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Reasoning with perceived information: A case for nonmonotonic reasoning

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Abstract

Several nonmonotonic logics have been proposed and studied in the context of human commonsense reasoning. Commonsense reasoning involves reasoning with perceived information which is often incomplete. The existing logics can handle defeasible nonmonotonic inferences. We propose a modified first-order logic so that defeasible beliefs can also be handled. The modification is in the form of a set of proper axioms to handle belief revision, and a modified *modus ponens* to capture nonmonotonic reasoning. The proposed logic provides a basis for contextual reasoning and also attempts to capture the notion of 'forgetfulness'. The proposed logic also permits property inheritance with exceptions from multiple more general concepts. The consistency of the theory generated using modified first-order logic is also examined.

Key words: Nonmonotonic reasoning, nonmonotonic inference rule, nonstandard logic, default rules.

1. Introduction

Commonsense reasoning involves reasoning with perceived information and incomplete information. We find various formalisms proposed to handle incomplete information in the literature[1–5]. We feel that human commonsense reasoning involves reasoning with defeasible beliefs. This may be primarily because our perception can be erroneous and the world around us is dynamic. Existing formalisms are able to revise default inferences only. We are of the opinion that every belief may need a revision.

Reasoning and learning are two closely related activities. In some sense, information generated *via* reasoning is the same as the information generated *via* learning. In both these activities, domain may be complete or incomplete. In the kind of reasoning proposed in this paper, the domain is assumed to be complete and no attempt is made to generalize the information in order to complete it. In every learning situation, the learner transforms the information provided by the environment into something which is stored for further use. The nature of this transformation determines the type of strategy used. Several basic learning strategies have been distinguished[6]: rote learning, learning by instruction, learning by deduction, learning by analogy, and learning by induction. The latter subdivides into learning from examples, and learning by observation and discovery. Deductive learning includes many truth-preserving transformations such as knowledge reformulation. In this sense, this paper can also be viewed as suggesting transformation rules for learning by deduction.

Belief is our perception of the fact. For example, when one observes a particular thing, he may perceive it as an animal while the fact may be that it is a statue. Our belief is normally biased and may change with time. We use beliefs in reasoning as though they are facts. Default is an abstraction or generalisation of beliefs. These generalisations help us in dealing with incomplete information.

In order to allow belief revision, we must be able to remember past perceptions. The perceptual process[7] involves two different but related functions:

(i) that of the sensory pattern, which provides a psychological basis for perceiving; and
(ii) that of another factor which constructs sensory pattern into something having a significance which goes beyond its immediate sensory character.

Remembering involves

(i) An original sensory pattern.
(ii) An original psychological orientation or attitude.
(iii) The persistence of this orientation or attitude in some setting which is different from original at least in a temporal sense.
(iv) The organisation, together with orientation or attitude of psychological material.

Material remembered usually has to be set in relation with other material and in most complete cases must be dated, placed, and must be given some kind of personal mark. We represent the perceived information that takes into account the above requirements.

Commonsense reasoning involves reasoning with vast amount of knowledge. In order to achieve parsimony in knowledge representation, normally inheritance hierarchies are employed. Even though simple inheritance representation can be dealt with easily, multiple inheritance provides tremendous representation flexibility.

In this paper, we discuss a formalism which permits belief revision and property inheritance with exceptions from multiple more general concepts. The rest of the paper is organised as follows. In Section 2, we discuss commonsense theory. Section 3 describes commonsense reasoning. In Section 4, we discuss multiple inheritance. Section 5 contains proposed modified first-order logic. Section 6 describes a belief revision scheme. In Section 7, we discuss some properties of the modified first-order logic.

2. Commonsense theory

Commonsense theory is simple[8]. If one wishes to know something not yet known about the world, he has to open his eyes and look around. And we have to listen to noises, especially those made by other people. Thus our sensory experiences—seeing, hearing, smelling, feeling, and tasting—are our source of knowledge. What sort of things our knowledge of material things are. If there is such a knowledge, it will be at the very least knowledge based upon the senses[9,10]. When we use our vision to look at an object, to see what is around us, to know what is there outside us, it is the sense data with which we are first acquainted and which we first come to know. On this view, our knowledge of material objects would be first and foremost knowledge of sense data. Whenever we know that some proposition about a material object is true, there is always some sense datum which is *a* subject of the proposition. When we see something, we are describing a sense datum—its color, shape and size. Sense data are the objects of our direct or immediate perception and they are in some sense the ultimate subjects of judgements about objects of the senses. What we actually see, when we look at something is quite different from what we may infer. The different modes of sensory experience—seeing, hearing, smelling, and feeling—are similar in the sense that the same kind of relation is involved in seeing objects, hearing sounds, feeling and smelling things. If we are to know anything of the world external to ourselves, we must use our senses to perceive that world. The relation which we have through our senses to their immediate objects of perception, no matter what sense or what sort of object is involved, is the relation of immediate or direct apprehension. Even though the sense datum is identical with the surface of the material object, different perceivers see it differently.

Commonsense may mean beliefs we have as a result of the nature and constitution of the mind which therefore can be expected to be found in every normal mind, regardless of its lack in special training and experience[11]. Or, commonsense may mean, a common 'turn of mind', not the content of beliefs that are universally held, but 'a way of thinking' followed by all minds by their very nature. Or, lastly, commonsense may mean the shared set of beliefs of men or of a fairly large group of men, without implying that these beliefs are 'born' in their minds or present in every man's mind. The minds of most men, or perhaps even of all sane men, act in more or less the same way is hardly disputable. It seems reasonable to suppose that our minds, like our bodies, have a tendency to behave in much the same way under similar conditions.

No two minds see exactly the same thing; there is always a slight difference in what is called *perspective* when two are said to be seeing the same thing[12]. What is seen, the sense data, is conditioned by the fact that what men see depends upon their sense organs, their nerves, and their brains. Therefore, although the world as seen by a particular man would exist if that man were not there at that time, we can reasonably suppose that some aspects of the universe existed from that point of view though no one was perceiving it. *Sensibilia* are what would be sensed if a mind and proper sense organs connected to it occupied that point of view. There are infinitely many such points of view. Suppose that we now think of the collection of all the appearances at one place and at one time. This can be called a *perspective*. Each observer has a space private to him in which all his sense data appear and the totality of these data at any time make up the perspective. On the other hand, the thing or object in the world is a bundle of all the events which consists of the various appearances of it—the sum of all its actual and possible appearances.

Sense data are things of whose existence we can be certain. They are the blocks out of which the world can be reconstructed with the help of mathematical logic and certain other building blocks. How sense data are related to the external world and how to use our sense experience to construct our knowledge, Carnap[13] tells us that the system can have as basic elements, elementary experiences. The point of his procedure is to exhibit the fact that all the concepts of social and natural sciences may be defined in terms of these elementary experiences. Thus the system has autopsychological basis (meaning what is unique to an individual).

Among the principles that Maritian[14] says are parts of commonsense are 'the data of senses' (for example, those bodies possess length, breadth, and height), 'self-evident axioms' (for example, every event has a cause), and 'consequences immediately deducible from these axioms'.

3. Commonsense reasoning

Commonsense reasoning involves reasoning with information perceived through our senses. The amount of information available for reasoning is incomplete[15]. In order to carry out reasoning with incomplete information, we have to assume suitably the missing details. These assumptions are normally suggested by defaults which tell us how to fill up the gap in knowledge. The information perceived depends on the context or the situation or the surroundings and may be erroneous. Even if the information perceived is perfect, the external world which we are perceiving is dynamic. Hence there is a need for us to revise our perceived information in order to keep our information up-to-date and in pace with the external world. The information perceived which may require revision may be called as belief. Our beliefs are context-dependent *i.e.*, our perception is normally biased and affected by the context. The change in surroundings or the dynamics of the external world may establish a new or different context. So, our belief may change when the context is altered or it may change with time even if the context remains the same. We may have contradictory beliefs in different contexts. Default is an abstraction or generalisation of beliefs (in other words, belief is like data and default is like knowledge). The defaults may vary from individual to individual and defaults of an individual may vary with time and an individual may employ different defaults in different contexts.

Among the characteristics of commonsense reasoning, the following four are worthy of note[16]:

First, commonsense beliefs tend to be habitual and imitative. They rest on customs and tradition, and are sometimes stated as proverbs or axioms.

Second, commonsense reasoning is often vague and ambiguous. It is superficially grounded and may vary from individual to individual, from group to group, or from place to place. In a complex and rapidly changing world, it is frequently inadequate to meet or cope with new and infamiliar situations unless the beliefs are suitably revised.

Third, commonsense belief in considerable part is untested belief. It may be that 'first look' is not always correct, and that things are not always that appear to be.

Fourth, commonsense reasoning is seldom accompanied by explanations of why things are as they are alleged to be.

Hence, if commonsense reasoning were to serve some useful purpose, commonsense beliefs need constant and careful reexamination.

We illustrate the above points with the following examples:

(1) The sentence 'Birds can fly' is not synonymous with 'All birds can fly' because there are exceptions. If we are told about a particular bird Tweety, we would be justified in assuming that it can fly. If we learn later that 'Tweety is a penguin', then the derived inference 'Tweety can fly' is withdrawn. This works fine if 'Tweety is a bird' is a fact. If it were a belief then the fact may be 'Tweety is a monkey' and we need to withdraw 'Tweety is a bird' and 'Tweety is a penguin' so that meaningfully we can assert 'Tweety is a monkey' and derive further inferences about Tweety.
(2) While we are passing through a forest, we believe that there could be harmful animals like lions, tigers, etc. Based on this belief we may perceive an inanimate object appearing like lion as a lion and may act violently towards it. However, on further examination, we may change this belief to ascertain that the object is indeed inanimate and may react differently.
(3) We may perceive that John likes Mary at a particular instance and the fact is that John likes Mary at that instance. However, we may perceive at a later instance in time that John does not like Mary which again is a fact and our reasoning at this new instance is controlled by this fact.
(4) While we are in a forest, we may perceive a rope as a snake and while we are in our room we may perceive a snake as a rope.
(5) We feel that a congenital blind man or inhabitant of Antarctica may not have 'Birds can fly' as a default.

So we have a scenario in which our beliefs are based on perception and we have to reason with defeasible beliefs.

4. Multiple inheritance

Inheritance hierarchies are a classical mechanism in artificial intelligence. In such cases, properties can be associated with the most general objects for which they are valid. The transitivity of *ISA* relation implicit in hierarchies allows the propagation of these properties to more specific objects. Artificial intelligence research has often emphasised the need for multiple inheritance where a more specific object may inherit information from several more general concepts[17,18]. A further requirement is that they should allow exceptions. Exceptions are fairly easy to deal with in simple inheritance systems. Multiple inheritance without exception is easy to deal with theoretically. The combined structure, multiple inheritance with exceptions, however, offers, a number of unpleasant and challenging surprises.

In systems that permit multiple inheritances, the inheritance tree is replaced by directed inheritance graph. Any node in such a network may have multiple neighbours and the directed graph indicates the direction of inheritance. In order to provide a formalism for property inheritance with exceptions from multiple more general concepts, we propose a multiple inheritance network as follows. Each node in the network is connected to its neighbours *via* the following operators:

$\supset$: allows the inheritance of properties from nodes representing more general concepts (*ISA* properties).
$\Rightarrow$: allows the inheritance of natural properties of the node.
$>$: allows the inheritance of the default properties of the node.

The semantics of these operators is exactly identical to the semantics of the usual implication. Multiple operators are necessary to overcome some of the problems associated with multiple inheritance. In the following sections, we discuss the relevance of these operators in property inheritance. The property inheritance requires that some properties of some ancestors must be preferred over others. In other words, default properties must be inherited only after inheriting natural and *ISA* properties. The inheritance rule is completely captured in the inference rule discussed in section 5.

5. Modified first-order logic

We start with first-order theory (FOT). The problem in employing FOT for commonsense reasoning is that it is monotonic in behaviour. The monotonicity of FOT is essentially because of two reasons: Firstly, we cannot delete premises from within FOT. Once they are asserted, they remain. Secondly, *modus ponens* which is used to derive theorems is monotonic. It always tries to add a theorem. In order to achieve complete nonmonotonic behaviour, we introduce proper axiom schemas that allow us to treat premises, which represent beliefs, nonmonotonically, and modify the monotonic *modus ponens* into nonmonotonic *modus ponens*. We discuss, in the following, the modified FOT. We borrow from FOT everything except the inference rule *modus ponens*. To this, we add a set of proper axioms and a nonmonotonic *modus ponens* inference rule. α keeps track of belief revisions and β suggests that we do 'forget' certain things and recast them again and γ indicates the context.

We call the set of proper axioms *Mundus sensibilis.*

$$(\forall\alpha\beta\gamma)(B(P,\alpha,\beta,\gamma)\supset P_{\alpha\beta\gamma})$$

$$(\forall\alpha\beta\gamma)(P_{\alpha\beta\gamma}\wedge\neg L(P_{(\alpha+1)\beta\gamma})\wedge\neg L(P_{1(\beta+1)\gamma})\wedge odd(\alpha)\wedge\neg L(P_{0\beta\gamma})\supset(C_\gamma\supset P))$$

$$(\forall\alpha\beta\gamma)(P_{\alpha\beta\gamma}\wedge\neg L(P_{(\alpha+1)\beta\gamma})\wedge\neg L(P_{1(\beta+1)\gamma})\wedge even(\alpha)\wedge\neg L(P_{0\beta\gamma})\supset (C_\gamma\supset\neg P))$$

$$(\forall\gamma\delta)(B(C,\gamma,\delta)\supset C_{\gamma\delta})$$

$$(\forall\gamma\delta)(C_{\gamma\delta}\wedge(\forall\tau)(\neg L(C_{\tau(\delta+1)}))\supset C_\gamma)$$

where $\alpha,\beta,\gamma,\delta\in\omega$ and ω is $\{0,1,2,\ldots\}\tau\in N$, a finite subset of ω.

ISA modus ponens (*ISA–MP*) replaces monotonic *modus ponens* of FOT.

From P, infer every Q such that $P \supset Q \wedge \neg L(\neg Q)$,
infer every $Q \in NMTC_{\Rightarrow}(P)$,
infer every $Q \in NMTC_{>}(P)$.

Notes

(1) $P_{\alpha\beta\gamma}$ and $C_{\gamma\delta}$ are special symbols and do not figure in any theorem.
(2) P is a first-order predicate representing a belief or a default.
(3) $B(P, \alpha, \beta, \gamma)$ is a second-order sentence and for a given P and γ, α is incremented from 0 onwards till some maximum is reached when β (indicating relearning) is incremented. $B(P, \alpha, \beta, 0)$ indicates a context independent default or context independent belief. $B(P, 0, \beta, \gamma)$ indicates our voluntary or involuntary intension of withdrawing belief in P. We observe that this attempts to model 'forgetfulness' which is a feature of human commonsense reasoning. The act of 'forgetting' means that some perceived information is no longer available for reasoning. Let us assume that P is a perceived information which is indicated by $B(P, 1, 1, \gamma)$. When $\neg P$ is perceived, P should not be used for further reasoning. This can be achieved by asserting $B(P, 2, 1, \gamma)$ which derives only $\neg P$. This corresponds, in some sense, to falsifying belief in P. When P is forgotten (in humans, this can happen because P was perceived long time back), neither P nor $\neg P$ is available for reasoning. This can be achieved by asserting $B(P, 0, 1, \gamma)$. Observe that the act of 'forgetting' is not automatic, but so is perception.
(4) $B(C, \gamma, \delta)$ is a second-order sentence. C_γ indicates a particular context and δ the number of times we have switched from one context to another. C_0, which is always true, takes care of context independent defaults and beliefs. At any point in time, only one $C_\gamma, \gamma > 0$, is true.
(5) The context of belief assertions is explicitly input in the form $B(C, \gamma, \delta)$.
(6) L is a modal (belief) operator[3] defined as:

$$L(P) \in T \quad \text{if} \quad P \in T$$

and

$$\neg L(P) \in T \quad \text{if} \quad P \notin T$$

where T is any theory.

(7) Finiteness of τ indicates that we allow only finite number of contexts.
(8) $NMTC_{\supset}(P)$ is a nonmonotonic transitive closure with respect to the operator $\supset$ and is defined as follows:

$$NMTC_{\supset}(P) \equiv R^1_{\supset} \cup R^2_{\supset} \cup R^3_{\supset} \cup \ldots$$

where

$$R^1_{\supset} = \{Q \mid P \supset Q \wedge \neg L(\neg Q)\}$$

and

$$R^i_{\supset} = \{Q \mid P \in R^{i-1}_{\supset} \wedge P \supset Q \wedge Q \notin \bigcup_{j=1}^{i-1} R^j_{\supset} \wedge \neg L(\neg Q)\}.$$

(9) $NMTC_{\Rightarrow}(P)$ is a nonmonotonic transitive closure with respect to the operator $\Rightarrow$ and is defined as follows:

$$NMTC_{\Rightarrow}(P) \equiv R^1_{\Rightarrow} \cup R^2_{\Rightarrow} \cup R^3_{\Rightarrow} \cup \ldots$$

where

$$R^1_{\Rightarrow} = \{Q | P \Rightarrow Q \wedge \neg L(\neg Q)\}$$

and

$$R^i_{\Rightarrow} = \{Q | P \in R^{i-1}_{\Rightarrow} \wedge P \Rightarrow Q \wedge Q \notin \bigcup_{j=1}^{i-1} R^j_{\Rightarrow} \wedge \neg L(\neg Q)\}.$$

(10) $NMTC_{>}(P)$ is a nonmonotonic transitive closure with respect to the operator $>$ and is defined as follows:

$$NMTC_{>}(P) \equiv R^1_{>} \cup R^2_{>} \cup R^3_{>} \cup \ldots$$

where

$$R^1_{>} = \{Q | P > Q \wedge \neg L(\neg Q) \wedge \neg IS-A(\neg Q) \wedge \neg KIND-OF(P, \neg Q)\}$$

and

$$R^i_{>} = \{Q | P' \in R^{i-1}_{>} \wedge P' > Q \wedge Q \notin \bigcup_{j=1}^{i-1} R^j_{>} \wedge \neg IS-A(\neg Q) \wedge$$
$$\neg KIND-OF(P, \neg Q)\}.$$

(11) $IS-A(Q)$ is defined as follows:

For any

$$P \in T, \quad \text{if } Q \in NMTC_{\supset}(P) \vee Q \in NMTC_{\Rightarrow}(P) \text{ then } IS-A(Q) \in T.$$

For all

$$P \in T, \quad \text{if } Q \notin NMTC_{\supset}(P) \wedge Q \notin NMTC_{\Rightarrow}(P) \text{ then } \neg IS-A(Q) \in T.$$

(12) $KIND-OF(P, Q)$ is defined as follows:

For any

Q' such that $Q' \in INHERITORS(P) \wedge Q' \notin T$,

if $Q \in NMTC_{>}(Q')$ then $KIND-OF(P, Q) \in T$.

For all

Q' such that $Q' \in INHERITORS(P) \wedge Q' \notin T$,

if $Q \notin NMTC_{>}(Q')$ then $\neg KIND-OF(P, Q) \in T$.

(13) $INHERITORS(P)$ is defined to be a set of all Q such that Q is below P in the inheritance network.

For example, consider the hypothetical network shown in fig. 1 and its equivalent axiomatic representation shown in fig. 2. Let $Q_1(T)$ and $Q_2(S)$ be the additional premises. Then the theorems generated are as follows: $P_1(T)$, $P_2(T)$, $P_3(T)$, $Q_{11}(T)$, $Q_2(S)$, $P_6(S)$, $P_4(T)$, $R_2(T)$. At this stage, $INHERITORS(R_2(T))$ includes the set $\{P_4(T), P_5(T), P_2(T), P_3(T), Q_1(T)\}$ and so when we attempt to generate theorems from $R_2(T)$, $\neg P_{52}(T)$ does not get generated. Further theorems generated are $R_{21}(T)$, $P_5(T)$, $P_{51}(T)$, $P_{52}(T)$, $R_2(S)$, and $R_1(T)$.

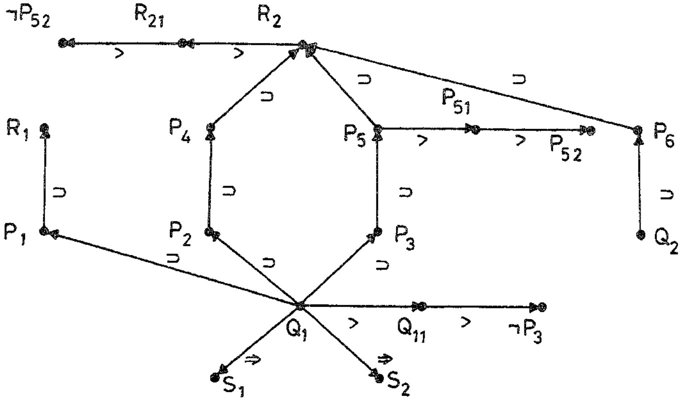

FIG. 1. Hypothetical inheritance network.

$(\forall x)(Q_1(x) \supset P_1(x))$	$(\forall x)(P_1(x) \supset R_1(x))$
$(\forall x)(Q_1(x) \supset P_2(x))$	$(\forall x)(P_2(x) \supset P_4(x))$
$(\forall x)(Q_1(x) \supset P_3(x))$	$(\forall x)(P_4(x) \supset R_2(x))$
$(\forall x)(Q_1(x) > Q_{11}(x))$	$(\forall x)(R_2(x) > R_{21}(x))$
$(\forall x)(Q_1(x) \Rightarrow S_1(x))$	$(\forall x)(R_{21}(x) > \neg P_{52}(x))$
$(\forall x)(Q_1(x) \Rightarrow S_2(x))$	$(\forall x)(P_3(x) \supset P_5(x))$
$(\forall x)(Q_{11}(x) > \neg P_3(x))$	$(\forall x)(P_5(x) > P_{51}(x))$
$(\forall x)(Q_2(x) \supset P_6(x))$	$(\forall x)(P_{51}(x) > P_{52}(x))$
$(\forall x)(P_6(x) \supset R_2(x))$	$(\forall x)(P_5(x) \supset R_2(x))$

FIG. 2. Equivalent axiomatic representation.

6. Belief revision scheme

At present, we are not intereseted in revising the belief incrementally. Belief revision takes place when a contradiction to the current belief is perceived. We assume that when such a revision takes place, the appropriate belief premises are available for affecting revision. This may involve generating $B(P, 0, \beta, \gamma)$ for some P indicating that we are no longer interested in P and we want to withdraw it.

One important reason for doing all this is to have a consistent set of beliefs. We propose a hierarchy of contexts and beliefs to keep the consistency under control. The context parameter tends to model the conjecture that human beings believe in contradicting things. We feel that human beings believe in contradicting things in different contexts. Within a single context, contradiction can arise because of error in perception. In this case, we have two alternatives: one is to revise the belief and the second is to withdraw it. So, the possible solution is to either revise or withdraw the false beliefs, and assert new supposedly true beliefs.

We now look at the behaviour of the nonmonotonic *modus ponens* which tends to capture default reasoning. We have to worry about two things: Firstly, if we do not deal properly with default inferences, default reasoning can introduce inconsistency when more specific information is perceived. Secondly, premises implying ISA hierarchy must be explicitly or implicitly ordered. An explicit ordering of defaults which captures ISA hierarchy can be seen in the following[2]:

$$\forall x. \neg\, ab\, aspect1\, x \supset \neg\, flies\, x \tag{1}$$

$$\forall x. bird\, x \supset ab\, aspect1\, x \tag{2}$$

$$\forall x. bird\, x \wedge \neg\, ab\, aspect2 x \supset flies\, x \tag{3}$$

$$\forall x. penguin\, x \supset ab\, aspect2 x \tag{4}$$

$$\forall x. penguin\, x \wedge \neg\, ab\, aspect3 x \supset \neg\, flies\, x \tag{5}$$

If we have only *bird Tweety*, we can derive *flies Tweety*. But if we add *penguin Tweety*, we cannot derive, using (3), even though *bird Tweety* is present, *flies Tweety*, because $\neg\, ab\, aspect2$ *Tweety* is not true. So, when we have more specific information, the explicit ordering prevents the inference using more general information. In this particular example, $aspect2x$ orders the defaults $bird x \supset flies x$ and $penguin x \supset \neg\, flies x$. If we do not do this, then either *bird Tweety* would derive *flies Tweety* or *penguin Tweety* would derive $\neg$ *flies Tweety*. In modified FOT, we keep $bird x \supset flies x$ and $penguin x \supset \neg\, flies x$ as they are and we do not introduce any abnormal aspects. Indirectly, $ISA - MP$ produces the desired effect. The absence of abnormal aspects disturbs the explicit ordering of defaults. We circumvent this problem by prefering to use the most specific information available[19].

For example, let us assume that we have the following:

$$\forall x. bird\, x \supset flies\, x \tag{6}$$

$$\forall x. penguin\, x \supset bird\, x \tag{7}$$

$$\forall x. penguin\, x \supset \neg\, flies\, x. \tag{8}$$

If we perceive *bird Tweety*, then we can derive *flies Tweety*. Later if we perceive it as *penguin Tweety*, then in order to prefer more specific information, we need to withdraw *bird Tweety* and assert *penguin Tweety*. This can be achieved by asserting $B(P, 0, \beta, \gamma)$ where P is *bird Tweety*, thus effectively withdrawing it and asserting $B(Q, 1, 0, \gamma)$ where Q is *penguin Tweety*. Now, using (8), we derive $\neg$ *flies Tweety* and $ISA - MP$ cannot derive *flies Tweety* from (6).

An example of belief revision (see Theorem 7):

'Birds can fly'	(default)
'Penguins cannot fly'	(default)
'Penguin is a bird'	(ISA)

'Tweety is a bird'	(belief-1, rev-1)
'Tweety can fly'	(nonmonotonic inference)
'Tweety is a penguin'	(belief-2, rev-1)

This perception generates two belief premises: one, (belief-1, rev-0) to withdraw 'Tweety is a bird' and second, (belief-2, rev-1) to assert 'Tweety is a penguin'.

'Tweety cannot fly'	(nonmonotonic inference–previous inference is withdrawn)
'Tweety is a monkey'	(belief-3, rev-1)

This again generates a belief pair: one, (belief-2, rev-0), to withdraw 'Tweety is a penguin' and second, (belief-3, rev-1), to trigger the effect of 'Tweety is a monkey'. Thus, we cannot generate any inferences about Tweety as bird and we can generate further inferences about Tweety as monkey.

The crucial point is the generation of beliefs, usually a pair at a time, one to block the inferences from the false belief and second to assert the new belief. We think that such a belief revision happens automatically, unconsciously in human beings and it may not be possible to have logical systems that can generate such beliefs automatically (implementations may employ some heuristics to achieve the desired effect).

Consider the following situation:

P: 'It is raining',
Q: 'It is sunny'.
$Q \supset \neg P$ a common observation.

Let $B(P, 1, 0, \gamma)$ and $B(Q, 1, 0, \gamma)$ be two beliefs. From the proper axioms, we have

$B(P, 1, 0, \gamma) \supset P_{10_\gamma}$ and $B(Q, 1, 0, \gamma) \supset Q_{10_\gamma}$. These further generate P and Q, respectively.

When these beliefs are asserted in that order, we find no contradiction and when we generate the closure, $L(P) \in T$ since $P \in T$. Therefore, $\neg P$ is not derived. So $ISA - MP$ takes care of consistency. On the other hand, if the situation were, to start with, 'It is raining' and after a while rain stops and we have 'It is sunny'. In order to achieve what we have in mind, the following must be inputted:

$B(P, 0, 0, \gamma)$ to withdraw P and

$B(Q, 1, 0, \gamma)$ to generate Q.

Finally, $ISA - MP$ derives $\neg P$.

Consider again the hypothetical network shown in fig. 1. Let $\Gamma = \{Q_1(a), Q_2(b)\}$ be a set of premises. Let us compute Δ, the set of theorems closed under the inference rule $ISA - MP$.

Initially, $\Delta = \Gamma$.

Let us expand $Q_1(a) \in \Delta$ (it could have been $Q_2(b)$ as well). This involves applying $ISA - MP$ on $Q_1(a)$ and has three components:

(i) $C_1 = \{Q \mid Q_1(a) \supset Q \wedge \neg L(\neg Q)\}$

$= \{P_1(a), P_2(a), P_3(a)\}$ and $\Delta = \Delta \cup C_1$.

(ii) $C_2 = NMTC_{\Rightarrow}(Q_1(a)) = \{S_1(a), S_2(a)\}$ and $\Delta = \Delta \cup C_2$.

(iii) $C_3 = NMTC_{>}(Q_1(a))$. The possible candidates are $\{Q_{11}(a), \neg P_3(a)\}$. But as $IS-A(P_3(a))$ is *true* (since $P_3(a) \in NMTC_{\Rightarrow}(Q_1(a))$), $\neg IS-A(P_3(a))$ is *false* and so is $\neg IS-A(\neg\neg P_3(a))$. Therefore, $\neg P_3 \notin C_3$. Hence $C_3 = \{Q_{11}(a)\}$ and $\Delta = \Delta \cup C_3$.

Now we can select any theorem $\in \Delta$ and expand it. Say, we select $P_2(a)$ and then $P_4(a)$. So, $\Delta = \Delta \cup \{P_4(a), R_2(a)\}$. Let us investigate when $R_2(a)$ is selected for expansion. $C_1 = \phi$ and $C_2 = \phi$. $C_3 = NMTC_{>}(R_2(a))$. The possible candidates are $\{R_{21}(a), \neg P_{52}(a)\}$. Consider, $KIND-OF(R_2(a), P_{52}(a))$. $INHERITORS(R_2(a))$ is the set $\{Q_1(a), P_2(a), P_3(a), P_4(a), P_5(a)\}$. Let Q' be $P_5(a)$ (see section 5). Then $NMTC_{>}(P_5(a)) = \{P_{51}(a), P_{52}(a)\}$. Therefore, $KIND-OF(R_2(a), P_{52}(a))$ is *true* and $\neg KIND-OF(R_2(a), P_{52}(a))$ is *false*. Hence, $\neg KIND-OF(R_2(a), \neg\neg P_{52}(a))$ is *false* and $\neg P_{52}(a) \notin C_3$. Therefore, $C_3 = \{R_{21}(a)\}$ and $\Delta = \Delta \cup C_3$. Similarly, the remaining theorems of Δ can be expanded to complete it.

7. Some results

Theorem 1: The system has no redundant implicative operators.

Proof: In the proposed logical system, we have three different implicative operators, *viz.*, $\supset$, $\Rightarrow$, and $>$. The semantics of all three operators is equivalent to that of logical implication. The need for different implicative operators can be seen from the following. Let P and Q be any two adjacent nodes in the inheritance network. We are interested in the kinds of properties P can inherit from Q. Depicting the possible kinds as set-theoretic relations, we have only four possible cases to consider as shown in fig. 3.

Clearly, in case 1, P inherits nothing from Q and this case can be safely ignored.

Case 2 deals with the natural properties of node P. The important reason why we maintain inheritance network is to achieve parsimony in representation. Hence, we rarely find nodes being duplicated. This suggests that between any two nodes P and Q of the inheritance network, there is something that makes node P distinct from node Q. In other words, there is a set of natural properties of P that uniquely identifies P. Even though the relationship is bidirectional, from the point of view of inheritance, it is enough if we consider the implication from P towards its natural properties. We denote such an implication by $\Rightarrow$.

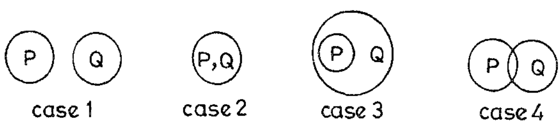

FIG. 3. Possible relations between P and Q.

Case 3 deals with ISA inheritance where P inherits nonmonotonically the default properties of its ancestor Q and inherits monotonically Q and its natural properties. This is clearly indicated by the set-theoretic relation 'P is a subset of Q' and the implication is from P towards Q. We denote such an implication by $\supset$.

Case 4 deals with partial implication and we prefer the direction of implication from P towards Q since we are interested in the properties that P can inherit from its neighbours. The implication is partial since there is a case of P being true and Q not being true. This characterises the default properties that P can inherit. We denote such an implication by $>$.

Hence the theorem.

Theorem 2: $ISA-MP$ subsumes MP.

Proof: By MP, we mean monotonic *modus ponens* of FOL which is as follows: From P, $P \rightarrow Q$, infer Q where $\rightarrow$ is the standard logical implication of FOL.

$ISA-MP$, on the other hand, infers from three operators. One natural mapping from $ISA-MP$ to MP would be to map the operator $\supset$ of $ISA-MP$ to $\rightarrow$ of MP. This means that in the network we do not have default property operator $>$ and we can imagine that the natural property $\Rightarrow$ is also represented using the operator $\supset$. Hence, when the nodes in the network are connected using only the operator $\supset$, then $ISA-MP$ behaves much like MP since both $NMTC_{>}(P)$ and $NMTC_{\Rightarrow}(P)$ are null. The monotonicity of $\supset$ virtually rules out the applicability of L operator. Note that the set of inferences made using the $ISA-MP$ is a subset of the set of inferences made using MP in general. In the monotonic case, these two sets are equal. Hence the result.

Lemma 0: $ISA-MP$ permits simple inheritance.

Proof: Consider a simple hierarchy

(k_1) $P_1 \supset P$

(k_2) $P_2 \supset P_1$

$\vdots$

(k_n) $P_n \supset P_{n-1}$

P_n

Proof is based on induction on the level of ISA hierarchy. At the lowest level, it follows from Theorem 2.

Let it be true at any level i.
To show that it is true at level $i+1$ also.
At level i, we have from (k_i) and P_i, and $ISA-MP$, we get P_{i-1}.
Hence the result.

Theorem 3: $ISA-MP$ permits simple multiple inheritance.

Proof: A simple multiple inheritance can be viewed as multiple simple inheritances. Let $S_1, \ldots, S_m$ be m simple inheritances with respect to P.

i.e.,

$$
\begin{aligned}
&P \supset Q_1 \supset \ldots \quad S_1\\
&P \supset Q_2 \supset \ldots \quad S_2\\
&\qquad\qquad\qquad \vdots\\
&P \supset Q_m \supset \ldots \quad S_m
\end{aligned}
$$

which says that P inherits properties from m simple inheritances. $ISA-MP$ ensures that P inherits from all its neighbours $Q_1, \ldots, Q_m$ and lemma 0 ensures further inheritances from $Q_1, \ldots, Q_m$.

Theorem 4: Hierarchy of operators ensures correct inheritance.

Proof: Any node P in the network can inherit the properties from its neighbouring nodes connected to it *via* one of the operators $\supset$, $\Rightarrow$, and $>$. The proper inheritance demands that natural and ISA properties be preferred over default properties. And $ISA-MP$ does exactly this. Hence the result.

Theorem 5: $ISA-MP$ permits multiple inheritance.

Proof: Follows from theorems 3 and 4.

Theorem 6: The set of proper axioms does not introduce any inconsistency.

Proof: Proper axioms are as follows:

$$(I) \qquad (\forall\alpha\beta\gamma)(B(P,\alpha,\beta,\gamma) \supset P_{\alpha\beta\gamma})$$

$$
(II) \qquad \begin{cases}
(\forall\alpha\beta\gamma)(P_{\alpha\beta\gamma} \wedge \neg L(P_{(\alpha+1)\beta\gamma}) \wedge \neg L(P_{1(\beta+1)\gamma}) \wedge odd(\alpha)\\
\qquad\qquad \wedge \neg L(P_{0\beta\gamma}) \supset (C_\gamma \supset P))\\
(\forall\alpha\beta\gamma)(P_{\alpha\beta\gamma} \wedge \neg L(P_{(\alpha+1)\beta\gamma}) \wedge \neg L(P_{1(\beta+1)\gamma}) \wedge even(\alpha)\\
\qquad\qquad \wedge \neg L(P_{0\beta\gamma}) \supset (C_\gamma \supset \neg P)).
\end{cases}
$$

Consider, any i and γ with $\beta = 0$

$$B(P, i, 0, \gamma).$$

This presumes that we already have,

$$
\begin{aligned}
&B(P, 1, 0, \gamma)\\
&B(P, 2, 0, \gamma)\\
&\quad\vdots\\
&\quad\vdots\\
&B(P, i-1, 0, \gamma).
\end{aligned}
$$

From (I) we get, $S1 = \{B(P,1,0,\gamma),\ldots,B(P,i,0,\gamma),$

$$P_{10\gamma}, P_{20\gamma},\ldots,P_{i0\gamma}\}.$$

Note that $P_{10\gamma}, P_{20\gamma},\ldots,P_{i0\gamma}$ are obtained using $ISA-MP$. Generating closure of $S1$, we get

$$S2 = S1 \cup \{L(P_{10\gamma}), L(P_{20\gamma}),\ldots,L(P_{i0\gamma})\}$$
$$\cup \{\neg L(P_{\alpha\beta\gamma}) \mid P_{\alpha\beta\gamma} \notin S2\}.$$

Since P or $\neg P$ can be generated only from (II), to show that there is no inconsistency due to P or $\neg P$, we need to show that there is only one triplet $<\alpha,\beta,\gamma>$ satisfying (II).

Using (II) along with $S2$, we observe that for $0 \leqslant \alpha \leqslant i-1$, (II) is not satisfied because $L(P_{10\gamma}),\ldots,L(P(P_{i0\gamma}) \in S2$ and for $\alpha > i$, (II) is not satisfied because $P_{\alpha\beta\gamma} \notin S2$. When α is i, the antecedent, $P_{i0\gamma} \wedge \neg L(P_{(i+1)0\gamma}) \wedge \neg L(P_{11\gamma}) \wedge \neg L(P_{00\gamma})$ is true resulting in P or $\neg P$ depending on whether i is odd or even. Assume that, now we assert $B(P,0,0,\gamma)$.

We have to only check what happens to the antecedent of (II) when α is i. Clearly, $\neg L(P_{00\gamma})$ is false because $P_{00\gamma} \in S1$ and therefore $L(P_{00\gamma}) \in S2$. Therefore, for no $\alpha \geqslant 0$, (II) holds and we have effectively withdrawn P.

We increment β for two reasons. Firstly, when we have asserted $B(P,0,\beta,\gamma)$ and are currently perceiving P. Secondly, when we want to assert P irrespective of the current status of P.

Now consider $B(P,i,j,\gamma)$ for any γ. This again presumes that we have the following:

$$I3 = \begin{cases} B(P,k_0,0,\gamma), 0 \leqslant k_0 \leqslant K_0 \quad \text{or} \quad 0 < k_0 \leqslant K_0 \\ B(P,k_1,1,\gamma), 0 \leqslant k_1 \leqslant K_1 \quad \text{or} \quad 0 < k_1 \leqslant K_1 \\ \vdots \\ \vdots \\ B(P,k_{j-1},j-1,\gamma), 0 \leqslant k_{j-1} \leqslant K_{j-1} \quad \text{or} \quad 0 < k_{j-1} \leqslant K_{j-1} \\ B(P,1,j,\gamma) \\ B(P,2,j,\gamma) \\ \vdots \\ \vdots \\ B(P,i,j,\gamma). \end{cases}$$

From (I) we get,

$$S3 = I3 \cup \{[P_{00\gamma}], P_{10\gamma}\ldots,P_{K_0 0\gamma},$$
$$[P_{01\gamma}], P_{11\gamma}\ldots,P_{K_1 1\gamma},$$
$$\vdots$$
$$\vdots$$
$$[P_{0(j-1)\gamma}], P_{1(j-1)\gamma}\ldots,P_{K_{(j-1)}(j-1)\gamma},$$
$$P_{1j\gamma},\ldots,P_{ij\gamma}\}$$

where $[P_{0i\gamma}]$ indicates an optional entry.

Generating closure of $S3$, we get,

$$S4 = S3 \cup \{[L(P_{00\gamma})], L(P_{10\gamma}), \ldots, L(P_{K_0 0\gamma}),$$
$$[L(P_{01\gamma})], L(P_{11\gamma}), \ldots, L(P_{K_1 1\gamma}),$$
$$\vdots$$
$$L(P_{1j\gamma}), \ldots, L(P_{ij\gamma}), \}$$
$$\cup \{\neg L(P_{\alpha\beta\gamma}) | P_{\alpha\beta\gamma} \notin S4\}.$$

Again using (II), we observe that only for the triplet $\langle i, j, \gamma \rangle$, the antecedent, $P_{ij\gamma} \wedge \neg L(P_{(i+1)j\gamma}) \wedge \neg L(P_{1(j+1)\gamma}) \wedge \neg L(P_{0j\gamma})$, is true resulting in P or $\neg P$ depending on whether i is odd or even.

Since in all cases, for only one triplet $\langle \alpha, \beta, \gamma \rangle$ the antecedent is true, the proper axioms cannot generate any inconsistency.

Theorem 7: The system reasons with the most specific information in the corresponding *ISA* hierarchy.

Proof: Consider a simple *ISA* hierarchy where

$(K_0)\, P_1$ is a subclass of P_0,

$(K_1)\, P_2$ is a subclass of P_1,

$\vdots$

$(K_{n-1})\, P_n$ is a subclass of P_{n-1}.

At the lowest level, it is trivially true.

Induction hypothesis: Let this be true at any level i. The n-level *ISA* hierarchy can be represented as follows:

$$(K_0) \quad P_1(X) \supset P_0(X)$$
$$(K_1) \quad P_2(X) \supset P_1(X)$$
$$\vdots$$
$$(K_{n-1}) \quad P_n(X) \supset P_{n-1}(X)$$
$$(K_n) \quad P_n(X)$$

to show that it is true at level $(i+1)$ also.

Let $B(P_i, \alpha, \beta, \gamma)$ represent the belief in P_i. From P_i, (K_{i-1}) and using $ISA-MP$ we can derive P_{i-1} and so on. When we perceive more specific information P_{i+1}, we withdraw P_i by asserting $B(P_i, 0, \beta, \gamma)$ and assert $B(P_{i+1}, 1, 0, \gamma)$. Thus at the level $i+1$, system reasons with the most specific information in the corresponding ISA hierarchy.

Definition 1: Given that $A \cup \{P\}$ and $A \cup \{Q\}$ are consistent, P and Q are said to be contradictory beliefs if the theory generated from $A \cup \{P, Q\}$ is the entire language, where A is a set of consistent beliefs.

Lemma 1: If the set of current beliefs is consistent then the theory is consistent.

Proof: We generate theorems from the current beliefs using inference rules. Therefore, in order to prove the lemma, we need to show that inference rules do not introduce any inconsistency. Obvioulsy, the nonmonotonic inference rule $ISA-MP$ and universal generalisation cannot introduce any inconsistency.

In the following, we assume that A is consistent and $A \cup \{P, Q\}$ is inconsistent.

Lemma 2: If $A \cup \{P\}$ is consistent and P and Q are contradictory beliefs then $A \cup \{\neg P, Q\}$ is consistent.

Proof: Obvious.

Lemma 3: If $A \cup \{P\}$ is consistent and P and Q are contradictory beliefs then $A \cup \{Q\}$ is consistent.

Proof: Obvious.

Theorem 8: If we perceive Q then we can believe in Q.

Proof: If there is any $P \in$ set of current beliefs such that P and Q are contradictory beliefs, then we input a pair of beliefs; one to either revise P generating $\neg P$ or to withdraw P, and second to assert Q. From lemmas 2 and 3, this cannot introduce any inconsistency. Lemma 1 ensures that the theory is consistent.

Theorem 9: Belief revision cannot introduce any inconsistency.

Proof: Belief revision generates $\neg P$ and theorem 8 says that the contradictory beliefs of $\neg P$ are withdrawn to keep the theory consistent.

Theorem 10: Belief withdrawl cannot introduce any inconsistency.

Proof: The belief can be withdrawn for three reasons: one, we are in the process of correcting the erroneous perception; second, the premise no longer holds because the external world has changed; third, the 'act of forgetting' may assert $B(P, 0, \beta, \gamma)$. Proof follows from the fact that if A is consistent then any subset of A is also consistent, and lemma 1.

Theorem 11: If P is any theorem, then there is no theorem Q such that P and Q are contradictory beliefs.

Proof: Follows from theorems 8, 9, and 10.

Theorem 12: The theorems of our logical system are the nonmonotonic inferences from $A \cup \{P\}$ or $A \cup \{\neg P\}$ or A.

Proof: For any belief $B(P, i, j, \gamma)$, from theorem 6, we can have only one of P or $\neg P$ or *nothing*. The proper axioms have no role to play in the nonmonotonic inference—in this respect, they are a sort of meta-axioms. So we are left with either $A \cup \{P\}$ or $A \cup \{\neg P\}$, or A, and a nonmonotonic inference rule $ISA-MP$. Hence the result.

8. Conclusion

We have presented a problem in human commonsense reasoning that involves defeasible beliefs. Our approach to the solution is based on proposing modified first-order logic and reasoning with human-oriented beliefs and permitting property inheritance with exceptions from multiple more general concepts. The crucial aspect is that an external object/activity could generate multiple but related beliefs so that our false beliefs about the object can be withdrawn and a new supposedly true belief can be asserted. Reasoning with multiple inheritance requires reasoning with the most specific information. This is made possible by permitting belief revision so that when more specific information is available, more general information can be withdrawn. The system retains all the previous (may be false) beliefs and makes available only current belief for reasoning, somewhat akin to human brain/mind structure. The salient features of the proposed approach include:

1. The system attempts to model 'contextual reasoning' and 'forgetfulness'.
2. It handles defeasible beliefs.
3. The nonmonotonic behaviour is characterised by using a modified version of *modus ponens* as inference rule.
4. The defaults can be used in the reasoning process without including any abnormal aspects.
5. The most specific information in the *ISA* hierarchy is used for reasoning.
6. The system permits property inheritance with exceptions from multiple more general concepts.

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