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Robust multichannel image restoration using Markov random field

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Abstract

This paper addresses the problem of color image restoration, distorted by intra- and inter-channel blur, and corrupted by additive noise. The image is modeled as Markov random field, and color image restoration is cast as a maximum *a posteriori* (MAP) estimation problem. We propose a first-order interchannel interaction model for restoration. We compare this model with the results when interaction is not considered. The proposed model is fairly general, and the results are satisfactory even when interchannel degradation parameter is unknown.

Keywords: Image processing, color images, image restoration, Markov random field, multichannel images.

1. Introduction

The aim of image restoration is to recover the original from the degraded image, corrupted by noise. Typically, degradation is modeled by a linear blur. Sources of blur are motion blur, out-of-focus blur, atmospheric turbulence, and others. Noise may arise from digitization and quantization, transmission and recording medium, thermal noise, etc. Classical methods of image restoration[1,2] assume the blurring operation is exactly known a priori. Observed image is then de-convolved using known blurring function. This approach basically involves solution to Fredholm integral equation of the first kind, which are usually ill-conditioned.[3] Such ill-conditioning can be overcome by regularization procedures.[4]

Restoration of monochrome images, when the blurring function is known, is well addressed in the literature. Some of the techniques are listed elsewhere.[5–7] Results for color image restoration are not all that satisfactory mainly because the psychology of color image perception in human beings is not fully understood.

Nonlinear model for color images has been reported in literature. Panjwani and Healy[8] propose a scheme for segmentation, wherein a color pixel at each location (i, j) is a linear combination of color components of the neighbors. The neighborhood includes both within and across the channel neighbors. In the above, the number of parameters involved is large. Galatsanos *et al.*[9] extend the model given by Andrew and Hunt[1] to include the interchannel parameters in H. They use a 'weighted 3D operator defined by $3 \times 3 \times 3$ convolution mask' and use a least squares approach for energy minimization. Here again the number of parame

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ters is large. Zhu and Galatsanos[10] use the model proposed by Galatsanos *et al.*[9] They use deterministic multichannel filters and compare the results obtained by cross-validation methods. Tom *et al.*[11] use the same model of Galatsanos *et al.*[9] for identification and restoration within and across the channel blurs. They use maximum likelihood objective functions and then the expectation maximization algorithm for optimization. Markov random field (MRF) model for color images was suggested by Daily[12] for segmentation.

We propose a simplified model[13] using lesser number of parameters. We use a probabilistic approach and global minimization, namely, simulated annealing. The parameters for our purpose can be estimated by the method of Nanda *et al.*[14]

In this paper, we assume color planes interact with each other. To account for the interchannel blurring, we propose an interchannel interaction model, the first-order interchannel interaction (FOII). Then, a probabilistic approach is used by modeling the color image as MRF. Restoration problem is then cast as a maximum a posteriori (MAP) estimation problem.[15] The proposed model is quite robust and works well even when the amount of interchannel degradation is not taken into consideration, or when the degradation takes place in some other coordinates. In general, the energy function will be nonconvex with multiple local minima and nonunique global minima. We use simulated annealing (SA) algorithm with inverse log cooling schedule for energy minimization.

2. Image model

Let X be the lexicographically ordered (row-transposed stacking) vector for an $M \times M$ image.[1]

Definition 1: *X is a Markov random field if and only if*

$$P[X_{i,j} = x_{i,j} \mid X_{k,l} = x_{k,l}, \forall (k, l) \neq (i, j)] = P[X_{i,j} = x_{i,j} \mid X_{k,l} = x_{k,l}, (k, l) \in \eta_{i,j}] \tag{1}$$

where P[|] is the conditional probability and $\eta_{i,j}$ is the neighborhood of (i, j). The neighborhood condition is translation independent except at boundaries where a free boundary assumption is made.

Now, according to the Hammersly and Clifford theorem,[15, 16] $P[X = x]$ can be written as:

$$P[X = x] = \frac{1}{Z}\exp(-U(x)). \tag{2}$$

The normalizing constant Z (the partition function) is given by

$$Z = \sum_{\text{all config.}x} \exp(-U(x)) \tag{3}$$

and $U(x)$ is the (Gibbs) energy function given by

$$U(x) = \sum_{c \in \mathscr{C}} V_c(x) \tag{4}$$

with $\mathscr{C}$ being the set of all *cliques*.[16] A typical unconditional problem would be to estimate a configuration x, such that $P[X = x]$ is maximized, or equivalently, $U(x)$ is minimized.[17]

We extend the monochrome image observation model as given in Andrews and Hunt[1] for the color image as:

$$Y^c = \mathbf{H}X^c + N^c \text{ for } c = 1, 2, 3 \tag{5}$$

where c is the color *plane*. For a color plane c, Y^c is the observed image, X^c the original image, and N^c the corrupting noise vector which is assumed to be independent of X^c. **H** is the blurring matrix which is assumed to be the same for all color planes. Note that for an image of size $M \times M$, X^c, Y^c, and N^c, all are lexicographical-ordered column vectors of size $3M^2 \times 1$.

The structure of X is

$$X = [X_{0,0}\ X_{0,1} \ldots X_{M-1,M-1}]^T \tag{6}$$

where

$$X_{i,j} = [x^r(i,j)\ x^g(i,j)\ x^b(i,j)]^{\mathrm{T}},\ 0 \le i, j \le M-1. \tag{7}$$

The structure of Y and N is similar to that of X.

H is a $3M^2 \times 3M^2$ matrix, whose structure is similar to the one given in Galatsanos *et al.*[9] and Bhat and Desai.[18] The exact structure of **H** is as given below

$$\mathbf{H} = \begin{array}{cccccc} \mathbf{H}_\xi & \mathbf{H}_1 & \mathbf{H}_1 & 0 & \mathbf{H}_1 & \mathbf{H}_1 \\ \mathbf{H}_1 & \mathbf{H}_\xi & & & 0 & \mathbf{H}_1 \\ & \vdots & & & & \\ \mathbf{H}_1 & \mathbf{H}_1 & & \mathbf{H}_1 & \mathbf{H}_1 & \mathbf{H}_\xi \end{array} \tag{8}$$

for an $M \times M$ image

where

$$\mathbf{H}_\xi = \begin{pmatrix} \overline{\mathbf{H}}_\xi & \overline{\mathbf{H}}_1 & \mathbf{H}_1 & & \overline{\mathbf{H}}_1 & \overline{\mathbf{H}}_1 \\ \overline{\mathbf{H}}_1 & \mathbf{H}_\xi & \mathbf{H}_1 & \mathbf{H}_1 & 0 & \mathbf{H}_1 \\ & & \vdots & & \vdots & \\ \mathbf{H}_1 & \mathbf{H}_1 & 0 & \overline{\mathbf{H}}_1 & \overline{\mathbf{H}}_1 & \mathbf{H}_\xi \end{pmatrix} \tag{9}$$

and $\overline{\mathbf{H}}_1$ is 3×3 identity matrix.

Structure of $\mathbf{H}_1$ will be the same as that of $\mathbf{H}_\xi$ with $\overline{\mathbf{H}}_\xi$ replaced by $\overline{\mathbf{H}}_1$. The **H** matrix is appropriately normalized.

The term ξ decides the amount of interchannel blurring operation. At the pixel level, the above model can be re-written as

$$y^c(i,j) = \sum_{(k,l)\in \boldsymbol{S}} h(k,l)x^c(i-k, j-l) + n^c(i,j) \tag{11}$$

where $\boldsymbol{S}$ is the support of the point spread function (PSF).

Let $X = [X^1\ X^2\ X^2]^{\mathrm{T}}$, and $Y = [Y^1\ Y^2\ Y^2]^T$. We assume that X is an MRF; thus, X has the probability distribution given in eqn (2). Pictorially, the interchannel blurring (for red color plane) is shown in Fig. 1.

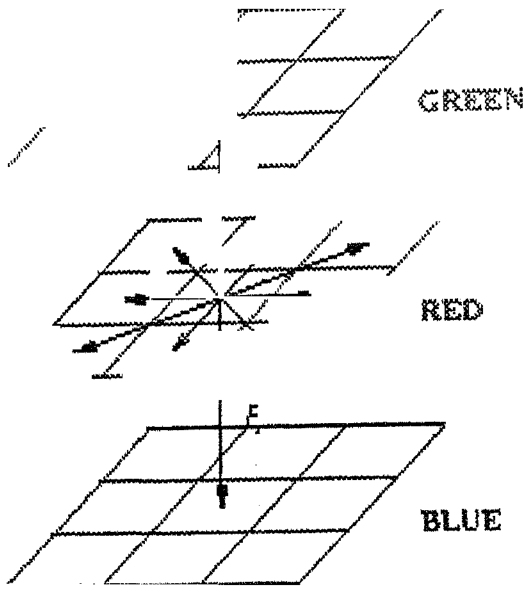

FIG. 1. Degradation model.

With color image observation model as described above, the color image restoration problem can be cast as:

Estimate X such that $P[X = x|Y = y]$ is maximized with respect to x^c, $c = 1, 2, 3$.

We further make the assumption that $N = [N^1\ N^2\ N^3]^T$ is normally distributed with zero mean and covariance matrix $\sigma^2 I$ (I being a $3M^2 \times 3M^2$ identity matrix). Moreover, N is statistically independent of X.

In general, the a priori energy function $U(x)$ will have the three color planes, X^1, X^2 and X^3, nonlinearly interacting with each other. Moreover, to take care of discontinuities in the color planes, one could also incorporate horizontal and vertical line fields[16] corresponding to each color plane. Thus, the most general form for the a posteriori energy ($U_p(x)$) function could be expressed as

$$U_p(x) = U(x, l, v) + \frac{\|Y - \mathbf{H}X\|^2}{2\sigma^2}$$

with

$$U(x, l, v) = f(x^1, x^2, x^3, l^1, l^2, l^3, v^1, v^2, v^3)$$

where l^c and v^c represent, respectively, horizontal and vertical line fields corresponding to the color plane c.

We propose to solve $U(x, l, v)$ by two different methods:

1. In this form we assume that the color planes are uncorrelated:

$$U_1(x,l,v)=\sum_{c=1}^{3}U_1\left(x^c,l^c,v^c\right) \tag{12}$$

and

$$U_1(x^c, l^c, v^c) =$$

$$\Sigma_{i,j}\,\mu\left[\left(x^c_{i,j}-x^c_{i,j-1}\right)^2\left(1-v^c_{i,j}\right)+\left(x^c_{i,j}-x^c_{i-1,j}\right)^2\left(1-l^c_{i,j}\right)\right]+\gamma\left[l^c_{i,j}+v^c_{i,j}\right] \text{ for } c=1,2,3, \tag{13}$$

where

$$l^c_{i,j}=1\left(\left|x^c_{i,j}-x^c_{i-1,j}\right|-\theta^c_l\right) \text{ and } v^c_{i,j}=1\left(\left|x^c_{i,j}-x^c_{i,j-1}\right|-\theta^c_v\right.$$

with θ's being the respective thresholds.† The term in the first bracket of the energy function signifies the interaction between the neighboring pixels. The second bracket term provides the penalty for every discontinuity created and prevents spurious discontinuities.

2. In this form, we take into account first-order interchannel interaction between the color planes.

$$U_2(x,l,v)=\sum_{c=1}^{3}\sum_{d=1}^{3}\sum_{i,j}\mu\left[\left\{\left(x^c_{i,j}-x^c_{i-1,j}\right)\left(x^d_{i,j}-x^d_{i-1,j}\right)\left(1-l^c_{i,j}\right)\left(1-l^d_{i,j}\right)\right\}+\right.$$

$$\left.\left\{\left(1-v^c_{i,j}\right)\left(1-v^d_{i,j}\right)\left(x^c_{i,j}-x^c_{i,j-1}\right)\left(x^d_{i,j}-x^d_{i,j-1}\right)\right\}\right]+\gamma\left[l^c_{i,j}+v^c_{i,j}+l^d_{i,j}+v^d_{i,j}\right] \text{ for } c=1,2,3. \tag{14}$$

Minimization of the *a posteriori* cost function can be done using a variety of minimization algorithms. We opted for the simulated annealing algorithm because it guarantees convergence in probability.

3. Simulation

We have considered a linear blur of size 5×5. Then, the observation model (11) can be rewritten as (for $\xi = 1$)

$$y^r(i,j)=\frac{1}{27}\sum_{(k,l)=0}^{(k,l)=5}\left[h(k,l)x^r(i-k,j-l)\right]+x^g(i,j)+x^b(i,j)+n^r(i,j). \tag{15}$$

This is the equation for the red plane. Similarly, equations for green and blue planes are also defined.

Here, we report the simulation results for the FOII energy functions given by (14) and compare its performance with the usual linear energy function given by (13).

For simulation purpose, we used a uniform, space invariant blur. White Gaussian noise was added (with $\sigma = 10$) separately on R, G and B. This generates blurred noisy (degraded) image. We have worked on a 64×64 synthetic image and 128×128 Lisa image. Values of μ, γ and $\theta^c_l=\theta^c_v=\theta$, respectively, were 0.025, 200.5 and 15 for synthetic image and 0.75, 150

†$1(z) = 1$ if $z > 0$ and 0 otherwise.

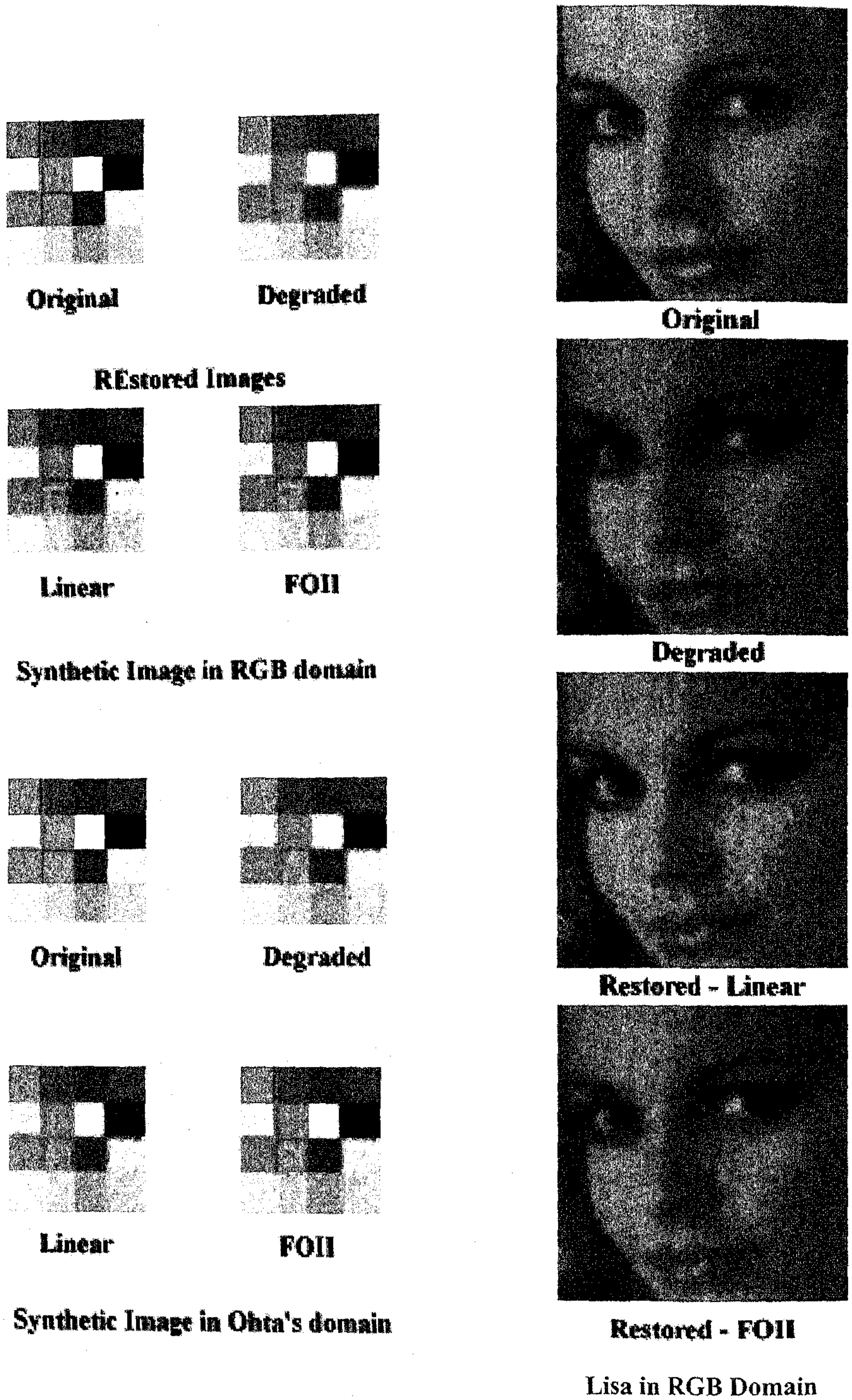

FIG. 2. Simulation results.

and 20, respectively, for Lisa image. The number of iteration-stopping criterion for SA was 1250 for both the images. The initial temperature was 5.15 and inverse log cooling schedule was used. The value ξ selected was 1. All the parameters were selected by trial and error method. *Even though the value of* $\xi = 1$ *was selected during the degradation process, while restoring,* ξ *was selected to be* 0 (*zero*). Simulation results are shown in Fig. 2.

To validate the performance, signal-to-noise ratio (SNR) of degraded and estimated images is computed as ($\hat{x}$ being the estimated value of x):

$$\mathrm{SNR} = 10 \log \frac{}{x - \hat{x}\|^{2}} \tag{16}$$

The results of our simulation are listed in Table I.

4. Discussion

As can be seen from Table I, the proposed FOII performs better than the linear model. However, when the degradation on each color plane is independent of the other ($\xi = 0$), it is found that the performance of the proposed model is almost similar to the linear model, which is expected.

The proposed model also works satisfactorily when degradation was done in other color coordinates. We linearly degraded in the YIQ and Ohta *et al.*'s I_1, I_2, I_3 coordinates.[19] The resulting image was transformed to *RGB* domain. This effectively will do the interchannel blurring with ξ unknown. Even then, satisfactory results were obtained with FOII model. Results are tabulated for synthetic image. Thus, the FOII model seems to be fairly general, independent of the value of ξ.

Table I
Results of simulation

S: Synthetic image, L: Lisa images

Table II
Synthetic image degraded in Ohta *et al.*'s (O) and YIQ (Y) coordinates

Table III
Lisa image degraded in Ohta *et al.*'s (O) and YIQ (Y) coordinates

Methodology	R		G		B	
	S	L	S	L	S	L
Degraded images	26.26	26.37	27.45	28.10	23.55	23.70
Linear	26.53	27.26	27.61	28.25	23.83	23.99
FOII	27.29	27.29	28.98	29.92	25.04	25.14

As mentioned earlier, $\xi = 1$ was selected during degradation, and $\xi = 0$ during restoration. It is found that the proposed model is quite robust. It is also found that the proposed algorithm works well irrespective of the exact knowledge of ξ.

However, this one is not an optimal model. The SNR improvements are not all that satisfactory for highly textured images (like wings of parrot and mandril image). This may be due to some of the information being irrecoverably lost because of the 5×5 blur. However, for comparatively smooth image (faces, for example) the proposed algorithm works satisfactorily.

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