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Design of power system stabilisers to minimise power fluctuations*

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Abstract

This paper presents an application of optimal control theory for the design of power system stabilisers (PSS) as a dynamic compensator with the objective of minimising the oscillations in the power output of generators. A single- and 13-machine system examples are given to illustrate the methodology. The results indicate that the new design approach is very effective in minimising power fluctuations in the system.

1. Introduction

Stability is an important characteristic of the operation of modern power systems. The use of fast-acting high-gain voltage regulators has improved transient stability but worsened the problem of sustained low-frequency oscillations exhibited in many interconnected power systems[1]. deMello and Concordia[2] were the first to study this phenomenon and identify the factors responsible for the oscillatory instability. The use of power system stabilisers (PSS) has been recommended to provide damping. These are auxiliary controllers which receive feedback from rotor speed, electrical power output or bus frequency and provide a supplementary stabilising signal to the excitation system of generators.

The design of PSS in the power industry has received wide attention[2,3]. The approach is to use a single machine infinite bus equivalent system model and apply classical control techniques. The design objective is to improve the damping torque. It has been shown[4]

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that this objective is equivalent to the assignment of poles corresponding to the rotor oscillations.

A major drawback of the existing approaches[2,3] is that the complexities of a multimachine system are ignored. Coordinated application of PSS may be required in many systems[5]. The classical control theory is also inadequate to design PSS in a large system where the interaction between various machines may have to be considered.

The application of pole assignment techniques using state-space formulation is feasible and there are many ways for the design of decentralised controllers as PSS[4,6]. However, a practical problem to be faced in the design is the exact specification of the closed-loop poles. This problem is yet to be resolved satisfactorily.

As the need for PSS is encountered due to the undamped low-frequency oscillations in transmission lines, it would be natural to design the PSS with the objective of minimising power oscillations. The design algorithm can be based on optimal control with dynamic output feedback[7,8]. The objective function selected represents the integral squared value of oscillations in power output of generators in the system.

The design procedure developed is presented here and applied to a single-machine system. The algorithm is also applicable to PSS design in large systems. The results for a 13-machine system are presented for illustration.

2. System model

The power system model is non-linear in general and has to be linearised for PSS design. This is best illustrated by taking the example of a single-machine system (fig. 1). The generator is represented by a third-order dynamic model considering only the rotor swing and the flux decay. The excitation system is represented by a simple model (fig. 2). This is typical of a static exciter.

The linear state space model of the system is given by

$$\dot{\hat{\underline{x}}} = [\hat{A}]\hat{\underline{x}} + \hat{\underline{b}}\hat{u} \tag{1}$$

$$\hat{y} = [\hat{c}]\hat{\underline{x}}$$

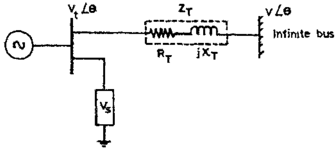

FIG. 1. Single-machine infinite bus system.

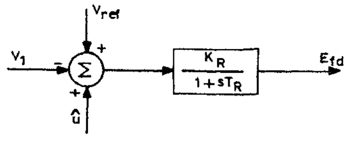

FIG. 2. Block diagram of excitation system.

where

$$\hat{x}^t = [\Delta\omega\, \Delta\delta\, \Delta I_d \Delta E_{fd}].$$

$\hat{u}$ represents the auxiliary stabilising input to the excitation system (fig. 2) and $\hat{y}$ the scalar output variable used as input to PSS.

The elements of $[\hat{A}, [\hat{b}]$ and $[\hat{c}]$ are functions of the system parameters and the operating point.

3. Design of optimal PSS

Design of PSS can be posed as a problem of determination of the feedback gains of an output feedback system, to minimise a performance index, as shown below.

Consider, for simplicity, a second-order PSS, with a single input and a single output. Its transfer function can be represented as

$$F(s) = \frac{\theta_0 s^2 + \theta_1 s + \theta_2}{s^2 + r_1 s + r_2}. \tag{2}$$

The following state and output equations can be written for the PSS:

$$\dot{z} = Sz + R\hat{y}$$

$$u = Qz + K\hat{y} \tag{3}$$

where $z \in R^2$ is the state vector, $\hat{y}$ the input signal to PSS and $\hat{u}$ its output signal. The matrices $\hat{S}$, $\hat{R}$, $\hat{Q}$ and $\hat{K}$ are defined below:

$$\hat{S} = \begin{bmatrix} 0 & 1 \\ -r_2 & -r_1 \end{bmatrix}; \quad \hat{R} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\hat{Q} = [(\theta_2 - \theta_0 r_2), \quad (\theta_1 - \theta_0 r_1)]$$

and

$$\hat{K} = \theta_0.$$

The linearised state-space model of the power system, given in eqn (1), is combined with eqn (3) to obtain

$$\dot{x} = Ax + Bu \tag{4}$$

$$y = Cx \tag{5}$$

$$u = Ky \tag{6}$$

where

$$x = [x^t z^t]^t; \quad y = [y z^t]^t$$

$$A = \begin{bmatrix} \hat{A} & 0 \\ 0 & 0 \end{bmatrix}; \quad B = \begin{bmatrix} \hat{b} & 0 \\ 0 & I \end{bmatrix}$$

$$C = \begin{bmatrix} \hat{C} & 0 \\ 0 & I \end{bmatrix}; \quad \text{and} \quad K = \begin{bmatrix} \hat{K} & \hat{Q} \\ \hat{R} & \hat{S} \end{bmatrix}.$$

The 3×3 matrix K, for the second-order PSS considered, can be expressed as

$$K = \begin{bmatrix} \theta_0 & (\theta_2 - \theta_0 r_2) & (\theta_1 - \theta_0 r_1) \\ 0 & 0 & 1 \\ 1 & -r_2 & -r_1 \end{bmatrix}. \tag{7}$$

The parameters of PSS can be determined if the matrix K is known. K will be determined to minimise the performance index

$$J = \int_0^\infty (\underline{x}^t Q \underline{x} + \underline{u}^t R \underline{u})\, \mathrm{d}t \tag{8}$$

where Q is a symmetric positive semidefinite matrix and R a positive definite matrix. The system of eqns (4) and (5) is assumed to be controllable and observable. With the output feedback of eqn (6), the closed-loop system is

$$\dot{\underline{x}} = A_c \underline{x} \tag{9}$$

where

$$A_c = (A + BKC). \tag{10}$$

The performance index J of eqn (8) can now be written as

$$J = \int_0^\infty \underline{x}^t (Q + C^t K^t RKC) \underline{x}\, \mathrm{d}t. \tag{11}$$

The performance index is clearly a function of the initial state $\underline{x}(0)$ of the system. The dependence of J on any particular $\underline{x}(0)$ can be eliminated by assuming the initial state to be a random variable, uniformly distributed over the surface of an n-dimensional unit sphere[7]. The problem then reduces to determination of K which minimises

$$J = \text{trace}\,[S] \tag{12}$$

where S satisfies the Lyapunov equation

$$A_c^t S + SA_c + (Q + C^t KRKC) = 0. \tag{13}$$

An algorithm for the solution of the above problem is given below:

1. Start with an initial guess $K^{(0)}$ of K such that $A + BK^{(0)}C$ is stable.
2. Compute S by solving Lyapunov eqn (13).
3. Compute matrix P by solving the Lyapunov equation

$$PA_c^t + A_c P + I = 0 \tag{14}$$

where I represents the identity matrix of appropriate dimension.

4. Compute the gradient

$$(\partial H/\partial K) = 2(-RKCPC^t - B^t SPC^t) \tag{15}$$

5. Update K according to

$$K^{(i+1)} = K^{(i)} - \alpha(\partial H/\partial K)|_{K=K^{(i)}} \tag{16}$$

where $0 < \alpha < 1$.

6. Go to step 2 until $\partial H/\partial K$ is close to zero or difference in the values of J obtained in two successive iterations is less than a prespecified value.

4. Numerical example

4.1. *System data*

For the system shown in fig. 1 the following data are considered. The machine and line parameters are given in per unit on machine base.

Generator parameters:

$$x_d = 1.72, \quad x'_d = 0.45$$

$$x_q = 1.68, \quad T'_{do} = 6.3\,\text{sec}$$

$$H = 4.0\,\text{sec}$$

$$\omega_o = 314\,\text{rad/sec.}$$

Transmission line parameters:

$$R_T = 0.024, \quad x_T = 0.6$$

$$G_S = 0.0, \quad B_S = 0.066.$$

Voltage regulator parameters:

$$K_R = 50.0, \quad T_R = 0.02\,\text{sec.}$$

Operating data: Generated power $= 1.0$ p.u.

at 0.9 p.f. lagging, $V = 1.0\angle 0°$

The matrices A and $\underline{b}$ of the system are

$$[\hat{A}] = \begin{bmatrix} 0 & -35.9593 & -15.8432 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & -0.3808 & -0.3463 & 0.3527 \\ 0 & 240.3313 & -544.6585 & -50 \end{bmatrix}.$$

$$[\hat{\underline{b}}'] = [\,0 \quad 0 \quad 0 \quad 2500]$$

The open-loop eigenvalues of $\hat{A}$ are $0.2079 \pm j6.1138$, -5.0065, -45.7555.

The complex pair of eigenvalues correspond to the rotor oscillation mode. The open-loop system is evidently unstable. This can be stabilised by employing a PSS.

5. Design of PSS

The PSS is designed as a dynamic compensator with the transfer function given in eqn (2). The input $\hat{y}$ to the PSS is assumed to be speed signal and the output is the control variable $\hat{u}$.

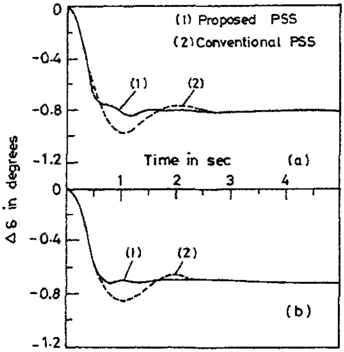

FIG 3. Response of $\Delta\delta$ for $\Delta V_{ref.} = 1.0$ p.u. (a) at full load, and (b) at half load.

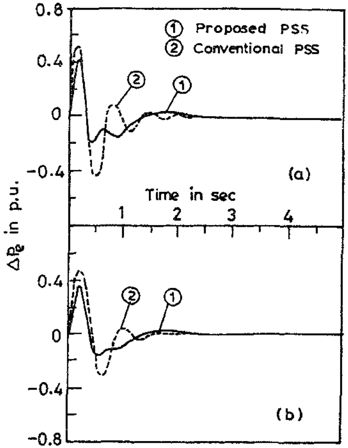

FIG. 4. Response of ΔP_e for $\Delta V_{ref} = 1.0$ p.u. (a) at full load, and (b) at half load.

Table I
Closed-loop eigenvalues of the system

With the proposed PSS design	With the conventional PSS design
-0.4807	-3.2061
$-2.0267 \pm j9.9036$	$-3.5741 \pm j12.6436$
$-2.4879 \pm j2.3790$	$-1.4278 \pm j3.0511$
-47.6506	-49.1365

The PSS is designed to minimise the power fluctuations. Hence the performance index is chosen as

$$J = \int_0^\infty (\Delta P_e)^2 \, dt = \int_0^\infty (\underline{x}^t Q \underline{x}) \, dt \tag{17}$$

where $Q = T^t T$, T is defined by

$$\Delta P_e = T\underline{x}. \tag{18}$$

For the example chosen

$$T = [0 \quad 0.9157 \quad 0.4034 \quad 0]. \tag{19}$$

The results after applying the algorithm given in the previous section are

$$F(S)=\frac{0.2565S^2+0.6733S+0.4839}{S^2+6.8141S+3.2345}. \quad (20)$$

The eigenvalues of the closed-loop system and those obtained with the application of the conventional PSS design[1] are shown in Table I.

The response of the closed-loop system to a step change in the voltage reference of AVR is shown in figs 3 and 4. They show the variations in the rotor angle and the generator power output for two operating conditions, namely, (i) full load, and (ii) half load. The PSS is designed at full load.

It is observed that the peak overshoots in the rotor angle and the power output are less in the case of the PSS design using optimal control compared with the conventional design. This is not surprising as the new PSS design is based on minimising power oscillations.

6. PSS design in large systems

The conventional PSS design is not applicable when large multimachine power systems are considered. The use of pole-assignment techniques for decentralised design of PSS is

Table II
Eigenvalues for the 13-machine system

Open loop	Closed loop	
	Pole assignment[10]	Optimal control
$0.02892 \pm j14.05587$	$-2.92632 \pm j14.10790$	$-2.89734 \pm j13.80945$
$0.04537 \pm j12.83184$	$-0.18524 \pm j13.37862$	$-1.61382 \pm j13.61650$
$0.00852 \pm j11.83472$	$-0.23075 \pm j11.77149$	$-0.19060 \pm j13.39934$
$0.00988 \pm j10.80273$	$-0.18680 \pm j10.70338$	$-0.37997 \pm j11.79321$
$0.00045 \pm j10.57437$	$-0.06316 \pm j10.56687$	$-0.35992 \pm j10.86744$
$0.00674 \pm j10.46237$	$-0.06807 \pm j10.50623$	$-0.34208 \pm j10.59898$
$0.01693 \pm j10.34604$	$-0.57632 \pm j10.40899$	$-0.12740 \pm j10.51181$
$0.01668 \pm j10.18523$	$-0.05326 \pm j10.26468$	$-0.53875 \pm j10.26825$
$0.00640 \pm j9.40696$	$-0.02639 \pm j9.41090$	$-0.22389 \pm j9.44422$
$0.01546 \pm j8.84131$	$-0.43355 \pm j8.77155$	$-0.35956 \pm j9.39171$
$0.03302 \pm j7.89500$	$-0.43318 \pm j7.90272$	$-0.28609 \pm j8.65416$
$0.05525 \pm j6.15929$	$-0.55035 \pm j6.17615$	$-0.27365 \pm j6.90138$
-5.78423	-0.38637	$-10.76684 \pm j9.05954$
-6.09830	-2.98356	$-1.81087 \pm j2.84426$
-6.88350	-3.50351	$-12.81823 \pm j2.07492$
-43.87653	-4.53175	-2.85494
-44.58668	$-7.22301 \pm j5.86688$	-3.11521
-44.99036	$-10.97008 \pm j8.82632$	-5.86962
0.00000	-10.19024	-7.10297
	-15.53490	$-48.46174 \pm j0.071491$
	-46.52583	-56.98209
	-47.92226	
	-56.93663	

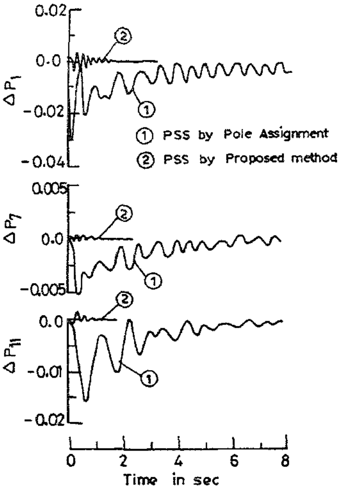

FIG. 5. Fluctuations in power output of generators in p.u.

complicated[6]. However, the proposed design is capable of extension to multimachine systems, as algorithmically, the method involves parameter optimisation. However, computational requirements increase considerably. In order to test the algorithm for the design of PSS in realistic systems, a 71-bus 13-machine system example is chosen. The system consists of five thermal and eight hydro units.

Based on eigenvalue sensitivities of the open-loop system[9] the PSS is chosen on three most effective machines, namely, machines 1, 7 and 11. Considering fourth-order models for these three machines and classical models for the rest the eigenvalue analysis is carried out. The open-loop eigenvalues are shown in Table II. This table also shows closed-loop eigenvalues when PSS are designed using a decentralised stabiliser design proposed[10] and for PSS designed using the proposed method with the objective of minimising

$$J = \int_0^\infty \Sigma(\alpha_i P_{ei}^2)\,dt,$$

where α_i is the weighting factor for machine i.

The responses for a unit step disturbance in the voltage reference to machine 1 is shown in fig. 5 where variations in the power outputs of the machines provided with PSS are shown (α_i is assumed to be equal to 1.0 for machines with PSS and zero for the rest).

Figure 5 also shows the responses for PSS designed using the technique given[10]. The differences in the responses for the two cases are dramatic and this observation applies even when other disturbances are considered.

7. Conclusions

A new method for design of power system stabilisers with the objective of minimising power oscillations is presented with two case studies. The method is general enough to include large system representations and avoid the complications inherent in techniques using pole assignment. The results show that the system performance with the PSS designed using this method is far superior to other methods.

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