J Indian Inst Sci, Sept.–Oct. 1999, **79**, 471–478

BOOK REVIEWS

Classical and spatial stochastic processes by Rinaldo B. Schinazi, Birkhauser Verlag AG, Klosterberg 23, CH-4010, Basel, Switzerland, 1999, pp. 192, sFr. 98.

Markov chains on a discrete state space is a part of the staple diet of all probabilities. Not surprisingly, there are many texts on the subject, beginning with the old classics by Kemeny, Snell and Knapp, David Freedman, etc. Typically, these will go through the 'standard stuff' such as classification of states, stationary distributions, etc. and then take up some special themes such as connections with probabilistic potential theory. Markov chains also remain one of the most useful topics in probability from an application point of view and there is another family of texts that begin with Markov chains, leading to queuing theory, branching processes, etc. (The text by Sheldon Ross is an excellent specimen of this genre.) More recently, ideas originating in Markov chain theory have also found many applications in statistical physics, broadly in the areas of percolation and interacting particle systems. The distinguishing feature here is a new spatial dimension which is not a part of classical Markov chain theory. For example, in interacting particle systems, one plants a particle per site in a spatially distributed manner. These evolve as a coupled Markov chain. What makes it different is the presence of two 'infinities': There is already the classical time asymptotics, characterizing stationary or otherwise behavior as time tends to infinity, and now there is also the thermodynamic limit as the spatial extent (in other words, number of sites) increases to infinity in a controlled manner. Entirely new phenomena surface in the latter limit, such as the existence of multiple phases. Literally, it's a whole new dimension!

These areas have undergone explosive growth since the 80s and continue to do so unabated, as even a cursory glance at the leading probability journals would indicate. There have been excellent texts on the different strands of this research, such as the books by Liggett and Durrette on interacting particles, and by Grimmiett, Durrett, Kesten on percolation. What's missing is a clean passage for a novice from the more traditional Markov chain theory to these exotic and exciting topics. The book under review is aimed at filling this gap, perhaps the first one to do so.

The book begins, as one might expect, with an overview of classical Markov chain theory, covering classification of states, birth and death processes, random walks, stationary distributions, various passage times, etc. This is all in discrete time and is followed by a chapter on continuous time birth and death processes. The last four chapters constitute what may be termed as 'special topics', these being the topics overlapping with statistical physics alluded to above. The first of these chapters is on percolation. It introduces percolation on Z^d and on a tree, characterizing two critical exponents of the latter. The chapter that follows is a very brief one on cellular automata, taking up a specific instance thereof and analysing it using a renormalization argument. The next chapter is longer, devoted to continuous time branching random walk and leading to an analysis of its phase transitions. The last of the special topics is the contact process on a homogeneous tree, again with an emphasis on the associated phase transitions. An appendix collects together some basic probability theory required as a backdrop.

The chapters on 'special topics' are rather brief. (The whole book is quite slim, for that matter.) The author's aim clearly has been not to present any of these in any great detail, but to provide a flavour of what they entail and to fix in readers' minds a few key ideas like the phase transitions, renormalisation, critical exponents, etc. He clearly has in mind the book's role as being a stepping stone to more detailed accounts of these subjects. I think that he has done a very good job of providing a user-friendly conduit from classical Markov chain theory to these exotic and exciting themes lying in the region of overlap between probability theory and statistical physics. Such a 'guided passage' was badly needed and this book has filled up an important gap in the spectrum. A recommended reading for all those who have been curious about these subjects, but have felt intimidated by the jargon gap. Also, an excellent basis for a 'topics' course, perhaps in conjunction with one or more standard texts.

School of Technology and Computer Science
Tata Institute of Fundamental Research
Mumbai 400 005

V. S. BORKAR

Computational conformal mapping by Prem K. Kythe, Birkhauser Verlag AG, P.O. Box 133, CH-4010 Basel, Switzerland, 1998, pp. 480. sFr. 128.

This book is devoted to a classical topic in the study of complex-valued functions of a single complex variable. More precisely, it is concerned with conformal maps which are analytic maps with nowhere vanishing derivative. This volume presents modern developments of the subject with necessary review of the classical theory. The main issues considered are: construction of conformal maps, practical iterative methods, their numerical analysis and computational aspects using the software Mathematica. Since these numerical algorithms are based on a formulation of the problem under consideration in terms of Fredholm integral equations, the book gives a numerical treatment of these equations. This is a fact which is not explicit from the title of the work. Though there are plenty of books dealing with numerical aspects of integral equations, there are not many with the same motivation and scope as the present book.

Techniques of conformal maps are in use for a long time in various areas: partial differential equations, fluid mechanics, elasticity, acoustics, to name a few. For instance, by means of this technique, it is possible to transform a boundary value problem posed on a geometrically complicated domain to a simpler standard domain. This has enormous advantages in problems which are domain sensitive. In fact, discretization of domains in such problems (for the implementation of finite difference/element method) introduces large errors. This is avoided if we could transform the problem to a simpler domain. After solving the problem on the simpler domain, we must translate our results on the problem domain. Thus there is need to obtain conformal transformations mapping problem domain onto simpler domains and vice versa. Both cases are investigated by the author. Yet another advantage of the conformal mapping method arises in optimal design problems of engineering structures. The optimization is done among the family of sets. The big handicap is that a family of sets is not a vector space. One of the ideas to alleviate this difficulty is to substitute sets with functions on a fixed standard domain. Needless to mention that the set of such functions carries a vector space structure. Conformal mapping techniques, when applicable, are of great help in this process. The limitation of the method is also quite well known: conformal mapping method is applicable only in two dimension. In higher dimensional problems, we need its generalizations such as quasi-conformal mappings, a topic which is not touched upon in this book.

Two-dimensional model problems arising in elasticity require conformal transformations of the interior of a bounded domain onto the unit disc. On the other hand, fluid flows past airfoils require that the exterior of a bounded domain be transformed onto the exterior of the unit disc. Transformations of both types are studied in the text.

Let us now quickly glance through the contents of the book. There are 14 chapters excluding Chapter 0 which presents historical background of the subject. Though it is not organized in this way, the contents can naturally be divided into four parts: Part One would consist of Chapters 1-6 dealing with classical developments and Part Two will include Chapters 7-9, 11 and 13. These later chapters describe the formulation in terms of various integral equations and alogorithms for solving them. Chapter 12 dealing with the effects of corner and pole-type singularities on the formulation in terms of integral equation and the subsequent numerical algorithms will constitute Part Three. Finally, Chapters 10 and 12 dealing with applications will form Part Four. Two examples of applications are considered. In Chapter 10, transformations mapping airfoil onto unit disc are obtained. Chapter 14 looks at the possibility of using conformal maps to advantage in adaptive grid generation problems in complex geometries which arise in finite difference/element methods.

In Chapter 1, Riemann mapping theorem which shows the conformal equivalence of a simply connected region different from the whole of complex plane with the unit disc is stated. This result has been the turning point for many subsequent developments in the theory of conformal mapping. In Chapters 2 and 3, Schwarz–Chirstoffel transformations mapping a polygon onto the upper half-plane are described along with additional computational issues. Chapter 4 is based on a minimization principle for conformal maps due to Bieberbach. This principle suggests automatically some canonical algorithms to compute conformal maps. Chapter 5 presents a perturbation method to construct conformal maps from a nearly circular region onto a circular disk. Relationship between Green's function of a boundary value problem and conformal map is explored in Chapter 6. Computational algorithms based on it are also given.

Chapters 7–9 form the core of the book wherein several integral equations arising in the conformal mapping theory are discussed. In these equations, the unknown is the so-called boundary correspondence function from which the conformal map can be obtained by means of evaluation of certain integrals. The integral equations obtained are mainly Fredholm equation of first kind (e.g. Symm's equation) or of second kind (e.g. Theodorsen's equation). Iterative procedures to solve these equations are described. Separate treatment for multiply-connected regions is given in Chapters 11 and 13.

The organization of the book is quite good. However, some misprints and vague passages have crept into the text and the agile reader should be aware of them. The subject matter of the book is very important both from a theoretical and a practical point of view. Suggested problems with hints and many computational tasks which can be implemented using Mathematica make the book attractive and interesting for students and teachers alike. Since many parts of the book dealing with classical aspects of conformal maps are sketchy, it is felt that the reader must have prior knowledge of these aspects. The title is recommended for students, teachers and researchers interested in complex variable techniques.

IISc-TIFR Mathematics Programme
IISc Campus
Bangalore 560 012

M. VANNINATHAN

Modern probability theory, 3rd edition, by B. R. Bhat, New Age International (P) Ltd, 4835/24, Ansari Road, Darya Ganj, New Delhi 110 002, 1999, pp. 328, Rs. 175.

This is the third edition of a nice introductory book on probability theory. Its intended readers are the post-graduate students of statistics in Indian universities. The author does not presuppose measure theory but develops it as the book unfolds. In that sense it is somewhat similar to Billingsley's classic.[1]

The author starts with chapter 0 (a new chapter in this edition) explaining the general concepts of probability theory in an informal style. Next four chapters are devoted to defining σ-field, random variables, probability space, distribution function and moments. These chapters also define measurable functions and develop the Lebesque–Stieltjes integral. The basic inequalities: c_r, Holder's and Jensen's inequality are also proved. The presentation is precise and terse. Although there are enough problems, for a book at this level there should be more examples.

Different convergence concepts are covered in Chapter 6 and 8. Chapter 7 develops the concept of characteristic functions. Independence is studied in Chapter 9. Borel cantelli lemma and Kolmogorov's 0–1law are also proved. Kolmogorov's inequality, a.s. convergence of series, and the various forms of law of large numbers (weak as well as strong) are fully developed in Chapter 10. Law of iterated logarithm is briefly mentioned and covered in the exercises at the end of the chapter. Central limit theorem is the topic of Chapter 11. Scalar as well as vector-valued r.v.s are covered. The problems extend the results to the stable distributions and define the domain of attractions for the general case. Chapter 12 is on conditioning with a very brief section on martingales. Radon Nikodym derivative is defined. The last chapter explains rudiments of finite-state Markov chains. Some of the proofs of the measure-theoretic concepts are relegated to the appendices at the end.

A few suggestions for the next edition. The problems at the end are used to extend and introduce new concepts that are developed in the main text. But the examples as well as the problems are all at the purely formal level. It would be nice to use the concepts developed on some applied problems. Since the book is specifically designed for statisticians, at least some applications to statistics should be provided. Some results on renewal theory and random walks should be included. They have numerous applications in other applied fields. Today, the most important part of modern probability theory is martingales. Some of its main theorem are provided in the exercises but they should be included in the main text. The section should be substantially expanded with various examples to show the power and generality of martingale tools.

In summary, it is a nice text but its value will be grately enhanced if the above-mentioned topics are included/extended. In fact, then it can be useful to engineers and applied scientists also.

Reference

1. BILLINGSLEY, P. *Probability and measure*, Wiley, 1979.

Department of Electrical Communication Engineering VINOD SHARMA
Indian Institute of Science
Bangalore 560 012

The language of physics by Elizabeth Garber, Birkhauser Verlag AG, Klosterberg 23, CH-4010, Basel, Switzerland, 1998, pp. 424, sFr. 118.

This book is on the history of the development of mechanics, mathematics and physics starting from the middle of the eighteenth century up to the period of the first World War. More precisely, it traces various events in the scientific development during the above-mentioned period and explains how a discipline nowadays called theoretical physics came into existence. Today, it is somewhat easy to define the functions of a theoretical physicist and the differences between him and a mathematician. Though it is becoming increasingly hard to define precisely the interfaces between pure mathematics, applied mathematics, mechanics and theoretical physics, the respective departments and budgets remain separate. However, when, for instance, Newton's *Principia* was written, this distinction did not exist and the subject was then called natural philosophy. The part of science known as physics in the middle of eighteenth century existed mainly in the form of experiments, observations and measurements. It is then interesting to ask and analyze how when and why the change took place and theoretical physics was born.

The present volume seeks to answer the above question by analyzing various events that took place in Europe. The scene is set in the following countries of Western Europe: France, Britain and Germany. The actors in the story are the dominating personalities of the era in the field: d'Alembert, Euler, Bernoulli, Lagrange, Fourier, Laplace, Poisson, Thomson, Stokes, Maxwell, Helmholtz, Einstein, Poincaré, Hilbert and many others.

The contents of the text are divided into four parts:

Part I : Eighteenth century science
Part II : Transitions 1790–1830
Part III : Transformations 1830–1870
Part IV : Conclusions and epilogue

In the first part, using the example of the wave equation and scientific developments around it, the author explores the meaning of mathematics and physics in the eighteenth century. During the period 1790–1830, the boundaries of European countries were redefined and the pursuit of science followed new patterns. On the other hand, the significance of the period

1830–1870 is more subtle. As the author explains, it is during these years that the creation of theoretical physics occurred within the cultural contexts of Britain and Germany. While it could not be born in the then prevailing culture of French science, it emerged in Britain during 1840–1860 in the works of Stokes, Thomson and Maxwell. The author convincingly demonstrates that all essential elements of theoretical physics were already in place by the year 1870. The last part covers the years 1870–1914 and examines the love–hate relationships that existed between mathematicians and physicists up to the outbreak of the first World War.

Even though there are many narratives on the subject, the author's analysis is very original. In her way of treating history of sciences, not only intellectual changes but also social and cultural changes are also taken into account. These latter aspects were omitted in earlier versions of the history. Even though mathematics is accepted as the language of physics, many earlier historians of science did not consider the role of mathematics in the development of scientific ideas. In this text, however, mathematics is given the centre stage. What is meant by mathematics today cannot obviously be taken as its definition in the eighteenth or nineteenth century. One has to understand the meaning of mathematics, its standards of proof, the mathematical definition of a solution to problems and other yardsticks, which were existent during the period under consideration. This takes the author to a long tour where the reader sees beautiful analysis of great personalities, their contributions, endless debates and controversies and physical background of the problem, physical interpretation of the solution, comparison with experiments, etc.

Concerning general features of the book, let me say that the organization of the text and the presentation of the events are praise-worthy. Several references are given in the footnotes on the same page and this is less taxing on the reader. Inclusion of several criticisms, disputes, disagreements, debates and controversies makes the narration very interesting. Apart from historians of science, this volume will be of interest to mathematicians and physicists as well.

IISc-TIFR Mathematics Programme M. VANNINATHAN
IISc Campus
Bangalore 560 012

Mutational and morphological analysis by J-P. Aubin, Birkhauser Verlag AG, Klosterberg 23, CH-4010 Basel, Switzerland, 1998, pp. 472, sFr. 148.

First of all, let me mention that contrary to what the title may indicate, this is not a book on biology; this is a mathematics book. However, the notions introduced and the tools developed in it will find applications in biology as well as in many other areas (image processing, shape optimization, numerical analysis, control theory, economics, front propagation problems, etc). Not many applications are included here and the author has postponed it to a forthcoming monograph. One may then be curious to know what the subject matter of the book which has so many applications could be. It is calculus: notion of derivatives and formulation of differential equations. History of mathematics shows that this had been the concern of many towering figures: Fermat, Newton, Leibniz, Bernoulli and so on. One may then ask about the new contribution and the understanding this book has brought to the field of calculus. It defines the notion of the derivative of a map between two spaces without linear structures. In a consistent and a very convincing way, it shows that linearity is not really indispensable for designing differential calculus. What is the need to develop calculus without the presence of linearity and where does it arise?

Classically, the problem of describing the motion of particles on a manifold-motivated calculus on manifolds is a well-developed subject. An essential technique used in it is linearization. This fails if the manifold is not smooth. Convexity, if present, can be exploited to advantage to define generalized derivatives, sub-differentials, etc. On the other hand, let us consider viability problems of the following type:

$$x'(t) = f(x(t), u(t)),\ u(t) \in U(x(t)),\ x(t) \in K(t). \tag{1}$$

Here, $x(t)$ denotes the state of the system, $u(t)$ a feedback control acting on the system, $U(x)$ a set of admissible controls depending on the state and finally $K(t)$ a compact set varying with time. We recognize that the first two conditions in (1) constitute a classical feedback control problem when $U(x)$ is a singleton. Thus, the novelty in (1) is the third condition called viability constraint. Given the initial state in $K(0)$, one is required to find $\{x(t), u(t)\}$ satisfying (1). This class of problems arises in many situations and the author of the present book has already written a monograph on the subject (cf: J-P. Aubin, *Viability theory*, Birkhauser, 1991). The main notion introduced there is that of the graphical derivatives for the set-valued map $t \rightarrow K(t)$. If, however, $K(t)$ is not explicity given and only the law of its evolution is known then the above notion is not adequate to formulate a differential equation describing the evolution of $K(t)$.

A somewhat dual difficulty arises in the shape optimization problems where the so-called cost functional J is a real-valued function defined on a class of sets. This is an example of a set-defined map. Since there is in general no obvious linear or convex structure present, it is not clear how to go about writing down optimality conditions for such problems. A partial solution to overcome the above difficulty is provided by Hadamard's method which essentially reduces the problems of domain variations to that of a class of functions which obviously has a linear structure. This idea was later generalized with the introduction of the concept of shape derivatives and transitions among subsets. Understanding sets through functions has been in the tradition of several mathematicians. Let me cite a few functions associated with sets: characteristic functions, indicators, gauges, support functions, distance functions, projections, etc.

A general question emerges encompassing the two situations described above: under what conditions can one define derivative of a map $f: E \rightarrow F$ between two metric spaces? The classical idea of manifold structures is not useful in the above situations. Inspired from the notion of transitions among subsets, the author gives an affirmative answer to the question if the spaces carry what are called mutational structures. Examples of mutational spaces include $K(X)$, the class of non-empty compact subsets of a finite-dimensional vector space X. One main source of difficulties with space $K(X)$ is that they do not inherit all structures carried by the basic space X. Fortunately, they are metric spaces with respect to Pompeiu–Hausdorff metric and they carry mutational structures. Using the concept of derivatives (called mutations) in $K(X)$, one can formulate meaningful differential equations, called morphological equations describing set evolution.

Iterated function systems so popular in the literature defining various fractal attractors are discrete versions of morphological equations. The discrete version is easy but the continuous version is more sophisticated.

Once morphological equations are formulated, several questions from dynamical systems suggest themselves. Cauchy–Lipschitz Theorem, Nagumo Theorem regarding existence of solutions, Invariant Manifold Theorem regarding qualitative behaviour of solutions and Lyapunov method regarding asymptotic stability of solutions are some of the issues addressed in the volume. In addition, some properties specific to morphological dynamics such as intersectable and confined evolutions are also its concern. Let us recall that in passing from smooth manifolds to non-smooth structures, cone structure of the tangent space is preserved even though vector space structure may be lost. Thus, the concept of normals and proximal normals dual to tangency concept can be developed. This makes the dual formulation of morphological equations possible.

The present monograph introduces and develops the above ideas in a systematic way with a lot of motivating arguments. Even for a person who is not a specialist in this field, the book is very informative and it will be a pleasure to read many parts of the book along with earlier monographs of the author.

In conclusion, this monograph on set-evolution reports on recent progress in dynamical systems. Because of inherent instability and the resulting bifurcation and chaos, point-wise formulation of dynamics may lose its significance. In such cases, the asymptotic state of the system is typically described by a compact invariant set (called attractor) which is not a single point. Under these circumstances, it is natural to consider the evolution of sets induced from that of the evolution law for points. Passing from set-evolution to that of functions associated with sets is reminiscent of field-theoretic point of view in physics. However, the approach followed in this text is different in the sense that the analysis is confined to sets and their evolution. The author is an acknowledged expert in this discipline of set-valued analysis in which he has already written several articles and monographs. There is no doubt that the notions introduced and the theory developed here will find wide applications in many fields. This volume is therefore strongly recommended to scientists interested in various aspects of dynamical systems.

IISc–TIFR Mathematics Programme
IISc Campus
Bangalore 560 012

M. VANNINATHAN

Published by Prof. M. S. Shaila, Editor, *Journal of the Indian Institute of Science*, Bangalore - 560 012;
Typeset by Creative Literati Pvt. Ltd., Bangalore - 560 043. Tel. Nos: 5070110, 5454047;
Printed at Lotus Printers, Bangalore - 560 044. Tel. 3380167, Fax: 3357378.