J. Indian Inst. Sci., Sept.–Oct. 1996, 76, 615–620

Spatial coherence enhancement through source masking: Spectral modifications introduced by the mask

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Received on February 10, 1995, Revised on September 13, 1995.

Abstract

We report spectral modifications introduced by uniformly redundant array (URA) when it is used as a source mask to enhance spatial coherence of a partially coherent light source on propagation. In the far field the URA introduces increasing spectral broadening and shift as the source correlation l is varied from 300 to 600 μm. However, spectral broadening in the far field did not follow any set pattern to prove a reciprocity relation between the enhancement of spatial coherence and decrease in temporal coherence

Keywords: Spatial coherence, temporal coherence, imaging impulse response

1. Introduction

In a recent work[1] we have described the application of a certain mask for the enhancement of spatial coherence in the far field of a partially coherent source. The mask used was the decoding mask of coded aperture imaging of X-ray sources and X-ray illuminated objects called the uniformly redundant array (URA)[2]. The URA, like other codes of X-ray imaging, has a sharp autocorrelation. It is this property of URA that is utilized in the reported work of coherence enhancement. For, the sharp autocorrelation of the mask results in a flat and uniform power spectral density on Fourier transformation. Employing Van Cittert–Zernike theorem it is easy to see that the sharp autocorrelation of the URA also results in a flat and uniform mutual intensity in the far field of the source masked by the URA. The main results of the work reported in Boopathi *et al.*[1] are: (i) The mutual intensity in the far field is given by the Fourier transform of the autocorrelation of the mask weighted by the correlation function of the input source. (ii) As the correlation length of the input source is increased from $l = 100\ \mu$m the mutual intensity in the far field increased correspondingly until it asymptotically approached its coherent limit of unity for the input correlation length of $l \simeq 800\ \mu$m. (iii) Even for very low correlation lengths of the starting source ($l = 100\ \mu$m) the effect of the mask in enhancing coherence in the far field was significant. (iv) The theoretical results were verified experimentally.

Masks used for coded-aperture X-ray imaging were developed essentially to enhance the energy throughput of the existing pinhole cameras. These masks did produce a gain in light throughput, but the image resolution obtained was poorer in comparison to that from the pinhole camera. The imaging impulse response given by the autocorrelation of the mask is not an ideal δ-function but has finite width and shape. This impulse response function imposes spatial frequency attenuation and cut-off on the object being imaged. Analogously when these masks were used to enhance spatial coherence of light on propagation we believe that we must have lost something on another score. The present work is a theoretical investigation aimed at ascertaining the effect of the code on the spectrum of light in the far field.

It was first shown by Wolf[3,4] that, in general, the normalized spectrum of light does not remain unchanged on propagation. Unless the correlation across the source is such that the degree of spectral coherence depends on the light frequency only through $k(\rho_2-\rho_1)$ (where $k = \omega/c$, ρ_1 and ρ_2 are position vectors of two representative points at the source plane, ω is the angular frequency of light and c, its velocity) the spectrum of light in the far field will be different from the source spectrum. Sources with degree of spectral coherence which are functionals of $k(\rho_2-\rho_1)$ are said to satisfy a certain *scaling law*. These results were also experimentally verified by Morris and Faklis[5].

Recently it was also shown by Foley[6,7] and Kandpal *et al.*[8] that an aperture placed on a source that obeys the scaling law can make it to violate the scaling law. The light in the far field from the aperture is found to have a spectrum shifted from the original source spectrum. The amount of spectral shift is shown to be proportional to a/l where a is the linear dimension of the aperture and l, the correlation length of source. (For a recent review of methods of introducing spectral shift see Kandpal *et al*[9]). The URA is also an aperture which renders the input source violating the scaling law and the light from it on propagation will have a spectrum that is different from the input spectrum.

In this paper we make use of the results of Foley[6,7] to investigate the spectral modifications introduced by the URA. Since the URA is made up of a collection of subapertures, the far-field spectrum, as we shall see, is both shifted and broadened with respect to the original source spectrum. The amount of broadening and shift depends also on the source correlation length. We show here that the enhancement of spatial coherence in the far field introduced by the URA is accompanied by a broadening and shift in the far-field spectrum compared to source spectrum.

In Section 2, a theoretical expression for the far-field spectrum of an aperture is arrived at. It is shown that this spectrum is different from the original source spectrum and is dependent on: (i) the correlation of the source, and (ii) the observation position in the output plane. Using this expression the far-field spectrum is evaluated in Section 3 for an URA illuminated by a partially coherent source with Gaussian spectral intensity. The central wavelength was chosen to be 541.8 nm and the full width as half maximum (FWHM) of the spectral profile was kept at 9 nm. The source correlation length is varied from 100 to 600 μm and the corresponding spectral shift and increase of spectral bandwidth are calculated. In Section 4, we give our concluding remarks.

2. Far-field spectrum of the aperture

The optical system shown in Fig. 1 has an input source code (SC) illuminated by a Gauss correlated source with complex degree of spatial coherence

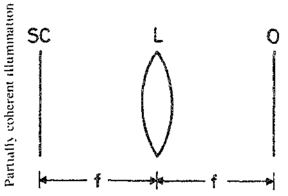

FIG. 1. The optical system SC: source code, L: Fourier transforming lens, and O: observation plane.

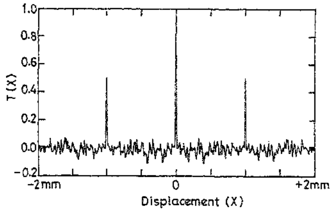

FIG 2. The computed autocorrelation of the URA along the horizontal cross section (eqn 5).

$$\mu(x_1,x_2)=\exp\left[-(x_2-x_1)^2/2l^2\right]. \tag{1}$$

Here l is the correlation length of the source. If we assume that $S_l(\omega)$ is the source spectrum of light illuminating the source code then the cross-spectral density W of the light at x_1 and x_2 in the source code is given by

$$W(x_1,x_2,\omega)=S_l-(\omega)\mu(x_2-x_1). \tag{2}$$

If the aperture amplitude transmittance is $t(x)$ then under paraxial approximation the spectrum of light at the back-focal plane of the lens L is given by the formula[7]

$$S(\rho,f,\omega)=(1/\lambda f)^2\int_{URA}\int_{URA}W_1(x_1,x_2,\omega)\,t^*(x_2)\exp\left[-(jk/f)(x_2-x_1).\rho\right]dx_1dx_2$$

$$=S_l(\omega)(1/\lambda f)^2\int_{URA}\int_{URA}\mu(x_2-x_1)t(x_1)t^*(x_2)\exp\left[-(jk/f)(x_2-x_1).\rho\right]dx_1dx_2. \tag{3}$$

Here k is the modulus of the propagation vector of light, f, the focal length of the lens L, and p, the position vector of a typical point at the back-focal plane. The source spectrum $S_l(\omega)$ after multiplication by the result of the double integral gives the spectrum of light in the output plane. We can simplify the integral in eqn 3 by the following substitution: $x=x_2-x_l$ and $X=\frac{x_1+x_2}{2}$.

The integral then becomes

$$(1/\lambda f)^2\int_{URA}\int_{URA}\mu(x)t\left(X+\frac{x}{2}\right)t^*\left(X-\frac{x}{2}\right)\exp\left[-(jk/f)x.\rho\right]dxdX. \tag{4}$$

The integral with respect to X gives the autocorrelation of the mask, *i.e.*,

$$T(x)=\int t\left(X+\frac{x}{2}\right)t^*\left(X-\frac{x}{2}\right)dX. \tag{5}$$

So the modification is effected through the Fourier transformation of the autocorrelation of the mask multiplied by the complex degree of spatial coherence of the source.

The autocorrelation of the mask is shown in Fig. 2. Apart from the central peak and the side peaks there is a train of pseudo-random subpeaks of average height that is approximately 10% of the central peak. It is these pseudo-random subpeaks (which will be absent in the case of a pinhole) that prevent the URA from attaining the resolution that the pinhole is capable of. We observed that the central peak of the autocorrelation of the URA is about 30 μm in an autocorrelation of 4 mm width.

In eqn 3 which evaluates the far-field field spectrum from the URA, the autocorrelation of the URA is multiplied by $\mu = \exp[-(x)^2/2l^2]$ before Fourier transformation. As l, the source correlation, is increased from 100 μm, the contribution from the pseudo-random peaks of the autocorrelation in the evaluation of eqn 3 steadily increases. This contribution to the factor multiplying and modifying the source spectrum also contributes to the shift and broadening of the source spectrum provided l is in a suitable range that makes the input source partially coherent. The term $\left(\frac{1}{x^2 f^2}\right)$ present in eqn 3 also contributes to the spectral shift towards blue. The role played by l in determining the amount of spectral shift and broadening in the far field can be verified by the numerical evaluation of eqn 3. This we do in the next section.

3. Numerical experiments

Equation 3 is numerically evaluated to get the spectrum of light, $S(\rho, f, \omega)$, at the output plane of Fig. 1. The autocorrelation of the URA is approximated to a nearly circular symmetric central peak surrounded by an array of pseudo-random subpeaks. In evaluating eqn 3 we have considered only a 1–d cross section of the autocorrelation. The aperture in the input is the decoding URA of Canon and Fenimore[2], a distribution of pixels of +1 and −1 amplitude transmittance. The URA used has a mosaic of the basic pattern which has 73 × 71 subcells. The mosaic pattern we had used has 146 × 142 subcells and has an overall dimension of 2 × 2 mm. As stated earlier, the input source spectrum was assumed to be Gaussian with a peak at 541.8 nm and FWHM = 9 nm and complex degree of spatial coherence $\mu = \exp[-(x_2-x_1)^2/2l^2]$. The focal length of the lens L (Fig. 1) is 380 mm. The correlation length l is varied from 100 μm (low spatial coherence) to 600 μm (beyond which the illumination is assumed to be coherent) since the basic subcell of the URA used is 15 × 15 μm. The source spectrum is evaluated at the output plane at $\rho = 0$ (on axis) and at off-axis points up to $\rho = 5$ mm. Our first observation is that when we suppressed the pseudo-random peaks and considered only the central peak of the autocorrelation, the far-field spectrum was unchanged compared to the source spectrum except for constant spectral shift ($\simeq$ 0.1 nm) throughout the output plane. Hence without pseudo-random peaks the URA should behave exactly like a pinhole. The inability to produce wider masks which on the one hand have sharp autocorrelation peaks similar to the pinhole and on the other are without the pseudo-random peaks results in the observed spectral changes in the far field. The amount of shift and broadening are reported below.

The spectral shift in the far field is summarized in Table I and the broadening in Table II. From Table I we see that at the on-axis point ($\rho = 0$) the shift is very little and

Table I
Spectral shift at the output plane

Source correlation l (μm)	Spectral shift (nm) at different ρ (mm) values					
	0	1	2	3	4	5
600	0.05	0.22	0.55	0.71	0.35	0.75(r)
500	0.05	0.17	0.36	0.43	0.47	0.03(r)
400	0.05	0.12	0.20	0.21	0.30	0.16
300	0.05	0.10	0.11	0.11	0.21	0.16
200	0.05	0.09	0.08	0.09	0.14	0.10
100	0.05	0.06	0.06	0.07	0.06	0.08

r: spectrum shifted towards red.

Table II
Spectral broadening at the output plane

Source correlation l (μm)	Increase in spectral bandwidth (nm) at different ρ(mm) values		
	3	4	5
600	0.31	0.70	0.83
500	0.15	0.28	0.58
400	0.02	0.04	0.23

does not vary with l. At the off-axis points (starting with $\rho = 1$ mm till $\rho = 5$ mm, beyond which we did not compute) the amount of shift showed a tendency to increase with l. All the shifts were towards blue except at the 5 mm point. At $\rho = 5$ mm the shift was towards blue for $l = 100$ to 400 μm and then for $l = 500$ μm and 600 μm it was towards red. Also it is to be noted that for values of l up to and below 300 μm the shifts are quite small ($\simeq 0.1$ nm).

The blue shift at the axis is due to $1/\lambda^2 f^2$ multiplier in eqn 3. As one moves away from the axis, blue shift is due both to $1/\lambda^2 f^2$ and the contribution from integration. The integration is a Fourier transform of the weighted autocorrelation of the mask. When the source correlation is relatively small the contribution from pseudo-random peaks will be negligible and the result of integration in eqn 3 is a decreasing function in the variable $\rho/\lambda f$. This function contributes to the blue shift. When l is reasonably large, at large ρ values, this function undergoes slope reversals. In this range of ρ, when the function on the right-hand side of eqn 3 undergoes its first slope reversal, the output spectral shift is towards red.

Table II gives the increase in FWHM of the far field in comparison to the source spectrum. Since the increase was noticed to be very small at and around the on-axis point the table gives the spectral broadening from $\rho = 3$ to 5 mm. For l below 400 µm the broadening was negligibly small. For these values of l, multiplication by $\exp[-(x)^2/2l^2]$ suppresses the contributions from the source spectrum. For $l = 400$ to 600 μm there was significant spectral broadening which steadily increased as we moved away from the axis (Table II).

The maximum broadening was about 0.83 nm at $\rho = 5$ mm at source correlation of $l = 600$ μm and is shown in Fig. 3 as a typical example of shifted and broadened spectrum along with the original source spectrum.

4. Conclusions

The major observations of Boopathi *et al.*[1] were that (i) as the source correlation l was increased from 100 to 600 μm, $|\gamma(\Delta r)|$, the normalized mutual coherence function[1] also

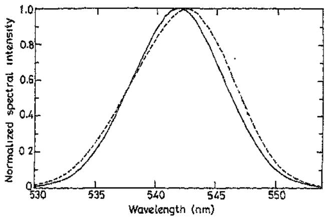

FIG. 3. A typical shifted and broadened spectrum (dashed line) at an off-axis point ($\rho = 5$ mm) of the output plane compared to the source spectrum (continuous line). Source correlation length $l = 600$ μm.

increased, and (ii) the increase in $|\gamma\ (\Delta r)|$ was more significant away from the axis (*i.e.*, $\Delta r \geq 1$ mm). We see from results here that these improvements in $|\gamma\ (\Delta r)|$ are accompanied by shift and broadening of the output spectral profile. The broadening in spectral bandwidth at any off-axis point increased with l. Also the spectral bandwidth for a constant value of l increased as we moved away from the axis beyond, say, $\rho = 3$ mm. The spectral shifts too increased with l at any off-axis point, and it showed a similar increase for any l as we moved away from the axis. The shifts were always towards blue except for $l = 500$ and 600 μm when the spectrum $\rho = 5$ mm shifted towards red. Even though at any off-axis point for l going from 400 to 600 μm the increase in $|\gamma\ (\Delta r)|$ is accompanied by an increase in spectral bandwidth, these changes did not follow any set pattern to arrive at a reciprocity relation between increase in $|\gamma\ (\Delta r)|$ and decrease in temporal coherence as a consequence of spectral broadening.

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