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## Short Communication

# A note on the nonlinear vibrations of rectangular plates with parabolically varying thickness

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**Abstract**

An analysis of the large amplitude vibrations of rectangular plates of parabolically varying thickness is presented. The method is based on Berger's assumption of neglecting the second invariant of the middle surface strain in the expression corresponding to the total potential energy of the system, in conjunction with a Galerkin procedure.

**Key words:** Parabolically varying thickness, large amplitude, frequency, taper constant.

## 1. Introduction

Investigations relating to nonlinear vibration are few in comparison with those for linear cases. This is, probably, due to the difficulties involved in solving the nonlinear differential equations. Berger[1], however, has proposed an approximate method to solve such problems which is simple and accurate for all practical purposes. Nash and Modeer[2] extended this technique offered by Berger to a dynamic case which was subsequently followed by different authors[3–6].

The work presented in this paper is to study the large amplitude vibrations of rectangular plates with parabolically varying thickness by means of Berger's method in combination with a Galerkin procedure.

## 2. Basic equations and their solutions

Let a flat rectangular plate with parabolically varying thickness and of length '$2a$' and of breadth '$2b$' be subjected to a normal load '$q$'. The origin of the cartesian

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coordinate system is located at the centre of the plate. The governing differential equations for the deflection function '$w$' of the plate exhibiting large deflection may be put in the following forms:

$$\nabla^2 (D\nabla^2 w) - (1-\nu)\left[\frac{\partial^2 D}{\partial x^2}\frac{\partial^2 w}{\partial y^2} - 2\frac{\partial^2 D}{\partial x \partial y}\frac{\partial^2 w}{\partial x \partial y} + \frac{\partial^2 D}{\partial y^2}\frac{\partial^2 w}{\partial x^2}\right]$$

$$- \frac{12CE}{1-\nu^2} f(t) \nabla^2 w - q = 0 \tag{1}$$

$$e = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{1}{2}\left\{\left(\frac{\partial w}{\partial x}\right)^2 + \left(\frac{\partial w}{\partial y}\right)^2\right\} = \left(\frac{c}{h}\right) f(t) \tag{2}$$

where $D = Eh^3/12\,(1-\nu^2)$ is the flexural rigidity of the plate, $E$ is the Young's modulus, $e$ is the first invariant, $u$, $v$ are the inplane displacements along $x$- and $y$-directions, respectively; $h = h(x, y)$ is the thickness of the plate at a point $(x, y)$ from the origin, and $\nu$ = Poisson's ratio and $\rho$ is the density of the plate material and '$C$' is a constant of integration $f(t)$ being an unknown function of time to be determined.

Let the law of thickness variation be $h = h_0 (1 + \beta\, x^2/a^2)$.

If the problem be restricted to the finding of the fundamental mode of frequency only one can set the expression for $w$ compatible with the boundary conditions for a plate with hinged immovable edges as

$$W = \cos\frac{\pi X}{2}\cos\frac{\pi Y}{2} F(t) \tag{3}$$

where $F(t)$ is an unknown function of time and $X = x/a$, $Y = y/b$ and $W = w/h_0$ have been introduced in their nondimensional forms. The actual analysis for obtaining the time differential equation as well as the evaluation of the coupling constant '$C$' may be omitted for brevity (actual analysis is given in Ref. 6). The time differential equation may, thus, be put straight forward in the following form

$$\ddot{F}(t) + A\,F(t) + BF^3(t) = Q. \tag{4}$$

## 3. Deductions

### *Free linear vibration*

The frequency parameter $\Omega$ appears to be

$$\Omega^2 = 12\,(1-\nu)^2 \rho a^4 \omega^2/Eh_0^2 = (\pi/2)^2\, \gamma/(1 + 0{\cdot}1306\,\beta) \tag{5}$$

where $\omega$ is the circular frequency of the plate. The values of the frequency parameter have been computed and are displayed in Table I for different values of the taper constant $\beta$.

Table I

**Values of the frequency parameter $\Omega^2$ for different values of $\beta$ and aspect ratio $(a/b)$ Rectangular plate with hinged immovable edges**

| $R/\beta$ | −0·5 | −0·4 | −0·3 | −0·1 | 0·0 |
|---|---|---|---|---|---|
| 3/2 | 27·5367 | 34·2895 | 41·2170 | 56·2259 | 64·3053 |
| 1 | 4·8413 | 8·5765 | 12·3727 | 20·2552 | 24·3532 |
| 2/3 | 0·4234 | 2·1639 | 4·7917 | 10·0552 | 12·7022 |
| 1/2 | .. | 0·8426 | 3·0128 | 7·3442 | 9·5126 |
| $R/\beta$ | 0·1 | 0·3 | 0·4 | 0·5 | |
| 3/2 | 72·8120 | 91·2096 | 101·1232 | 110·6869 | |
| 1 | 28·5704 | 37·3966 | 41·9874 | 46·4077 | |
| 2/3 | 15·3688 | 20·7839 | 23·5438 | 26·1248 | |
| 1/2 | 11·6886 | 16·0775 | 18·2971 | 19·9523 | |

*Nonlinear static case*

If the inertia term in eqn. (4) is rejected the static behaviour of the plate can be obtained from the following equation

$$AF + BF^3 = Q. \tag{6}$$

The nonlinear static behaviour has been shownin Fig. 1.

*Nonlinear free and forced vibrations*

To avoid repetition one may be referred to Ref. 6 for the analysis of free and forced vibrations. The relative time pericds of free linear and nonlinear vibrations $(T/T^*)$ have been presented graphically (Fig. 2) against relative amplitude $(A_0/h_0)$.

## 4. Numerical results and discussion

The values of the frequency parameter have been computed for different values of taper constant $\beta$ and aspect ratio $a/b$. Table I shows that the frequency increases with the increase of $\beta$ while for the same value of $\beta$ the frequency decreases with the increasing value $(a/b)$.

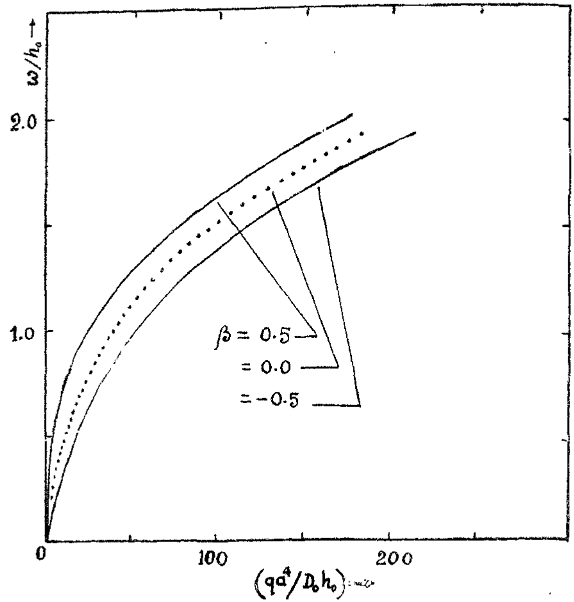


FIG. 1. Static behaviour of square plate with parabolically varying thickness.

Figure 2 depicts the relative period of vibrations in terms of the relative amplitude $(A_0/h_0)$ for $a/b = \frac{1}{2}$. In all calculations $\nu$ has been taken to be 0·3 and the values of $\beta$ ranges from $-0{\cdot}5$ to $+0{\cdot}5$. It is evident from Fig. 2 that the general trend is to decrease the period of nonlinear vibration with the increase of amplitude. Further, this decreasing tendency of $T^*$ is faster when $\beta$ decreases through negative values than when it does so through positive values. It has been further observed that the decreasing value of $(a/b)$ further accelerates this trend.

In conclusion, it appears that the nonlinear effects are stronger when $\beta < 0$ than the case when $\beta > 0$. This can be explained in the light of the fact that in the first case $\beta < 0$ the mass concentration near the supports decreases and the overall plate stiffness is decreased whereas in the latter case, as $\beta$ increases through positive value, the overall plate thickness is increased because of the increased thickness near the

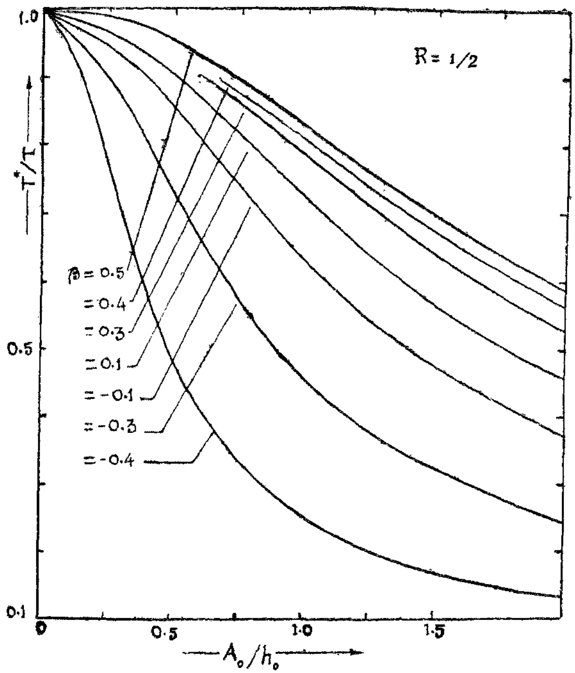


FIG. 2. Relative time period *vs.* relative amplitude for a rectangular plate with aspect ratio, 1/2.

boundary. In the first case the effect is to increase the frequency and the opposite holds when $\beta > 0$.

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