.
*<vsr@iimb.ernet.in>; # <shek@iimb.ernet.in>; †<mrr@iimb.ernet.in>

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Abstract

In this paper we address the issue of representation of cognitive processes of managers in organizations. This purpose derives from a need felt by theories that address strategic behavior of firms. We propose to represent the cognitive processes of managers (termed as beliefs, values, etc., in organizational science theory) as qualitative probabilistic networks (QPNs). QPNs have a topology similar to cognitive maps (directed-acyclic-graphs with qualitative signs attached to edges). Perceptions (concepts) of top management are represented as nodes, and beliefs about environmental uncertainties are quantified as probability estimates. We propose intuitive models on these QPNs. These models result from studying human behavior under uncertainty with the help of psychological experiments. During the experimentation we observe two patterns of inference that subjects resort to, while inferencing under uncertainty. We incorporate these patterns into the existing theories of inter-causality. This is done by defining patterns with the help of certain probabilistic criteria. Finally, we demonstrate the applicability of these intuitive patterns in quantitative belief propagation in a cognitive map abstracted from the annual report of a company from the Indian automobile industry.

Keywords: Qualitative probabilistic networks, belief propagation, stochastic simulation, cognitive maps, verbal protocols, inter-causal reasoning.

1. Introduction

The study of strategic behavior to environmental changes is very topical. Historically, issues related to this have been addressed by two different streams of theories in strategic management. They are: the industrial organizational (IO) economics stream[1], and the behavioral stream[2]. The distinguishing characteristic between these two approaches is primarily the type of information attended to (structured or unstructured), and the focus on the sources of information (internal or external to the organisation).

1.1. *Theories on strategic behavior of firms*

(i) *IO Theory of firm behavior*: The IO stream emphasizes on structured information like market shares, profitability, etc., in analyzing firm behavior/performance[1, 2]. The underlying premise here is that the market or the industry imposes selective pressures to which the firm must respond. In this approach, firms are assumed to be rational with an objective of allocating scarce resources to alternative ends, to maximize profits. However, managerial 'proactiveness'(or mindsets)-based competencies (or drawbacks) are not given any serious consideration. Firm-level behavior like limits of bounded rationality in humans[4], technological uncertainties, constraints and mobility factors, information asymmetries and other things are ignored[5].

In other words, the strategic behavior of a firm is analyzed with reference to the firm's performance in the industry (*i.e.*, external to the organisation).

(ii) *The behavioral approach*: Behavioral researchers argue that any firm's strategic behavior is the final form of managerial thought processes. Thus, reactions to changes in the environment (say, launch of new product) are the final outcome of managerial causal understanding of the environment. However, information on managerial thought processes is highly unstructured.

In broad terms, this stream of research suggests that cognitive processes of managers influence market performance. This approach is developed on the limits of bounded rationality model in individuals, advocated by Simon[4]. Cyert and March[2] developed the behavioral theory of firms based on this model. According to them, an organization is an adaptively rational system that learns from experience. The key assumption here is that firms need not be rational, or, in other words, profit maximizing need not be the sole objective of a firm. The underlying factors that have an important role in firm behavior are issues like 'organizational slack'[2].

Thus, the main emphasis in this approach is on the cognitive processes that take place in the minds of managers which affect the strategic behavior of firms. Hence, any organization's strategic response is a reflection of change in the top management's mental models (cognitive processes), with respect to significant changes in the environment[6,7]. Top managers, with their perceptible filters, may generate unique information that enables them to effectively interpret the firm's environment with respect to opportunities and threats. In other words, the central thesis of this approach is 'cognitive processes' direct strategic behavior. Researchers[8–10] in strategic management of late have emphasized on this missing role of beliefs, values and culture of an organisation in evaluating firm performance (which have unstructured information content). The above observations can be conceptually represented as in the framework given in Fig. 1.

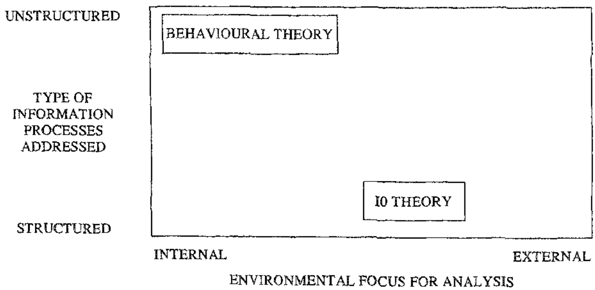

FIG. 1. Conceptual framework

Thus, to analyze strategic behavior, an important prerequisite that one needs to address is the issue of 'representation of information'. There are various mathematical tools and techniques to represent structured information. Modern-day decision-support systems and spread sheets have provisions to deal with these structured information[11]. Various studies[1, 3] have used these representation tools to understand strategic behavior. However, little research has been done on representational issues of unstructured information which is central to understanding strategic behavior of firms. In the following section we cite the research done on representation of unstructured processes in various fields.

1.2. *Representation of unstructured processes*

Representing qualitative information like human decision-making processes, values, beliefs, etc., has been a challenging task for researchers interested in strategic management [8, 9]. Also, many decision problems encountered in organizational decision making are unstructured in the sense that predetermined algorithms are not available for their solution[12]. According to Mintzberg *et al.*[13] decision processes in organizations are primarily unstructured. Prior to addressing the knowledge representation issue, we consider the process of knowledge elicitation.

One of the major difficulties that has been stated by researchers who are interested in knowledge representation is the process of explicating knowledge. For most human activities there is little formal documentation of relevant knowledge and procedures. Thus, the important step in the process of studying representation of cognitive processes of managers is extraction of relevant knowledge from the managers.

The process of knowledge elicitation, however, has been addressed by cognitive psychologists whose primary objective is to understand the human mind. Hoffman[14] compares various elicitation methods with respect to the number of knowledge elements elicited per minute in humans. Burton[15] compares four methods (formal interviews, protocol analysis, laddered grid, and card sort) to elicit information about how undergraduate geology students distinguished various types of rocks. It has been found that protocol analysis produced fewer rules than the three other elicitation methods.

Researchers interested in modeling cognitive processes of humans found that mathematical models developed to capture human behavior and the true cognitive processes mediating performance have been dissociating[16]. However, mathematical regression models have remarkably been able to reproduce the final outcome. The experiments of Einhorn *et al.*[17] show that how rules extracted from think-aloud protocols on subjects making judgements could predict observed judgements similar to that of linear mathematical models. Larcker and Lessig[18], however, found that process models were superior in reproducing observed judgements in comparison with regression models. Linear models fail to give the 'intermediate steps' (the process) involved in arriving at the final outcome.

In the research involved in strategic management, various representation techniques are being used to capture the cognitive processes in managers (see Fiol and Huff[19] for greater details). Mapping of cognitive processes based on causal assertions (termed as cognitive maps) made by managers is one such popular technique.

The other important issue, in addition to those stated above, which is essential to representing strategic behavior of firms is the representation of uncertainty. According to researchers in organizational science, beliefs of top management include perception of top management with respect to environmental uncertainties. Traditionally, representation of uncertainty has been in the form of probabilistic networks. In this paper, we adapt this approach for analyzing strategic behavior of firms.

1.3. *Organisation of the paper*

In the following section, we give a brief overview of cognitive maps, Bayesian belief revision and qualitative probabilistic networks (QPNs), and their related concepts. We then define intuitive models which are based on an experiment done on understanding decision process in human beings. Here we detail the protocol experiments conducted, and the associated observations made from the transcripts. We then propose intuitive models which are directed by certain probabilistic criteria. We incorporate the intuitive models into the general characterizations for inter-causal reasoning as detailed by Druzdzel and Henrion[20]. We also elicit the applicability of these intuitive models for belief propagation. Further, we demonstrate quantitative belief propagation scheme developed on the basis of proposed intuitive models. In other words, we demonstrate the applicability of models based on human intuition in the construction of belief networks, and in representing the decision processes of managers. Finally, we conclude by summarizing the main contributions of this paper.

2. Network representation of beliefs and uncertainty

2.1. *Cognitive maps*

Cognitive maps provide graphical descriptions of the unique ways in which individuals view a particular domain[21,22]. The term 'cognitive map' has been used to describe several forms of diagrammatic representation of an individual's cognition. Causal mapping is one such technique[19]. These are essentially network representation of beliefs of individuals.

Technically, cognitive maps are directed graphs with signed edges. Formally, a cognitive map, M, is represented as a pair (C,E), where C is a set of nodes representing concepts, and E, a set of signed edges. The possible signs, e, attached to an edge are usually '+' and '–', although sometimes edges with sign '0' are also present. For more details on the semantics of the cognitive maps, refer Wellman [23].

The intention behind drawing a cognitive map is to describe an individual's conscious perception of the environment. However, the aim is not to map an individual's entire set of beliefs, or to present a model that simulates actual cognition. Typically, in practice, the map is restricted to a particular domain. This is done by filtering in details that relate to specific situations or detailed instances of the individual's experience, from a general set of observations. Typically, cognitive maps have as many as 100 such conceptual nodes.

Barr *et al.*[6] have used cognitive maps of top management to understand the process of strategic behavior of firms. In their research, they have tried to give causal explanations to firms' behavior with respect to decision process of top management. Cognitive maps are of potential interest to organization theorists because they can be used for graphical display of the process of strategic behavior. Barr *et al.*[6] and Stubbart and Ramprasad[24] have used cognitive mapping technique to identify key assertions, and represent them as a directed graph. In their study, these maps have been used to abstract changing managerial beliefs and their impact on strategic behavior of firms.

2.2. *Representation of uncertainty*

An important factor that is necessary to be looked at in capturing the decision process in managers is the representation of 'uncertainty'[25]. Complex systems like organizations and medical diagnostic systems exist in highly uncertain environments. This environmental uncertainty must be captured while studying the strategic behavior of firms. To study the strategic behavior of firms, capturing the top managements' subjective estimates of uncertainty along with representation of cognitive processes is essential.

Traditionally, representation of uncertain information has been a great challenge to computer scientists working in the field of artificial intelligence. Probabilistic models and fuzzy logic-based models are some of the important representation schemes that have been proposed. Amongst the various probabilistic models, network-based representation is one of the most popular forms used in various applications. This is largely due to: (i) the inherent representation of Bayesian conditional independence in the topology itself, and (ii) the pictorial representation of the decision problem, thus eliciting the parameters explicitly. An important form of probabilistic networks is Bayesian belief networks (BBNs). For a complete overview of probabilistic network models, refer Pearl[26].

Cognitive maps are similar in topology to probabilistic networks. However, cognitive map representation does not have provision to capture uncertainty. A variation of BBNs is QPNs[27]. These networks need a graphical representation of domain knowledge which captures probabilistic conditional dependencies.

2.2.1. *Bayesian belief networks*

Bayesian belief networks (BBNs) have been developed as tools for capturing coherent probabilistic representations of uncertain knowledge[28]. Historically, BBN models were developed to represent a subjective view of a system elicited from a decision maker or domain expert. These representations are used when causal dependencies in a model are probabilistic. BBNs are represented as directed acyclic graphs. Nodes in these graphs are connected by edges which capture the conditional dependency. In addition, each node has a probability estimate attached to it. The probabilistic estimates in a BBN represent the subjective estimates of the domain expert. Henrion[28] has used this scheme to represent knowledge of apple tree root disorders.

If the nodes of a probabilistic network follow a distribution and the causal dependence is constrained by a qualitative sign (like in cognitive maps), we obtain, QPNs[27]. QPNs are signed directed acyclic graphs, with nodes denoting concepts and signed edges denoting abstract causal relations. Wellman[23] has demonstrated the efficient inferencing capability of QPNs.

2.3. *Qualitative probabilistic networks*

Formally, a QPN G can be represented as an ordered pair (V, Q), where V is a set of variables (represented as nodes in the graph) and Q, a set of qualitative relations among the variables[27]. All qualitative relations are expressed by signs {+, –, 0, ?}, the last sign, ?, denoting ambiguity. These networks support two types of qualitative relationships: qualitative influence and qualitative synergy. Qualitative influence captures the sign of direct influence between two variables, and corresponds to arcs in a belief network. We reproduce the definition of qualitative influence here. The reader is referred to Wellman[27] for further details and clarifications. All variables are assumed to be discrete variables, unless otherwise stated.

Definition 1 (*qualitative influence*): We say that a positively influences c, written $S^+(a, c)$, iff for all values $a_1 > a_2$, c_0 and X (where X is vector representing unspecified nodes in the network)

$$\Pr(c \geq c_0 / a_1, X) \geq \Pr(c \geq c_0 / a_2, X). \tag{1}$$

Vector X captures the status of all the irrelevant ancestor nodes (*i.e.*, nodes which are not under consideration for discussion). The inequality gets reversed for $S^-(a,c)$, and becomes equal for $S^0(a, c)$.

Note: Here we elaborate on the notations used in the above inequalities. Notation, $\Pr(c \geq c_0 /a, X)$ implies cumulative probability distribution given a and X. Thus, the inequality (1) is to be viewed in the first-order stochastic dominance sense[27].

We demonstrate the concept of qualitative influence using an example. Consider the graph given in Fig. 2. Assume that node c can take three values in the ordinal scale: c_0, c_1, and c_2 (where $c_0 \geq c_1 \geq c_2$). Similarly, let node a take two values: a_1 and a_2 (where $a_1 \geq a_2$). Suppose,

$\Pr(c = c_0/a_1) = 0.5$, $\Pr(c = c_1/a_1) = 0.3$, $\Pr(c = c_2/a_1) = 0.2$, and
$\Pr(c = c_0/a_2) = 0.4$, $\Pr(c = c_1/a_2) = 0.3$, $\Pr(c = c_2/a_2) = 0.3$.

Then, we have, according to definition 1,

$\Pr(c \geq c_0/a_1) = 0.5$, $\Pr(c \geq c_1/a_1) = 0.8$, $\Pr(c \geq c_2/a_1) = 1.0$, and
$\Pr(c \geq c_0/a_2) = 0.4$, $\Pr(c \geq c_1/a_2) = 0.7$, $\Pr(c \geq c_2/a_2) = 1.0$.

These satisfy the conditions for $S^+(a,c)$ (in the first-order stochastic dominance sense). Thus, we can say that there is a positive qualitative influence between nodes a and c.

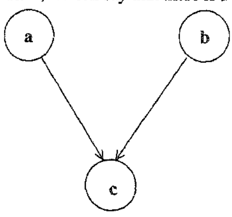

FIG. 2. An example of qualitative probabilistic network (two-cause network).

2.3.1. *Qualitative synergies*

The qualitative synergy property among the variables in a QPN takes two forms: additive synergy and product synergy. Additive synergy is used with respect to two direct ancestors of a variable. Positive additive synergy, $Y^+(\{a, b\}, c)$, captures the property that the joint influence of a and b on c is greater than the sum of their individual influences. Negative additive synergy, Y^-, and zero-additive synergy, Y^0, are defined analogously. We restate the definition from Wellman and Henrion[29] here.

Definition 2 (*Additive synergy*): Let a, b and x be the predecessors of c in a QPN. Variables a and b exhibit *negative additive synergy* with respect to particular value c_0 of c, written $Y^-(\{a, b\}, c_0)$, if for all $a_1 \geq a_2$, $b_1 \geq b_2$, and x

$$\Pr(c \geq c_0/a_1, b_1, x) + \Pr(c \geq c_0/a_2, b_2, x) \leq \Pr(c \geq c_0/a_1, b_2, x) + \Pr(c \geq c_0/a_2, b_1, x). \qquad (2)$$

Positive additive synergy, Y^+, and zero-additive synergy, Y^0, are defined by substituting $\geqq$ and $=$, respectively, for $\leq$ in (2).

The intuitive appreciation behind framing inequality, (2), is to capture the property that the joint influence of two causes when present is greater than the sum of individual influences[27].

Product synergy is defined in Henrion and Druzdzel[30]. This synergy has been used to derive sufficiency conditions for explaining away[29]. 'Explaining away' is a common form of inter-causal reasoning. It captures situations where an observed effect and an increase in the probability of occurrence of one of the causes brings down the likelihood of occurrence of all other causes. The generalizations on inter-causality, as propounded by Wellman and Henrion[29], primarily deal with two-cause single-effect nodes. In multi-cause nodes, the assumption is that irrelevant causes are instantiated (observed) to a particular value. We reproduce their definitions here:

Definition 3 (Product synergy I): Let a, b and x be the predecessors of c in a QPN. Variables a and b exhibit negative product synergy with respect to particular value c_0 of c, written $X^-(\{a, b\}, c_0)$, if for all $a_1 \geq a_2$, $b_1 \geq b_2$, and x

$$\Pr(c_0/a_1, b_1, x) * \Pr(c_0/a_2, b_2, x) \leq \Pr(c_0/a_1, b_2, x) * \Pr(c_0/a_2, b_1, x). \qquad (3)$$

Positive product synergy, written X^+, and zero product synergy, written X^0, are defined by substituting $\geq$ and $=$, respectively, for $\leq$ in (2). The irrelevant ancestor nodes are represented by x in the above equation.

Note: Here it may be noted that notation $\Pr(c_0/...)$ denotes a point estimate (unlike as stated earlier as a cumulative distribution given for additive synergy). Also, it may be noted that product synergy is defined with respect to each value assumed by common effect node c. There are, thus, as many product synergies as the number of values that variable c can assume. If c is a binary variable (say a Boolean variable), there are two product synergies, one for C (true) and another for C′ (false). While additive synergy has found applicability in reasoning in planning and monotone decision policies[27], product synergy has found applications in belief propagation[31].

Druzdzel and Henrion[20] have proved that an uninstantiated irrelevant cause node (x in the above case) affects the inter-causal relation between the observed cause nodes (nodes a and b here). They proposed a new definition for product synergy taking this aspect into consideration. We reproduce their definition here.

Definition 4 (Product synergy II) : Let a, b, x be direct predecessors of c in a QPN (Fig. 3) and y be direct predecessor to b. Let n_x denote the number of possible values of x. Variables a and b exhibit negative product synergy with respect to a particular value c_0 of c, regardless of the distribution of x, written $X^-(\{a, b\}, c_0)$, if for all $a_1 \geq a_2$ and for all $b_1 \geq b_2$, a square matrix $n_x * n_x$ matrix D with elements

$$D_{ij} = \Pr(c_0/a_1, b_1, x_i)*\Pr(c_0/a_2, b_2, x_j) - \Pr(c_0/a_1, b_2, x_i)*\Pr(c_0/a_2, b_1, x_j) \tag{4}$$

is half-positive semi-definite. If D is half-positive semi-definite, a and b exhibit positive product synergy written $X^+(\{a, b\}, c_0)$. If D is a zero matrix, a and b exhibit zero product synergy written as $X_0(\{a, b\}, c_0)$.

The necessary and sufficient conditions for explaining away to take place is given below (reproduced from Wellman and Henrion[29]). This general condition is valid for both definitions (3 and 4, above) of product synergy[20]. It is to be noted that the term 'half-positive semi-definiteness' is nothing but the co-positivity condition for a square matrix.

Theorem 1 (explaining away) : Let a, b and x be the predecessors of c (Fig. 3). A necessary and sufficient condition for $S^-(a, b)$ upon observation of c_0 is negative product synergy, $X^-(\{a, b\}, c_0)$.

2.4. *Belief propagation in probabilistic networks*

Belief propagation in probabilistic networks can be of two forms: qualitative (which is purely based on signs) and quantitative (where updation of subjective probabilistic estimates is done). For a quick overview for introduction to algorithms for inferencing in belief networks, refer Henrion[32]. Henrion classifies qualitative propagation techniques as 'weak'. This kind of belief propagation is resorted to when it is difficult to obtain a complete point-valued probability distribution. In this paper, we consider only quantitative belief revision.

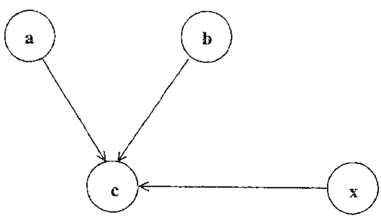

FIG. 3. An example of qualitative probabilistic network (three-cause network).

2.4.1. *Quantitative belief revision*

Primarily there are two methods of belief revision[32], namely, exact and approximate methods. Exact methods deal with studying the impact of evidence by explicitly computing the joint distribution over all variables as a product of all prior and conditional distributions. This method gets complicated in the case of multiply-connected poly trees (like cognitive maps). Lauritzen and Spiegelhatler's[33] clique-triangulation method and Shachter's[34,35] graph reduction method address belief revision in such graphs. However, these methods alter the topology of the graph, thereby leading to loss of information.

A completely different form of belief revision is done by employing simulation (Monte Carlo techniques), termed approximate methods by Henrion[32]. Pearl[36], and Chin and Cooper[37] have proposed stochastic simulation as one such method. The key advantage of these methods over exact methods is the complexity of the algorithm with respect to the size of the entire graph. However, the complexity of the algorithm is exponential in the number of observed (or evidence) nodes. The other advantage of this method over exact methods is that the topology of the graph is preserved. In this paper, we resort to stochastic simulation method for quantitative belief revision, as the primary objective of belief revision is to study the influence of evidence on all concepts (nodes). For this we need to preserve the topology of the graph.

3. Intuitive models

Studying human decision processes and human intuition under uncertainty has been the central focus of research to psychologists[16, 38, 39]. Issues related to this have also been studied by researchers in fields like artificial intelligence, primarily because they have been subjected to criticism of developing systems which are nonintuitive in nature[31,40]. This criticism stems from the classical 'expert system' paradigm that employs computer representations and inference mechanisms intended to emulate human reasoning processes. While there is ample evidence to show that normative schemes such as probability theory are poor models of human reasoning under uncertainty[41], expert systems (*viz.*, PROSPECTOR) built on these normative theories have also been found to be successful in replicating expert opinions[42].

3.1. *Literature survey*

Studying human decision process has become popular ever since Newell and Simon[38] conducted experiments to observe problem-solving strategies of humans. Currently, this approach is being widely used in fields where the objective has been to capture decision processes of experts[40,43]. While there is a vast literature on human judgement under uncertainty for simple inference problems (see Kahneman and Tversky[41] and Morgan and Henrion[44] for extensive review on this topic), little work has been done on issues related to cognitive processes in more complex situations, *i.e.*, multiple hypothesis and multiple evidence situations[30]. Henrion and Druzdzel[30] have used this methodology to study the inferencing process adapted by humans in uncertain situations. The main findings of their research work are the following[30]: (i) subjects generally use qualitative terms for probabilities, using quantitative information very rarely; (ii) subjects resort to causal reasoning in uncertain inference; (iii) subjects resort to two different strategies during uncertain inference, which are qualitative belief propagation and scenario-based reasoning.

While (i) and (ii) above are consistent with the earlier studies on intuitive reasoning[41,45], (iii) laid the foundation for explanation-based reasoning in uncertain inference[46]. The first strategy cited above in (iii), qualitative belief propagation, involves propagating the qualitative impact of an evidence from event to event, following causal and diagnostic relationships. In scenario-based reasoning, the reasoner identifies scenarios that are consistent with the causal explanations compatible with known evidence.

An important issue, as sighted earlier, is the capture of decision process in humans. Largely, researchers have used verbal protocols of subjects to capture these decision processes[16]. The methodology adapted captures the thinking process of subjects (humans) while they analyze a problem given to them. The subjects are requested to 'think aloud' the steps they have taken. These 'think aloud' (TA) protocols are then transcripted and analyzed to identify patterns of inference.

3.2. *Verbal protocol analysis*

Research on cognitive psychology has emphasized methods that rely wholly on external observations. The predominant form of these observations has been through verbal reports articulated by subjects.

Typically, verbal reports are elicited by asking the subjects specific questions. To answer such questions, as Ericsson and Simon[16] detail it, ... *the subject has to comprehend the question and transform it to retrieval cues that select the relevant information from the vast amount of information in the memory. In addition, the subject has to put the retrieved information into a sequential form that allows the generation of a coherent series of verbalization. However, the actual problem in these types of studies is the possibility that the information they retrieve at the time of the verbal report might be different from the information they retrieved while actually performing the experimental task.*

Typically, verbalization process is categorized into two types: concurrent and retrospective. In concurrent verbalization, the subject is instructed to 'think aloud' concurrently about the thought processes, while answering the question. In this method, the subject is asked not to describe or explain the thoughts. This is because, any such provocation would make the subject to attend to information not normally needed to perform the task. In such situations the sequence of thought process gets changed—altering the purpose of the experiment. Ericsson and Simon[16] classify the information needed for this type of experiment as 'short-term memory'-oriented experiments.

In retrospective verbalization, the subject is instructed to detail the cognitive processes after the experiment is over. Immediately after the task is completed, the particular subset of the sequence of thoughts occurring during performance of the task that is stored in 'long-term memory' is tapped [16].

Ericsson and Simon's[16] framework predicts a close correspondence between concurrently and retrospectively reported information. However, according to them, this correspondence is valid in task durations which are between 2 and 10 seconds.

The other issue in protocol analysis is the different effects of verbalization. These different effects, termed Types 1, 2 and 3, are formed on the basis of different types of instructions given to the subject to verbalize. The major distinction between these different types is: In Types 1 and 2, the instructions are to verbalize *per se* about the thought processes in general. In Type 3, the instructions are to verbalize specific information, such as reasons and explanations. Type 3 verbalization forces subjects to change their thought sequences in order to generate, and verbalize overtly the information requested. This does not occur in Types 1 and 2. For greater details, and for an excellent overview on protocol analysis, refer Ericsson and Simon[16].

3.3. *The experiment and findings*

In line with the above research, and based on earlier research done on human problem solving[38], we conducted protocol analysis on subjects to identify strategies they adapt to propagate belief revision. The main objective in conducting the experiment was to find patterns of inference adapted by humans while they propagate beliefs. We requested the subject to verbalize concurrently about his thought processes.

3.3.1. *The objective*

The objective of this experiment is to observe the patterns of inference in humans reasoning under uncertainty. This study investigates the following: what strategies (heuristics) do humans adapt for qualitative belief propagation?

3.3.2. *Methodology*

Subjects are given a problem and their concurrent verbalizations are recorded. The problem administered here is given in Appendix I. This problem set has been used by Henrion and Druzdzel[30] for their protocol analysis. However, the second problem has been modified to suit the profile of the subjects. The subjects are primarily graduate students. Ideal test conditions were provided. The test supervisor (one of the authors), however, periodically prompted the subjects to 'think aloud'. Other than this, there was no conversation of any kind between the subject and the supervisor. The verbalizations were taped and transcripted. For this purpose, the methodology recommended[30,38] was followed. The problem set which is used for this experiment is given in Appendix I. The transcripts of the verbal protocols of one of the subjects is given in Appendix II, which is indicative of protocols of the sample studied.

3.3.4. *Observations*

We refer to the transcript given in Appendix II to study patterns adapted by humans for inferencing under uncertainty.

(1) As observed by Henrion and Druzdzel[30] in their experiments, the subject (S2 here) resorted to qualitative belief propagation scheme. According to Henrion and Druzdzel[30], under this scheme, human beings propagate beliefs by updating information available in the evidence through causal chain. The verbal accounts (statements between C138 and

C148) of the subject demonstrate this aspect. Here the subject is trying to find causal explanations to observed events.

(2) The subject has used 'explaining away' when reasoning qualitatively. Explaining away, as detailed in Section 2, deals with reasoning about causes in the event of an observed effect. The verbal accounts between C49 and C57 corroborate this. Both observations, (1) and (2), substantiate the earlier work done by Henrion and Druzdzel[30].

(3) The other interesting observation we noticed, that forms the central theme of this paper, is the different types of causal clustering that the subject resorted to while propagating beliefs. Consider the verbal accounts (between C58 and C80) of the subject given in Appendix II. While reasoning with the help of uncertain information, the subject used combinations (similar but not identical, to logical AND and OR) in clustering causes, of which he does not have any extra information. The subject also articulated that he resorted to this kind of reasoning because he does not know how each cause acts in accordance with other causes. Similarly, while providing causal explanations for Problem 2, the verbal accounts of the subject (between C145 and C160) detail about how the subject tries to cluster causes of an observed effect. He stated that the company can adapt different combinations of FOCUS, DIFFERENTIATION, and LOW-COST strategies. In other words, he reasons for situations where there are combinations like FOCUS–DIFFERENTIATION, LOW COST–DIFFERENTIATION, and LOW COST–FOCUS strategies. Given that the subject is not provided with any information about the causes (FOCUS, DIFFERENTIATION, and LOW-COST strategies), for the observance of the effect (MARKET LEADERSHIP), the subject resorted to these different possibilities. This process can be interpreted as follows: the subject was trying to generate alternate situations to determine the cause(s) for the observed effect. Among the possible set of causes for the observed effect, he was clustering the causes in a logical sense to explain the occurrence of effect node.

Similarly, while reasoning in a two-cause situation, *viz.*, the causal network that include A, B and C of Problem 1, the subject tried to generate situations such as assuming the effect node when both causes are absent. The subject here even tried to generate a scenario in which the observed effect could have been caused by an unknown event (external) which is not stated in the problem. Though at the outset, this scenario seems impossible, this does have a certain significance which we will explore further.

The subjects' behavior in this situation can be interpreted as that of establishing a functional relationship among the causes. He also assumes the impossible situation as detailed earlier—observation of the effect node when both causes are not observed. However, he ruled out this possibility later, after realising that B and C are the only possible causes. We make the following observation: the subject is generating all combinations that imply (similar to logical connectives like AND, OR...) causes for the effect. The central issue that is being highlighted here is that, even within multiple combinations that the subject generated (which closely follow the pattern of a logical AND, OR), there are certain impossible situations which the subject considers.

The practical significance of such impossible situations has been highlighted by expert system developers[28]. An important information that these developers seek from the expert is the chances of an effect being observed even when none of the causes are present. Pearl[26] uses the term 'leaky probabilities' to denote such estimates. The advantage of incorporating these probabilities is to make the model more robust. Usually, the numerical value of such estimates is of the order of, say, 0.001. However, there is little empirical justification for such concepts (Druzdzel–personal communication). Typically such effects can be classified as follows: either completely random events, or events about which very little information is available. Usually random events are events for which causes cannot be clearly delineated. The other type of events are those where there is lack of complete information. In the present context, there is very little information about event A, other than the fact that it is caused by B and C.

To put it succinctly: it is intuitively compelling on the part of the subject to resort to some sort of logical connectives while reasoning under uncertainty. This is particularly evident while propagating beliefs when very little information about the causes is available, given that the effect is observed.

The key findings based on our observations: (i) subjects resorted to causal reasoning during uncertain inference, (ii) there is support for 'inter-casual' reasoning as defined in Henrion and Druzdzel[30], and (iii) subjects used logical connectives similar to AND, OR while propagating beliefs in uncertain inference situations.

3.4. *The intuitive models*

From the above experiment, we can say that, in general, human beings intuitively adapt two patterns of inference, while propagating beliefs: AND and OR. The OR combination considered here is exclusive-OR rather than inclusive-OR. This is because, modeling inclusive-OR would be straightforward as it is a combination of AND and exclusive-OR.

In a multi-cause single-effect network, an AND pattern captures situations where all the possible causes need to be true (in a propositional sense) for the effect to be true. On the other hand, an XOR (exclusive-OR) pattern represents a situation where only one of the cause nodes is true for the effect node to be true. All through the discussion, we make the assumption that the probability values do not take extreme values (0 or 1) but are in the midst of the continuum[30]. Unless specifically stated, we assume that the probability values are point estimates, and not cumulative distributions.

Typically, causal networks can be segregated into two- and multi-cause networks. In two-cause networks, there are only two-cause nodes which lead to the effect node. In multi-cause networks, we consider situations where there are more than two cause nodes.

3.5. *Two-cause nodes*

Consider Fig. 2. The proposed intuitive models are defined on this network. After defining AND and XOR models, we generalize the conditions for inter-causal reasoning. Since the proposed models are not exhaustive, we can establish only one way implication. In other words, we show that if we find a causal network satisfying conditions for AND model, it can be proved that the inter-causal relationships between the two-cause nodes satisfies the conditions for positive product synergy (definition 4). However, it is important to note that in a causal network if it is observed that the cause nodes satisfy conditions for positive product synergy then it is not necessarily true that the network under consideration forms an AND model. Similar is the case for XOR synergy.

All variables are propositional, unless otherwise stated explicitly. Node a, for example, can take two values: a_1 and a_2. Thus, in a propositional sense, a_1 takes the value A (true) and a_2 takes the value A′ (false).

3.5.1. *Definitions*

Definition 3 *(XOR model)*: Let a, b be the predecessors of c in a QPN. Variables a and b form an XOR model with respect to a particular value c_0 of c, written XOR(a, b, c_0), if for all $a_1 > a_2$, and $b_1 > b_2$,

$$\Pr(c_0/a_1, b_2) \geq \Pr(c_0/a_2, b_2); \tag{1}$$

$$\Pr(c_0/a_1, b_2) \geq \Pr(c_0/a_1, b_1); \tag{2}$$

$$\Pr(c_0/a_2, b_1) \geq \Pr(c_0/a_2, b_2); \tag{3}$$

$$\Pr(c_0/a_2, b_1) \geq \Pr(c_0/a_1, b_1); \tag{4}$$

Example

This pattern of reasoning can be detailed with an example. Consider the example detailed above. Assume that node c represents market leadership, node a, differentiation strategy, and node b, cost leadership strategy. The probability of finding a firm having market leadership (*i.e.*, c = true, *i.e.*, takes the value c_0) will be the highest when it is observed that the firm follows either cost leadership strategy (when node a = true, *i.e.*, takes the value a_1) or differentiation strategy (when node b = true, *i.e.*, takes the value b_1). Thus, the probability of observing an effect node to be true will be the highest only when one of the cause nodes is true. The probability of finding a firm having market leadership is the lowest when it adapts both cost leadership and differentiation strategy because of economies of scale. Low-cost strategy is adapted by companies which have capabilities to produce. On the other hand, differentiation strategy is adapted by companies which have capabilities to give variety in a product. For such a strategy (differentiation), a company needs flexible manufacturing systems. Such companies cannot go for mass production which low-cost manufacturers adapt. Hence, finding a company that is a market leader, and which has adapted both differentiation and low-cost strategies, is uncommon.

The modeling of AND relationships, however, is not quite straightforward. Since the objective of framing these probabilistic models is to incorporate them into the generalizations to inter-causal reasoning in QPNs, we reason intuitively the rationale behind framing each equation.

Definition 4 (AND *model):* Let a and b be the predecessors of c in a QPN. Variables a and b form an AND model with respect to a particular value c_0 of c, written AND (a, b, c_0), if for all $a_1 > a_2$, and $b_1 > b_2$,

$$\Pr(c_0/a_1, b_1) \geq \Pr(c_0/a_1, b_2); \tag{5}$$

$$\Pr(c_0/a_1, b_1) \geq \Pr(c_0/a_2, b_1); \tag{6}$$

$$\Pr(c_0/a_1, b_2)\ R\ \Pr(c_0/a_2, b_2); \tag{7}$$

$$\Pr(c_0/a_2, b_1)\ R\ \Pr(c_0/a_2, b_2). \tag{8}$$

The inequality relation, R, is not defined for the reason that it can take any of the possible values: ≥, and ≤. However, there are certain limitations in each case to satisfy the conditions of inter-causality (given by definition 3, above).

Case 1: *When R takes the value* ≧

The relations (7) and (8) can be written as follows:

$$\Pr(c_0/a_1, b_2) \geq \Pr(c_0/a_2, b_2); \tag{9}$$

$$\Pr(c_0/a_2, b_1) \geq \Pr(c_0/a_2, b_2). \tag{10}$$

Here, we observe that the probability estimate which represents a situation where all the cause nodes need to be true for the effect node to be true (*i.e.*, $\Pr(c_0/a_1, b_1)$ is the highest among all the possible combinations). Equations 9 and 10 capture the situation where the probability of observing the effect node is true when both cause nodes are false is greater than or equal to probability of observing the effect when only one cause node is true. The intuition behind framing these sets of relations is that both cause nodes are functionally dependent on the observation of the effect node.

In this situation, the conditions for inter-causality are satisfied directly. Using relations 5, 6, 9 and 10, we can show that they satisfy the conditions for positive product synergy (definition 3).

Example

This inference can be explained with an example. Consider Fig. 2. Assume that node c represents sustainable competitive advantage, node a unique-resources (assets), and node b distinctive skills. For a company to have overall sustainable competitive advantage (*i.e.*, node c takes the value true), it is essential that it has both unique-resources (assets) and distinctive skills[47]. Thus, having just one of the two (either unique-resources (assets) or distinctive skills) will not help a firm build sustainable competitive advantage, or probabilistically, the chances of finding a firm having sustainable competitive advantage when it has either distinctive competence or unique-assets is less.

The above probability estimates have to be captured from the expert. Relations, however, are defined only when node c takes the value c_0 (true). The above relations need not be valid when node c takes the value c_1 (false). If c is multivalued, there could be an AND causal structure for a particular value of c, say c_i, but not for other values of c. As observed in the previous sub-section, this definition captures the situation when the observance of the effect node when none of the cause nodes is present. Pearl[36] and, Wellman and Henrion[29] term such estimates (*i.e.*, $\Pr(c_0/a_2, b_2)$) as 'leaky probabilities'.

Case 2: *When R takes the value of* ≦

In this situation, relations 7 and 8 take the form:

$$\Pr(c_0/a_1, b_2) \le \Pr(c_0/a_2, b_2); \tag{11}$$

$$\Pr(c_0/a_2, b_1) \le \Pr(c_0/a_2, b_2). \tag{12}$$

At the outset, the set of relations, 5, 6, 11 and 12, do not satisfy the conditions for inter-causality. We re-write the relations (11) and (12) in such a way that we can deduce the conditions under which the above set of relations defined for AND model satisfy conditions for positive product synergy (definition 3). We assume that the values of $\Pr(c_0/a_1,b_2)$ and $\Pr(c_0/a_2,b_1)$ are equal. An AND model satisfies the conditions of positive inter-causality only when the following relation is satisfied:

$$\frac{\Pr(c_0/a_1,b_1) * \Pr(c_0/a_2,b_2)}{\Pr(c_0/a_1,b_2)} > 1.0.$$

Rewriting the above, for the case where the variables a and b take multiple values (*i.e.*, when they are not binary),

$$\frac{\Pr(c_0/a_i,b_i) * \Pr(c_0/a_k,b_k)}{\Pr(c_0/a_i,b_k)} > 1.0.$$

where $a_i > a_k$, and $b_i > b_k$.

This case is intuitively more appealing than case 1 above as we are considering the situation where at least one cause-node is to be true for the effect node to be true.

Using the definitions for AND and XOR models given above for two cause-node situations, we generalize the conditions for inter-causal reasoning in the form of the following theorem. The proof is straightforward from the definitions given earlier.

Theorem 2a (Sufficiency conditions for product synergy): Let a and b be the predecessors of c (Fig. 2). A sufficient condition for $S^-(a, b)$ upon observation of c_0, is that the network of a, b, c should satisfy XOR(a, b, c_0) model.

3.6. *Inter-causal reasoning in multi-cause nodes*

As detailed earlier, causal networks topology can be segregated into two- and multi-cause networks. Using the definitions for AND and XOR models given above for two-cause networks, we generalize the conditions for inter-causal reasoning in multi-cause networks. In the proposed definitions, the causal structure considered is a two-cause single-effect network. Typically, multicause single-effect networks are of two types: (i) networks where the irrelevant cause nodes with respect to the effect node under consideration are uninstantiated, and (ii) networks where irrelevant cause nodes are instantiated. In the following subsection, we give definitions for situations where x (the irrelevant cause node) is uninstantiated. In such situations, we need to consider all possible values that node x can assume. The definitions for multi-cause networks, where we look only at situations where there are only three cause nodes, are detailed in Appendix IIa. There are two observations to be made here:

(1) All the above definitions have only one-way implications. In other words, if we observe that the conditional probabilities governing a network satisfy one of the proposed intuitive models, then we can make a statement about the status of product synergy. However, the converse need not be true. This means that if we observe two-cause nodes satisfying the conditions of negative or positive product synergy, we cannot conclude anything about the configuration of the causal structure of the network. This is primarily because multiple forms of causal networks exist for the same set of inter-causal conditions.

(2) The probabilistic criteria governing the proposed intuitive models need not necessarily result in AND or XOR patterns. In other words, in certain situations (particularly if nodes are multivalued), the probability estimates need not satisfy any of the proposed intuitive model criteria. This implies that in certain networks, the cause nodes and the effect node need not form either an AND or an XOR model.

4. Intuitive models in belief propagation

As detailed in Section 2, we observed that cognitive mapping technique can be used to capture cognitive arguments of an expert in a domain (here managers). We also saw that representation techniques like qualitative probabilistic networks have provision to capture subjective measures of uncertainty in the form of probability estimates. We proposed intuitive models on the QPNs based on the experiment detailed in Section 3. Here we discuss the applicability of the proposed intuitive models in studying belief revision.

We take a cognitive map constructed from the annual report of company A (for the year 1992-93) to exemplify the issues cited above (name withheld on request from the company). For the various concepts involved in a cognitive map, we captured subjective estimates of middle-level managers of the company through interviews. We took a segment of the cognitive map and converted it into a probabilistic network.

We conducted belief revision exercises on the probabilistic network developed to demonstrate the usefulness of such methods. We conducted stochastic simulation (pertaining to quantitative belief revision) exercise on a segment of the cognitive map using the methodology given in Pearl[36]. For the same map, we incorporated intuitive models while conducting stochastic simulation. We found that the convergence in the second case is faster to get approximate estimates.

4.1. *Research methodology*

The data source for constructing the 'raw' cognitive map is the annual report of the company. We used the methodology suggested by Barr *et al.*[6] in building the cognitive maps. Following this, we have interviewed the managers of the company and refined the cognitive map. This is primarily because the information about the company from the annual report provides an overview of its strategic thinking only. To understand the cognitive process of a manager in that company we have refined the 'raw' cognitive map with the managers' subjective view. We have interviewed for this purpose two managers, both fairly senior in rank with an average of about 8 years service with the company.

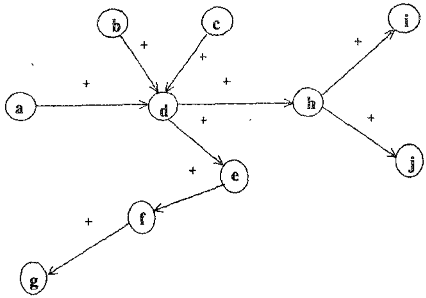

FIG. 4. Cognitive map abstracted from the annual report of Firm A.

a: New_industrial_policy and trade_reforms;
b: Competition_in_true_market_environment;
c: Economic_growth;
d: Introduction_of_new_products;
e: Upgradation_of_quality_and_manufacturing_standards;
f: Programme_of_plant_modernization_and_value_engineering;
g: Reduction_of_manufacturing_costs;
h: Achieving_technological_autonomy,_quality_consciousness;
i: Increase_the_crash_survivability_of_vehicles;
j: Improve_fuel_efficiency,_reduce_emission_levels;

Refer Fig. 4 for the resultant cognitive map generated, which is a segment of the complete cognitive map developed. On the completion of final cognitive map, we have interviewed the managers to capture their subjective estimates of relevant concepts in the cognitive map. We have captured the probability estimates with the help of linguistic table provided in Henrion and Druzdzel[30]. Table I gives the subjective estimates (conditional probabilities) of each manager. Using these values, we have conducted the belief propagation experiment.

4.2. *Stochastic simulation experiment (Quantitative belief propagation)*

Stochastic simulation is a method of computing probabilities by counting the fraction of time that events occur in a series of simulation runs. If a causal model of a domain is available, the model can be used to generate random samples of hypothetical scenarios that are likely to develop in the domain. The probability of any event or a combination of events can then be computed by recording the fraction of time it registers true in the samples generated[36]. Stochastic simulation on the cognitive map is conducted to generate a scenario. As discussed earlier, the scenarios generated on these mental models represent the top management's revised beliefs in that particular hypothetical situation. Given the subjective estimates of the nodes present in the cognitive map (Table I), the main task is to compute posterior probability estimate (or new belief) of every node in the map. Certain observed nodes are clamped to one of the two values: 0 or 1 (as all the nodes in this example are assumed to be propositional). The unobserved nodes are instantiated to some arbitrary initial state (0 or 1). The observed

Table I
Subjective probability estimates of managers

Pr(a) = 0.8;	Pr(e/d) = 0.9;
Pr(b) = 0.9;	Pr(e/–d) = 0.45;
Pr(c) = 0.85;	Pr(f/e) = 0.8;
Pr(d/a, b, c) = 0.9;	Pr(f/–e) = 0.45;
Pr(d/a, b, –c) = 0.3;	Pr(g/f) = 0.6;
Pr(d/a, –b, –c) = 0.5;	Pr(g/–f) = 0.3;
Pr(d/a, –b, c) = 0.4;	Pr(h/d) = 0.75;
Pr(d/–a, b, c) = 0.8;	Pr(h/–d) = 0.4;
Pr(d/–a, b, –c) = 0.7;	Pr(i/h) = 0.85;
Pr(d/–a, –b, –c) = 0.5;	Pr(i/–h) = 0.5;
Pr(d/–a, –b, c) = 0.3;	Pr(j/h) = 0.9;
Pr(j/–h) = 0.1;	

nodes that are clamped do not change the assigned values throughout the simulation run. All unobserved nodes change their state in accordance with the conditional probability dictated on it and the current state of the nodes that are in the Markov blanket. In a simulation run, the number of times each unobserved node takes a value of '1' is counted. This gives the conditional probability (which is the revised belief estimate) of that node for the given set of 'clamped' nodes (observed nodes). Thus, we get revised belief estimates for the nodes of the causal model in a hypothetical situation. These estimates are termed as 'BELIEFs'[26], and are not exactly posterior conditional probabilities. For greater details and examples, see Pearl[26,36].

The simulation exercise was done on an IBM-PC clone using C language. The average number of runs for convergence was around 500. Random numbers were generated using a randomized seed value, whose initial value was given by the user. Tables IIa and b give the final revised belief estimates for a hypothetical scenario with respect to the cognitive map shown in Fig. 4. We can generate more such scenarios by clamping the appropriate nodes.

Table IIa gives the belief values of all the unclamped nodes when we used Pearl's algorithm. The belief of node b when nodes a, c and j are clamped is written as BEL (B/A, C, J). As detailed above, this notation is just an alternate notation for posterior distribution of node A, under the given scenario. Table IIb gives the probability values when we used the intuitive models (Fig. 3) on node d and conducted stochastic simulation. As discussed in Section 2 (and as per Fig. 3), if each node represents a cognitive concept of the manager, with this methodology one can predict the managerial beliefs under various hypothetical situations. As discussed in Section 1, this kind of methodology is helpful in studying the strategic behavior of firms, as it facilitates capture of beliefs in managers.

From Table IIb it may be observed that, in the second method, the algorithm 'arrives' at the true estimates much faster than the first method. However, it can be observed that after five hundred runs, the second method does not have any added advantage as it gives similar result to that of Pearl's method[36]. Thus, the second method (that uses the intuitive models) arrives to the approximate values faster. In each run here, the algorithm tests for the intuitive patterns defined in Section 3. For the network given in Fig. 4, the algorithm checks for

Table IIa
Results of stochastic simulation experiment (Pearl's algorithm)

Clamped nodes	$A = 1; C = 1; J = 1$				
Posterior probabilities	*No. of runs*				
·	50	100	250	500	750
BEL(B/ A, C, J)	1.00	0.99	1.00	1.00	1.00
BEL(D/ A, C, J)	0.00	0.00	0.77	0.77	0.78
BEL(E/ A, C, J)	0.00	0.00	0.58	0.63	0.64
BEL(F/ A, C, J)	0.00	0.00	0.64	0.63	0.64
BEL(G/ A, C, J)	0.18	0.33	0.38	0.38	0.38
BEL(H/ A, C, J)	1.00	1.00	0.92	0.95	0.94
BEL(I/ A, C, J)	0.48	0.50	0.57	0.57	0.57

Table IIb
Results of stochastic simulation experiment (proposed model)

Clamped nodes	$A = 1; C = 1; J = 1$				
Posterior probabilities	*No. of runs*				
	50	100	250	500	750
BEL(B/ A, C, J)	0.98	1.00	1.00	1.00	1.00
BEL(D/ A, C, J)	0.00	0.68	0.77	0.77	0.78
BEL(E/ A, C, J)	0.00	0.00	0.59	0.63	0.64
BEL(F/ A, C, J)	0.00	0.00	0.64	0.63	0.64
BEL(G/ A, C, J)	0.18	0.33	0.36	0.38	0.38
BEL(H/ A, C, J)	1.00	1.00	0.92	0.95	0.94
BEL(I/ A, C, J)	0.48	0.50	0.57	0.57	0.57

intuitive patterns defined on node d, based on subjective probability estimates given by the managers. Thus the cause node b is made '0' or '1' based on the definitions given earlier. It is to be noted that nodes a, and c are clamped to '1'. While calculating the belief for node d, the algorithm checks for the status of cause nodes (a, b, and c), and identifies the pattern of inference that the causes form with respect to the effect node. As the cause nodes a and c are clamped to '1', the other cause node b is set to '0' or '1' (using the definitions given in Section 3), depending on the current value of node d. In this way, the number of runs required to arrive at an approximate value in calculating the beliefs of nodes b and d are fewer. This is because the value that node b takes is not dependent on the random number generator, but is based on the type of pattern it gets formed with respect to the other cause nodes. In large cognitive maps, where the emphasis is on finding approximate estimates only, we foresee the use of such intuitive models.

5. Conclusions

In addition to the conclusions given at the end of each section, we present a summary of the findings in this section. The critical issue that we have tried to address in this paper is representation of uncertain information in organizations. From the verbal protocols of subjects in an experiment we conducted, we found that humans resort to certain patterns of inference under uncertainty. We also found that human beings use logical connectives while propagating uncertainty. We formalized these observed patterns by proposing intuitive models with the help of certain probabilistic criteria. We incorporated these models into the existing generalizations on inter-causal reasoning defined on probabilistic networks. Finally, we applied this network representation in scenario generation.

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Appendix I

Introduction

(please read aloud. You may ask questions if you wish.)

You will be asked to solve two problems involving qualitative and quantitative reasoning. It is important that you solve the problems as well as you can, given the facts. Even more important is that you allow the experimenter to follow your approach to solving the problem. To accomplish this, you will be asked to 'think aloud' during the experiment.

Whenever you are asked to read something, please read aloud, if there is something you think about during the reading, please pause and say aloud whatever you think. Please, try to say aloud everything that comes to your mind, every thought you may have during the solution process. This includes even details of what may seem insignificant or even embarrassing. You do not have to explain why you are thinking that way. There is no need to interpret or justify your approach to the problem. Just say aloud what you are thinking at the moment. Be as spontaneous as possible.

If you are silent for more than a few seconds, you will be reminded to think aloud. Just to get an idea how the experiment will proceed try solving the following WARM UP problem (remember about thinking aloud).

Warm-up problem

(Please remember about reading aloud.)

John is now twice as old as Mary. In five years time this relation will not be true any more, but the age of their father, who is now 41, will be equal to the sum of the ages of John and Mary.

How old is Mary now?

The experiment

(Please read aloud. You may ask questions if you wish.)

You will be presented with two hypothetical situations where you will be asked to express your beliefs with respect to uncertain events. It is important that you estimate the probability of the events as precisely as you can, on the basis of facts.

The text of each of the problem consists of several pages. The first page will describe a certain situation. Every page will give you additional information, which is to be considered to be in a cumulative manner. No additional information will be given to you by the experimenter. However, if you think you cannot find the information you need with respect to estimating the probability, you are allowed to make any assumptions that you find necessary (both qualitative and quantitative). You are requested to mention them when thinking aloud.

You will be asked questions at the end of every page. You may use all the information available to you, given in the current page and also from the previous pages, while answering. Do not turn to the next page until you have finished answering all questions given for of the current page. You may go back and look at previous pages whenever you find it necessary. Even while re-reading the text, please read aloud.

Problem One

Effect A is observed sometimes. The only possible causes of A are B and C. B and C very rarely occur at the same time, but whenever one of them occurs, it is certain to cause A. C can be caused only by D. If D occurs, it is almost sure to cause C. D is not observable directly, but its presence can cause E. E has several possible causes, but all of them except for D, F and G are not probable in the present context. D, F and G need not occur together. The dependencies described above are the only dependencies among A, B, C, D, E, F and G.

A is observed

How probable is it that A is caused by B?

< Next Page >

Later E is also observed.

---

1. How and in what way does observing E affect the probability of A being caused by B you estimated in the question of the previous page?
2. Now, how probable is it that A is caused by B

< Next Page >

Later G is also observed.

---

1. How and in what way does observing G affect the probability of A being caused by B you estimated in the question of the previous page?
2. Now, how probable is it that A is caused by B?

Problem Two

Mr. A is a shrewd buyer. He buys a particular product either when the product is of good quality or when the price is less. It is also known to him that usually cheaper products are of poor quality. However, certain products are both cheap and are of good quality. He also knows that for a company to sell a product cheaper than its competitive brands, it has to be a low-cost producer. If he knows that a particular company is a low-cost producer, he is sure that the prices of this company's products are low. Typically, companies follow the following strategies to be the market leader: low-cost producer, differentiation (with respect to competition), focus (targeting on a particular segment only).

Mr. A visits the local supermarket for his weekly purchases. He decides to buy a product of a particular company.

How probable is that Mr. A would have decided to buy the product because it is of good quality

< Next Page >

Mr. A finds that the company which manufactured this product is the market leader.

1. How does this information affect the probability of Mr. A's decision to buy the product.
2. Now, how probable is that Mr. A has decided to buy because of good quality.

< Next Page >

Mr. A recalls that this company is focusing its products on teenagers and youth (adapting a focus strategy).

1. How and in what way do you think the knowledge about the company's strategy affect the probability of Mr. A's buying decision process.

2. Now how probable is it that Mr. A's decision to buy the product because it is of good quality.

Appendix IIa

PROTOCOL OF SUBJECT S2

Texts read by the subject
Texts spoken by the subject
(E: Texts spoken by the experimenter)
(comments on the protocol)
Brackets ( ) mark 5-second intervals.

C1: *Problem one. (pause) ( ) Effect A is observed sometimes. The only possible causes of A are B and C.*
C2: This looks like some kind of causal theory stuff. ( )
C3: (writing) A is caused by B and C.
C4: *B and C very rarely occur at the same time.*
C5: I can scribble on these sheets right?
C6: (E: sure no problem).

C7: But whenever one of them occurs ( ) it is certain to cause A.
C8: Looks like some kind of mutual exclusivity.
C9: ( ) or maybe that is an assumption.
C10: C can be caused only by D.
C11: (writing – draws a casual graph) Fine ( ).
C12: If D occurs it is almost sure to cause C.
C13: (writing) Can we assume the conditional probability is 1?
C14: This surely is a causal tree.
C15: *D is not observable directly but its presence can cause E.*
C16: What do you mean by not observing directly? (looks at the experimenter).
C17: (E: Some kind of a hidden cause – not observable).
C18: Oh it is a causal chain?
C19: D is not observable... can cause E.
C20: Right!
C21: E has several possible causes ( )
C22: Some kind of multiple causes
C23: (writing) but all of them except for D, F, G are not probable in the present context.
C24: So there are more causes which are not considered here.
C25: *usually D F G need not occur together ( )*
C26: ah! mutual exclusivity
C27: (writing) some kind of logical OR relation
C28: there are two types of OR right? inclusive and exclusive?
C29: *The dependencies above are the only dependencies among A B C D E F G*
C30: so they are exhaustive
C31: Let me cross check the information you gave to me ( )
C32: (pause).. Well
C33: I hope I have got the causal dependencies right
C34: These are the questions is it? (pointing to the information given at the bottom of the page)
C35: (E: they are observations)
C36: *Ok! A is observed*
C37: (writing) A is observed
C38: so what next
C39: *How probable is it that A is caused by B*
C40: Well (pause)
C41: from this graph... this says B and C cause A
C42: so just by observing A how can we say anything
C43: I can make assumptions right? (look at the experimenter)
C44: (E: yes and think aloud)
C45: Ok!
C46: (looks at the sheet on which the graph is drawn)... Well
C47: Assuming say ( ) that D is observed
C48: this makes C more probable to occur

C49: wait a minute ( ) it is given here that B and C occur very rarely together
C50: then in this case B is not the cause of A
C51: alternatively ( ) if...
C52: we assume B is observed
C53: then B is the cause of A
C54: we can also have a situation er... I mean hypothetically ( )
C55: given the incomplete information... (pause) say both A and C being not the cause for B ()
C56: otherwise... naturally the probability will be high
C57: Is that what you want? (looks at the experimenter)

C58: *Later E is also obseved. ( ) How and in what way does observing E affect the probability of A being caused by B you estimated in the question of the previous page?*
C59: (turns the previous page) Fine looks like I have to make assumptions again
C60: at the outset I would say it decreases
C61: *the only possible causes of A are B and C*
C62: E can be caused by D F and G
C63: naturally if E is observed ( ) assuming D F and G...
C64: D F G do not occur together
C65: If E is observed (pause) one of D F Gs has occurred
C66: assuming probability is 1/3... equally likely case
C67: If D has occurred ... then C has occurred (looks at the casual graph which he drew earlier)
C68: If C has occurred ... naturally B is not the cause for A.
C69: Alternatively ...
C70: If F has occurred ... then C would be the cause
C71: wait
C72: if F is observed ...
C73: B would be the cause ... because B and C rarely occur together
C74: similarly with G
C75: other combinations can also exist....
C76: either F G being observed and D not observed
C77: ... (looking at the graph drawn before him) this leads to higher probability
C78: if both D and F have occurred and G has not taken place
C79: ( ) then the probability becomes low
C80: ... anything is possible
C81: (looks at the experimenter) may be we should have a random number generator here (laughs)
C82: to decide which event is to be observed
C83: I feel here, it depends on how the causes act among themselves
C84: no information is being provided
C85: information on how the causes behave should be given
C86: is there any time limit? (looks at the experimenter)

C87: (E: no you can take as much time as you want)
C88: *How probable is it that A is caused by B*
C89: I think this question is repeated
C90: are you following the techniques of marketing research (laughs) cross-checking respondents?
C91: I would say low ( ) because ...
C92: as I explained ... it depends on whether D or F or G is the cause or ... combinations of them
C93: but assuming they could have occurred ...
C94: as E is observed
C95: naturally B is not the cause ... err low

C96: I hope you will listen to whatever you are taping here (laughs)
C97: *Later G is also observed*
C98: as I requested
C99: *How and in what way does observing G affect the probability of A being caused by B you estimated in the question of the previous page?*
C100: again I would say... odds in favour of C
C101: since you are asking about probabilistic or .... uncertainty
C102: ... think on those lines ( )
C103: I would say it gets reduced
C104: the more information you give me about the causes of E
C105: the less likely the cause of A would B be
C106: isn't it natural?
C107: *Now how probable is it that A is caused by B*
C108: I would say low.
C109: because ... you told that B and C rarely occur together
C110: naturally it is low.
C111: ...er ... fine that's it.

C112: *Problem two.*
C113: Am I supposed to do immediately or take a break.
C114: (E: Preferably do it right away).
C115: *Mr A is a shrewd buyer. He buys a particular product either when the product is of good quality or when the price is less. It is also known to him that usually cheaper products are of poor quality. However, certain products are both cheap and are of good quality. He also knows that for a company to ...*
C116: Let me organise the information. I know what you are going to ask me later.
C117: This looks like a market survey kind of thing.
C118: (writing) Mr. A buys a product based on good quality or cheaper price.
C119: (writing) cheaper products are of poor quality ... usually
C120: He is also not ruling out the possibility that some products can be cheap and of good quality ... a rarity in reality
C121: Anyway ... why do these kinds of experiments are based on ideal situations (laughs)

C122: *He also knows that for a company to sell a product cheaper than its competitive brands it has to be a low-cost producer*

C123: so the cause for cheaper products is the company is low-cost producer (writing)

C124: fine!

C125: *If he knows that a particular company is a low-cost producer he is sure that the prices of this company's products are low.*

C126: (writing) the conditional probability is 1... this is my assumption

C127: *Typically companies follow one of the following strategies to be a market leader, either low-cost producer or differentiators with respect to competition or focussing on a particular segment.*

C128: This looks like an example from Porter's framework.

C129: I make the assumption that the three strategies are mutually exclusive ... exclusive OR say (writing)

C130: *Mr. A visits the local supermarket for his weekly purchases; he decides to buy a product of a particular company*

C131: so our man goes to a grocery shop

C132: fine

C133: *How probable is that Mr. A would have decided to buy the product because it is of good quality*

C134: This looks exactly like the previous problem... more information

C135: no not information per se, but a realistic example

C136: Is it to cross-check my thinking (looks at the examiner)

C137: (E: no)

C138: *How probable is that Mr. A would have decided to buy the product because it is of good quality*

C139: It depends on what type of buyer is Mr. A

C140: I make an assumption that this guy prefers good quality to cheaper products ...

C141: assuming that good-quality products are priced high

C142: so it would be high ... say .8 types

C143: *Mr. A finds that the company which manufactured this product is the market leader*

C144: (looking at the graph) Well, I think I made a wrong assumption

C145: (writing) so it is observed that the company is the market leader

C146: *How does this information affect the probability of Mr. A's decision to buy the product*

C147: it depends on the strategy the company follows ... either low cost or differentiation or focus

C148: if it low cost ... then obviously the probability goes up because it is casually connected to cheaper product.

C149: on the contrary ... if the company adapts a differentiation and focus strategy ...

C150: it may not affect ...

C151: because he could have decided to buy due to good quality

C152: *Now how probable is that Mr. A has decided to buy because of good quality*

C153: (he traces with his pencil) naturally it goes down
C154: I would say the probability now would be... say 0.3

C155: *Mr. A recalls that this company is focusing its products on teenagers and youth adapting a focus strategy*
C156: (writing) So observed focus strategy
C157: *How and in what way do you think that knowledge about the company's strategy affect the probability of Mr. A's buying decision process*
C158: In this case I would say Mr. A would not buy ... because focus is not linked to cheaper products ... hence prompting Mr. A to buy
C159: wait... if I can make an assumption...
C160: companies adapt a combination strategy like focus–differentiation or focus-cost leadership or cost leadership–differentiation ... which are valid according to Porter
C161: so in this case I would say if focus and cost differentiation are present with the company ...
C162: then the probability actually goes up
C163: I cannot say anything of other cases
C164: we need more information
C165: so I would say... it may affect or may not
C166: Now how probable is it that Mr. A's decision to buy the product because it is of good quality
C167: it goes down... or it may not affect
C168: the probability would be 0.8 or becomes less ... case to case basis

APPENDIX IIb

Uninstantiated irrelevant cause nodes

Let the causes in a multi-cause-single-effect causal network be a, b and x, and let the effect node be c. Let a, b and c be propositional and x be multivalued. We use the following notation to generalize inter-causality relationships.

$$P = [\Pr(C/a_1, b_1, x_0)\ \Pr(C/a_1, b_1, x_1) \ldots \Pr(C/a_1, b_1, x_i)]$$

$$Q = [\Pr(C/a_2, b_2, x_0)\ \Pr(C/a_2, b_2, x_1) \ldots \Pr(C/a_2, b_2, x_i)]$$

$$A1 = [\Pr(C/a_1, b_2, x_0)\ \Pr(C/a_1,b_2,x_1) \ldots \Pr(C/a_1, b_2, x_i)]$$

$$A2 = [\Pr(C/a_2, b_1, x_0)\ \Pr(C/a_2, b_1, x_1) \ldots \Pr(C/a_2, b_1, x_i)]$$

$$X = [\Pr(x_0)\ \Pr(x_1) \ldots\ldots \Pr(x_i)]^T.$$

Positive product synergy (definition 4), according to the above notation, can be written as a square matrix $n_x * n_x$ matrix D, (where D is half-positive semi-definite matrix):

$$D \equiv (Q*X)^T*P*X - (A2*X)^T*A1*X \geq 0 \tag{7}$$

and whose elements are:

$D_{ij} = \Pr(c_0/a_1, b_1, x_i)*\Pr(c_0/a_2, b_2, x_j) - \Pr(c_0/a_1, b_2, x_i)*\Pr(c_0/a_2, b_1, x_j)$.

The inequality in the above (7) relation is reversed for negative inter-causality.

Definitions

Definition 4 *(and model* II*):* Let a, b and x be the predecessors of c in a QPN. Assume x to be uninstantiated and multivalued and has n_x states. Variables a and b form an AND model with respect to the observance of the effect node, *i.e.*, c = C, written AND(a, b, x, C), if for all $a > a_2$, and $b_1 > b_2$,

$$\begin{aligned}
&S(i)\ \Pr(C/a_1, b_1, x_i)*\Pr(x_i) \geq S(i)\ \Pr(C/a_1, b_2, x_i)*\Pr(x_i)\\
&S(i)\ \Pr(C/a_1, b_1, x_i)*\Pr(x_i) \geq S(i)\ \Pr(C/a_2, b_1, x_i)*\Pr(x_i)\\
&S(i)\ \Pr(C/a_2, b_2, x_i)*\Pr(x_i) \geq S(i)\ \Pr(C/a_1, b_2, x_i)*\Pr(x_i)\\
&S(i)\ \Pr(C/a_2, b_2, x_i)*\Pr(x_i) \geq S(i)\ \Pr(C/a_2, b_1, x_i)*\Pr(x_i) \qquad \text{(I)}
\end{aligned}$$

Notation S(i) denotes summation over all values of i, where i varies from 0 to n_x (n_x is the number of possible values that node x can take).

Note: Here we assume that the inequality in (I) is all $\geq$, as it is intuitively appealing unlike the relations given in two-node situation.

The above relations can be represented in matrix notation. Using the notation given above, (I) can be written as:

$$\begin{aligned}
&P*X \geq A1*X;\\
&P*X \geq A2*X;\\
&Q*X \geq A1*X;\\
&Q*X \geq A2*X. \qquad \text{(II)}
\end{aligned}$$

By matrix manipulation, we can show that relations in (II) satisfy relation in (7). In other words, networks which satisfy conditions for an AND model will possess positive product synergy between the cause nodes. However, it is to be noted that the converse need not be true.

Definition 4 (XOR *model* II*):* Let a, b and x be the predecessors of c in a QPN. Variables a and b form an XOR model with respect to the observance of effect node, *i.e.*, c = C, written XOR(a, b, x, C), if for all $a_1 > a_2$, and $b_1 > b_2$ (using matrix notations)

$$\begin{aligned}
&A1*X \geq P*X;\\
&A1*X \geq Q*X;\\
&A2*X \geq P*X;\\
&A2*X \geq Q*X. \qquad \text{(III)}
\end{aligned}$$

Similarly, we can prove that the above relations exhibit negative product synergy, as defined in definition 4 and written as inequality (7). Based on the above observations, we extend the generalizations for inter-causality (Theorem 1) by providing sufficiency conditions. These conditions are based on the proposed AND and XOR models. The following theorem captures these conditions. The proof, again, is direct from the arguments detailed above.

Theorem 2*b* (Sufficiency conditions for product synergy II): Let a, b and x be the predecessors of c such that a and b are conditionally independent given y, i.e., $S^0(a, b)$. Let x be an uninstantiated cause node and c be a propositional node. A sufficient condition for $S^-(a, b)$ upon the observation of node c (*i.e.*, c = C) is that the network of a, b, c should form an XOR(a, b, x, C) model.

Instantiated cause nodes

The generalizations are not straightforward if the irrelevant cause node (node **x** in the above case) is instantiated. In a multicause network, we need to find combinations of cause nodes with respect to observance (in a propositional sense) of the effect node. See Fig. 5 for a possible set of combinations when the relevant cause nodes (here a and b) form an XOR model with respect to the effect node (here c). The situation where the relevant cause nodes form an AND model with respect to the effect node can be similarly dealt. Irrelevant cause node, x, can be instantiated to one of the possible states: X or X′.

Theorem 3*a* (Product synergy III) Let a, b, x be the predecessors of c in a QPN. Assume all nodes are propositional. Let node x be instantiated to X′. Let nodes a and b form an XOR model with respect to c; nodes a and x form an XOR model with respect to c; and nodes b and x form an AND model with respect to c. The nodes a and b exhibit negative product synergy with respect to a particular value of c, say C, written $X^-(\{a, b, X', C)$, iff,

$$Pr(C/A, B', X')*Pr(C/A', B, X') \geq Pr(C/A, B, X')*Pr(C/A', B', X').$$

Theorem 3*b* (Product synergy III): Let a, b, x be the predecessors of c in a QPN. Assume all nodes are propositional. Let node x be instantiated to X. Let $S^-(b, c)$ be present between nodes b and c. Let nodes a and b form an XOR model with respect to c; nodes a and x form an XOR model with respect to c; and nodes b and x form an AND model with respect to c. Nodes a and b exhibit negative product synergy with respect to a particular value of c, say C, written $X^-(\{a, b, X, C)$, iff,

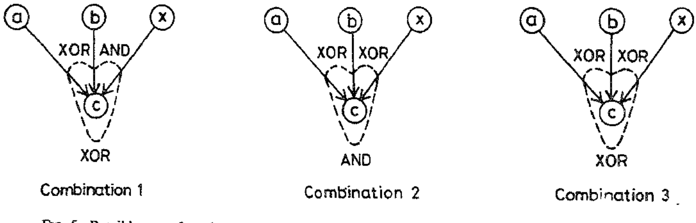

FIG. 5. Possible sets of combinations when nodes a and b form XOR model with node c.

$$\Pr(C/A, B', X)*\Pr(C/A', B, X') \geq \Pr(C/A, B, X')*\Pr(C/A', B', X').$$

Theorem 4 (Product synergy III): Let a, b, x be the predecessors of c in a QPN. Assume all nodes are propositional. Let node x be instantiated to X′. Let nodes a and b form an XOR model with respect to c; nodes a and x form an XOR model with respect to c; and nodes b and x form an XOR model with respect to c. The nodes a and b exhibit negative product synergy with respect to a particular value of c, say C, written $X^-(\{a, b, X', C)$, iff,

$$\Pr(C/A, B', X')*\Pr(C/A', B, X') \geq \Pr(C/A, B, X')*\Pr(C/A', B', X').$$

Proofs for Theorems 3a and b are given in Appendix Ia. Other theorems can be proved using a similar approach.

Note: There are two more combinations that can be formed, which however have the same inequality as given in Theorems 3a and b.

Combination 1: Let nodes a and b form an XOR model with respect to c; nodes a and x form an AND model with respect to c; and nodes b and x form an XOR model with respect to c. Nodes a and b exhibit negative product synergy with respect to a particular value of c, say C, written $X^-(\{a,b,X',C)$,

$$\Pr(C/A, B', X')*\Pr(C/A', B, X') \geq \Pr(C/A, B, X')*\Pr(C/A', B', X')$$

(which is the same as the relation in Theorem 3a).

Combination 2: Let nodes a and b form an XOR model with respect to c; nodes a and x form an AND model with respect to c; and nodes b and x form an XOR model with respect to c. Nodes a and b exhibit negative product synergy with respect to a particular value of c, say C, written $X^-(\{a, b, X, C)$,

$$\Pr(C/A, B', X)*\Pr(C/A', B, X) \geq \Pr(C/A, B, X)*\Pr(C/A', B', X)$$

(which is the same as the relation in Theorem 3b).

Apart from the combinations listed above not being exhaustive, there are certain combinations which are semantically contradicting. One of them is: nodes a and b forming an XOR model, nodes b and x forming an AND model, and nodes a and x forming AND model. In this situation, we can generalize the conditions for negative product synergy between nodes a and b only if $S^+(b, c)$ and $S^-(b, c)$ are present simultaneously. This contradicts the definition of qualitative influence. In addition, we can intuitively reason the invalidity of this combination.

Combinations resulting from nodes a and b being an AND pattern can be dealt with in a similar way.

Appendix IIb

Proofs for Theorems 3a and b are given below. Proofs for other theorems follow similar lines.

Theorem 3a (Product synergy III): Let a, b, x be the predecessors of c in a QPN. Assume all nodes are propositional. Let node x be instantiated to X′. Let $S^-(b, c)$ be present between nodes b and c. Let nodes a and b form an XOR model with respect to c; nodes a and x form an XOR model with respect to c; and nodes b and x form an AND model with respect to c. Nodes a and b exhibit negative product synergy with respect to a particular value of c, say C, written $X^-(\{a, b, X', C)$, iff,

$$\Pr(C/A, B', X')*\Pr(C/A', B, X') \geq \Pr(C/A, B, X')*\Pr(C/A', B', X').$$

Theorem 3b (Product synergy III): Let a, b, x be the predecessors of c in a QPN. Assume all nodes are propositional. Let node x be instantiated to X. Let $S^-(b, c)$ be present between nodes b and c. Let nodes a and b form an XOR model with respect to c; nodes a and x form an XOR model with respect to c; and nodes b and x form an AND model with respect to c. Nodes a and b exhibit negative product synergy with respect to a particular value of c, say C, written $X^-(\{a, b, X, C)$, iff,

$$\Pr(C/A, B', X)*\Pr(C/A', B, X) \geq \Pr(C/A, B, X)*\Pr(C/A', B', X).$$

Proof

Case 1: When node x is instantiated to X

From Theorem 2b above, as nodes a and b form an XOR model with c, there should be a negative inter-causal relationship present between the cause nodes. Thus, on the observation of the effect node, *i.e.*, when c takes the value C, then there will be a negative qualitative influence, *i.e.*, $S^-(a, b)$. Similarly, the other intercausal relationships that are present are $S^+(b, X)$ and $S^-(a, X)$. We try to generalize the inter-causal relations of the complete network.

Since $S^-(a, b)$, then by definition 1 (above),

$$\Pr(A/B', X, C) \geq \Pr(A/B, X, C).$$

This can be written as,

$$\frac{\Pr(A, B', C)}{\Pr(B', X, C)} \geq \frac{\Pr(A, B, X, C)}{\Pr(A, B, C)}.$$

This is equivalent to,

$$\frac{\Pr(C/A, B', X)*\Pr(A, B', X)}{\Pr(B', X, C)} \geq \frac{\Pr(C/A, B, X)*\Pr(A, B, X)}{\Pr(B, X, C)}.$$

which is equivalent to

$$\frac{\Pr(C/A, B', X)*\Pr(A, B', X)}{\Pr(C/B', X)*\Pr(B', X,)} \geq \frac{\Pr(C/A, B, X)*\Pr(A, B, X)}{\Pr(C/B, X)*\Pr(B, X)}.$$

Since nodes a, b and x are independent, the above inequality can be written as:

$$\frac{\Pr(C/A, B', X)*\Pr(C/B, X)}{\Pr(C/B', X)} \geq \Pr(C/A, B, X).$$

Since nodes b and x form an AND model, the ratio Pr(C/B, X)/Pr(C/B′, X) according to relations (3.1) and (3.2) will be greater than 1. Thus, the above inequality can be written as,

$$\Pr(C/A,B',X) \geq \Pr(C/A,B,X). \tag{IV}$$

However, the general conditions for the case when node a takes the value A′ is not straightforward. The additional condition we assume here is that there is S^{-}(b, c).

Starting from the definition of S^{-}(a, b), we have,

$$\Pr(A'/B, X, C) \geq \Pr(A'/B', X, C).$$

The subsequent simplification of the above relation gives rise to

$$\frac{\Pr(C/A',B,X)*\Pr(C/B',X)}{\Pr(C/B,X)} \geq \Pr(C/A',B',X).$$

From the definition of S^{-}(b, c) we have $\Pr(C/B', X) \geq \Pr(C/B, X)$. The above relation can be written as:

$$\Pr(C/A',B,X) \geq \Pr(C/A',B',X) \tag{V}$$

Multiplying relations IV and V, we get:

$$\Pr(C/A, B', X)*\Pr(C/A', B, X) \geq \Pr(C/A, B, X)*\Pr(C/A', B', X).$$

Case 2: When node x is instantiated to X′

If we assume that node x is instantiated to X′ by similar argument as above, we get

$$\Pr(C/A,B', X') \geq \Pr(C/A, B, X'). \tag{VI}$$

The other scenario that could happen in this network is that node a taking a value A′. From the definition of S^{-}(a, b), we have

$$\Pr(A'/B, X', C) \geq \Pr(A'/B', X', C).$$

If we proceed on similar lines as above, we get:

$$\Pr(C/A', B, X') \geq \Pr(C/A', B', X'). \tag{VII}$$

Multiplying VI and VII, we get

$$\Pr(C/A, B', X')*\Pr(C/A', B, X') \geq \Pr(C/A, B, X')*\Pr(C/A', B', X').$$