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Launching efficiency of overmoded dielectric rods excited in the HE_{lm} mode by a nonuniform magnetic ring source

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Abstract

The results of theoretical analysis of the problem of excitation of HE_{lm} modes on a cylindrical overmoded dielectric rod of circular cross-section excited by a concentric magnetic ring source of magnitude varying as a sinusoidal function of the peripheral angle leading to expressions for launching efficiency and far field patterns as a function of frequency, location of the source, characteristics of surface wave structures and higher order modes are presented Results of numerical evaluation of launching efficiency for the overmoded rods are reported in the form of graphs

Key words Launching efficiency, overmoded dielectric rods, and nonuniform magnetic ring source.

1. Introduction

Long distance propagation of microwaves by surface wave structures requires the knowledge of not only the surface wave characteristics of the structure guiding surface waves but also the efficiency of the launcher which transfers the energy from the source to the surface wave field Launching of surface waves has been studied thoroughly by Cullen[1,2] who solved the problem of exciting a plane surface wave from a narrow slot placed above the guiding surface This theoretical approach of Cullen has been extended for radial cylindrical surface waves by Fernando and Barlow[3] and Brown and Sharma[4] Wait[5] has studied theoretically the launching of surface waves by a magnetic ring source Collen[6] and Chatterjee[7,8] have also studied the problem of launching of a surface wave in detail

The subject of dielectric surface waveguides has been receiving attention for quite some time. Many of the theoretical and practical problems involved have been discussed in a very informative survey by Kao[9] Among them, the excitation of surface waves on these waveguiding structures is of much importance One of the simplest structures, which is of great practical utility is the circular dielectric rod The excitation of circularly symmetric surface waves on a dielectric rod by an elementary source for example, a magnetic current ring, have been investigated both theoretically and experimentally by Duncan[10] and Brown and Stachera[11] The excitation of the dipole HE_{11} mode is of great importance since it is the dominant mode and the easiest to excite The theoretical study of exciting the HE_{11} mode by a point dielectric dipole has been done by Gar Lam Yip[12] Cohn and King[13] have treated the source as a nonuniform magnetic ring and have studied the problem of excitation of the HE_{11} mode on a dielectric cylinder.

More recently, the demand for wider bandwidths for communication purposes has prompted research into the possible uses of dielectric surface waveguides for long distance telecommunication links at the millimetric and optical frequencies At these high frequencies a crowding of higher order modes is expected. A dielectric rod either when overdimensioned or when operated in a frequency range higher than the X-band supports a larger number of higher order modes An extensive study of the modes with respect to their effect of surface wave characteristics, power handling capability, radiation and gain has been done earlier by Dilli *et al*[11–18] The purpose of this paper is to obtain the launching efficiency and far field radiation patterns of overmoded rods by the method obtained by Cohn and King[13] as a function of source location, frequency and characteristics of the surface wave structures

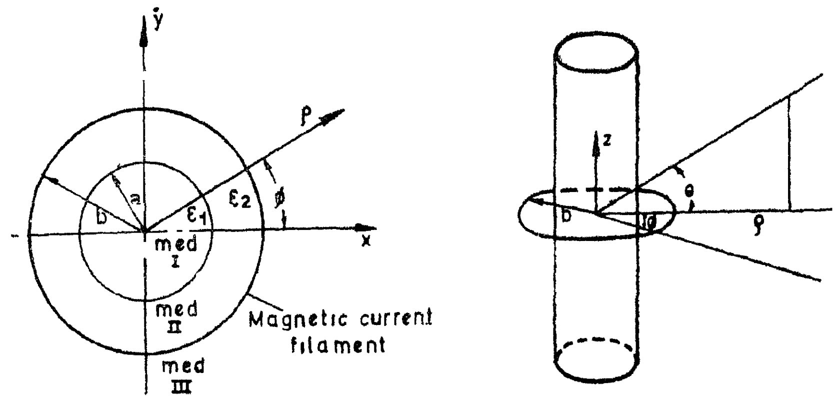
2. Co-ordinate system

The postulated method of exciting the dipole mode is physically realisable through the use of an annular slot in a conducting plane which is perpendicular to the axis of the dielectric rod This slot in turn would be energized from the opposite side of the conducting plane, with the prescribed angular variation by a circular waveguide in which the dominant TE_{11} mode is propagating The source, therefore, consists of an infinitesimally thin magnetic ring source which is concentric with and has a larger radius than the dielectric rod. Its magnitude is a sinusoidal function of the peripheral angle (ϕ). The magnetic ring source is specified by the following equation

$$\vec{I}_m = \vec{\phi}\delta(\rho - b)\,\delta(z)\sin\phi e^{-j\omega t} \tag{1}$$

The solutions to be sought in this problem are those in which the magnetic field component having the same co-ordinate direction as the assumed magnetic current source (H_ϕ) also has the same spatial variation as the source in that co-ordinate direction. Due to the character of the postulated solutions, the actual three-dimensional problem reduces to an equivalent two-dimensional problem.

The geometry and co-ordinate systems of the problem are shown in Fig 1 Medium I consists of the lossless dielectric rod of radius a and permittivity ϵ_1 Media II and III are free space and are divided by a hypothetical cylinder whose radius (b) is equal to the radius of the magnetic ring source The permittivity of Media II and III is that of free space (ϵ_0). The permeability of all three regions is that of free space (μ_0).



$\vec{I}_m = \vec{\phi}\, \delta(\rho - b)\, \delta(z) \sin \phi\, e^{-jwt}$

(a) (b)

FIG. 1 (a) End view, (b) Three-dimensional view of dielectric rod excited by a nonuniform magnetic ring source.

3. Field components

The components of the electromagnetic field generated by the magnetic ring source satisfy a set of equations analogous to the source free case

(i) Medium I The field in the guide (i e., for $0 < \rho < a$) is given by

$$E_{z11} = B_m J_1 (k_{1m}\rho) \sin \phi e^{-\gamma z} \quad (2a)$$

$$H_{z11} = C_m J_1 (k_{1m}\rho) \cos \phi e^{-\gamma z}. \quad (2b)$$

(ii) Medium II · The field between the current sheet and the guide (i e , for $a < \rho < b$) is given by

$$E_{z21} = [b'_{1m} H_1^{(1)} (k_{2m}\rho) + b'_{2m} H_1^{(2)} (k_{2m}\rho)] \sin \phi e^{-\gamma z} \quad (3\,a)$$

$$H_{z21} = [c'_{1m} H_1^{(1)} (k_{2m}\rho) + c'_{2m} H_1^{(2)} (k_{2m}\rho)] \cos \phi e^{-\gamma z} \quad (3\,b)$$

which represent standing waves.

(iii) The field outside the magnetic current sheet is ($\rho \geqslant b$)

$$E_{z31} = b_m H_1^{(1)} (k_{2m}\rho) \sin \phi e^{-\gamma z} \quad (4\,a)$$

$$H_{z31} = c_m H_1^{(1)} (k_{2m}\rho) \cos \phi e^{-\gamma z} \quad (4\,b)$$

which represent

(a) outward travelling wave for $u_2 > 0$, imaginary, (*b*) inward travelling wave for $u_2 > 0$ imaginary, and

(*c*) evanescent wave for $u_2 > 0$ real,

where $u_1 = k_{1m}\rho$ and $u_2 = k_{2m}\rho$

4. Boundary conditions

The boundary conditions are applied to the various field components in order to find the constants B_m, C_m, b'_{1m}, b'_{2m}, c'_{1m}, c'_{2m}, b_m, c_m

(i) At $\rho = a$,

$$E_{z11} = E_{z21}, \quad H_{z11} = H_{z21}, \quad E_{\phi 11} = E_{\phi 21}, \quad H_{\phi 11} = H_{\phi 21} \tag{5a–d}$$

(ii) At $\rho = b$

$$H_{z21} = H_{z31}, \quad E_{\phi 21} = E_{\phi 31}, \quad H_{\phi 21} = H_{\phi 31} \tag{6a–c}$$

(iii) The condition on E_{z21} at $= b$ is determined from the wave equation

5. Wave equation

The vector wave equation is

$$\nabla^2 \vec{E} + \omega^2 \mu\varepsilon \vec{E} = \nabla \times \vec{I}_m \quad \text{where } \vec{I}_m \text{ is given by eqn (1)} \tag{7}$$

The z-component of this equation is

$$\nabla^2 E_z + \omega^2 \mu\varepsilon E_z = \frac{1}{\rho} \frac{\partial}{\partial \rho} (\rho K_\phi). \tag{8}$$

Expanding the above equation

$$\frac{\partial^2 E_z}{\partial \rho^2} + \frac{1}{\partial} \frac{\partial E_z}{\partial \rho} + \frac{1}{\rho^2} \frac{\partial^2 E_z}{\partial \phi^2} + \frac{\partial^2 E_z}{\partial z^2} + \omega^2 \mu\varepsilon E_z$$

$$= - \frac{b}{\rho(\rho - b)} \delta(\rho - b)\, \delta(z) \sin \phi \tag{9}$$

since $\delta'(X) = -\delta(X)/X$ and the variation with respect to z is $e^{-\gamma z}$

Similarly for the H_z component the wave equation is

$$\frac{\partial^2 H_z}{\partial \rho^2} + \frac{1}{\rho} \frac{\partial H_z}{\partial \rho} + \frac{1}{\rho^2} \frac{\partial^2 H_z}{\partial \phi^2} + \frac{\partial^2 H_z}{\partial z^2} + \omega^2 \mu\varepsilon H_z = 0 \tag{10}$$

6. Evaluation of the boundary condition at $\rho = b$

Solutions are sought in which the various field components have the following functional dependence on

$$H_\phi = H'_\phi(\rho, z)\sin\phi, \quad H_\rho = H'_\rho(\rho, z)\cos\phi;$$
$$H_z = H'_z(\rho, z)\cos\phi, \quad E_\phi = E'_\phi(\rho, z)\cos\phi;$$
$$E_\rho = E'_\rho(\rho, z)\sin\phi, \quad E_z = E'_z(\rho, z)\sin\phi \tag{11a-f}$$

Therefore eqns (9) and (10) can be rewritten in the primed notation to take account of the prescribed variation

$$\frac{\partial^2 E'_z}{\partial\rho^2} + \frac{1}{\rho}\frac{\partial E'_z}{\partial\rho} + \frac{\partial^2 E'_z}{\partial z^2} + \left(\omega^2\mu\varepsilon - \frac{1}{\rho^2}\right)E'_z$$
$$= -\frac{b}{\rho(\rho-b)}\,\delta(\rho-b)\,\delta(z) \tag{12}$$

$$\frac{\partial^2 H_z}{\partial\rho^2} + \frac{1}{\rho}\frac{\partial H'_z}{\partial\rho} + \frac{\partial^2 H'_z}{\partial z^2} + \left(\omega^2\mu\varepsilon - \frac{1}{\rho^2}\right)H'_z = 0. \tag{13}$$

The method of integral transforms will be applied in solving these last two partial differential equations (eqns 12, 13). Let,

$$H'_q(\rho, z) = \int_{-\infty}^{\infty} H_{q1}(\rho, \gamma)\, e^{j\gamma z}\, d\gamma \tag{14 a}$$

where

$$H_{q1}(\rho, \gamma) = \frac{1}{2\pi}\int_{-\infty}^{\infty} H'_q(\rho, z)\, e^{-j\gamma z} dz \tag{14 b}$$

and

$$E'_q(\rho, z) = \int_{-\infty}^{\infty} E_{q1}(\rho, \gamma)\, e^{j\gamma z}\, d\gamma \tag{15 a}$$

where

$$E_{q1}(\rho, \gamma) = \frac{1}{2\pi}\int_{-\infty}^{\infty} E'_q(\rho, z)\, e^{-j\gamma z}\, dz \tag{15 b}$$

where q represents any of the co-ordinates ρ, ϕ or z.

Equation (14 a) is substituted into eqn. (13) and eqn (15 a) is substituted into eqn. (12), The resultant equations are multiplied by $1/2\pi\, e^{-j\gamma z}\, dz$ After integrating from $-\infty$ to $+\infty$ the following two equations are obtained

$$\left[\frac{\partial^2}{\partial\rho^2} + \frac{1}{\rho}\frac{\partial}{\partial\rho} - \gamma^2 + \left(\omega^2\mu\epsilon - \frac{1}{\rho^2}\right)\right] H_z(\rho, \gamma) = 0 \tag{16}$$

and

$$\left[\frac{\partial^2}{\partial\rho^2}+\frac{1}{\rho}\frac{\partial}{\partial\rho}-\gamma^2+\left(\omega^2\mu\epsilon-\frac{1}{\rho^2}\right)\right]E_z(\rho,\gamma)=-\frac{b\delta(\rho-b)}{2\pi\rho(\rho-b)} \tag{17}$$

The condition on E_z and therefore on E_{z1} at $\rho = b$ is determined by the wave eqn. (17) Multiplying each term in the equation by $d\rho$ and integrating over the interval $2\triangle$ from $\rho = b - \triangle$ to $\rho = b + \triangle$ we get

$$\left.\frac{dE_{z1}}{d\rho}\right|_{b-\Delta}^{b+\Delta}+\int_{b-\Delta}^{b+\Delta}\frac{1}{\rho}\frac{dE_{z1}}{d\rho}d\rho+(\omega^2\mu\epsilon-\gamma^2)$$

$$\int_{b-\Delta}^{b+\Delta}E_{z1}\,d\rho+\frac{1}{\rho}E_{z1}\Big|_{b-\Delta}^{b+\Delta}-\int_{b-\Delta}^{b+\Delta}\frac{1}{\rho}\frac{dE_{z1}}{d\rho}d\rho=-\frac{b}{2\pi}\int_{b-\Delta}^{b+\Delta}\frac{\delta(\rho-b)}{\rho(\rho-b)}\delta\rho \tag{18}$$

$$\text{RHS}=-\frac{b}{2\pi}\int_{b-\Delta}^{b+\Delta}\frac{\delta(\rho-b)}{\rho(\rho-b)}d\rho=\frac{b}{2\pi}\int_{-\Delta}^{\Delta}\frac{\delta'(\rho')}{(\rho'+b)}d\rho'$$

$$=-\frac{b}{2\pi}\left[\frac{d}{d\rho'}\left(\frac{1}{\rho'+b}\right)\right]_{\rho'-0}=\frac{1}{2\pi b}$$

where ρ' is a function of ρ.

Assuming E_{z1} is finite for all ρ and letting $\triangle \to 0$ and $\rho \to b$, the equation becomes

$$\left.\frac{dE_{z1}}{d\rho}\right|_{b+\Delta}-\left.\frac{dE_{z1}}{d\rho}\right|_{b-\Delta}+\frac{1}{b}E_z\Big|_{b+\Delta}-\frac{1}{b}E_z\Big|_{b-\Delta}=\frac{1}{2\pi b}$$

or

$$\left.\frac{dE_{z31}}{d\rho}\right|_b-\left.\frac{dE_{z21}}{d\rho}\right|_b+\frac{1}{b}E_{z31}-\frac{1}{b}E_{z21}=\frac{1}{2\pi b} \tag{6 a-d}$$

7. Determinantal equation

Applying the above conditions eqns. 5 (*a–d*), 6 (*a–d*) eight equations are obtained in terms of the eight arbitrary constants

(*a*) $C_m J_1(k_{1m}a) - c'_{1m}H_1^{(1)}(k_{2m}a) - c'_{2m}H_1^{(2)}(k_{2m}a) = 0$

(*b*) $B_m J_1(k_{1m}a) - b'_{1m}H_1^{(1)}(k_{2m}a) - b'_{2m}H_1^{(2)}(k_{2m}a) = 0$

(*c*) $-B_m\frac{\omega\mu_0}{k_{1m}}J_1'(k_{1m}a)+C_m\frac{\gamma}{ak_{1m}^2}J_1(k_{1m}a)+b'_{1m}\frac{\omega\mu_0}{k_{2m}}H_1^{(1)\prime}(k_{2m}a)+b'_{2m}\frac{\omega\mu_0}{k_{2m}}H_1^{(2)\prime}(k_{2m}a)$

$$-c'_{1m}\frac{\gamma}{ak_{2m}^2}H_1^{(1)}(k_{2m}a)-c'_{2m}\frac{\gamma}{ak_{2m}^2}H_1^{(2)}(k_{2m}a)=0$$

$$(d)\ B_m \frac{\gamma}{a k_{1m}^2} J_1(k_{1m}a) - c_m \frac{\omega \epsilon_1}{k_{1m}} J_1'(k_{1m}a) - b_{1m}' \frac{\gamma}{a k_{2m}^2} H_1^{(1)}(k_{2m}a) - b_{2m}' \frac{\gamma}{a k_{2m}^2} H_1^{(2)}(k_{2m}a)$$

$$+ c_{1m}' \frac{\omega \epsilon_0}{k_{2m}} H_1^{(1)\prime}(k_{2m}a) + c_{2m}' \frac{\omega \epsilon_0}{k_{2m}} H_1^{(2)\prime}(k_{2m}a) = 0$$

$$(e)\ b_{1m}' H_1^{(1)}(k_{2m}b) + b_{2m}' H_1^{(2)}(k_{2m}b) - b_m H_1^{(1)}(k_{2m}b) = 0$$

$$(f)\ b_{1m}' \frac{\gamma}{b} H_1^{(1)}(k_{2m}b) + b_{2m}' \frac{\gamma}{b} H_1^{(2)}(k_{2m}b) - c_{1m}' \omega \epsilon_0 k_{2m} H_1^{(1)}(k_{2m}b) - c_{2m}' \omega \epsilon_0 k_{2m} H_1^{(2)}(k_{2m}b)$$

$$- b_m \frac{\gamma}{b} H_1^{(1)}(k_{2m}b) - c_{1m}' \frac{\gamma}{b} H_1^{(1)}(k_{2m}b) = 0$$

$$(g)\ - b_{1m}' \omega \mu_0 k_{2m} H_1^{(1)\prime}(k_{2m}b) - b_{2m}' \omega \mu_0 k_{2m} H_1^{(2)\prime}(k_{2m}b) + c_{1m}' \frac{\gamma}{b} H_1^{(1)}(k_{2m}b)$$

$$+ c_{2m}' \frac{\gamma}{b} H_1^{(2)}(k_{2m}b) + b_m \omega \mu_0 k_{2m} H_1^{(1)\prime}(k_{2m}b) - C_m \omega \mu_0 H_1^{(2)\prime}(k_{2m}b) = 0$$

$$(h)\ - c_{1m}' \left[k_{2m} H_1^{(1)\prime}(k_{2m}b) + \frac{1}{b} H_1^{(1)}(k_{2m}b) \right] - c_{2m}' \left[k_{2m} H_1^{(2)\prime}(k_{2m}b) + \frac{1}{b} H_1^{(2)}(k_{2m}b) \right]$$

$$+ B_m \left[k_{2m} H_1^{(1)\prime}(k_{2m}b) + \frac{1}{b} H_1^{(1)}(k_{2m}b) \right] = \frac{1}{2\pi b} \qquad (19\ a\text{–}h)$$

8. Solution of the determinant

These eight equations, (19 *a–h*), are solved by setting the determinant $\triangle$, for need by the co-efficients equal to zero. After simplification,

$$\triangle = \frac{1}{b}\left(\frac{2}{\pi}\right)^2 \left[\omega^2 \mu_0 \epsilon_1 J_1(k_{1m}a) H_1^{(1)}(k_{2m}a) \frac{2j}{b} \right]^2 \left\{ \left[\frac{\epsilon_1}{k_{1m}} \frac{J_1'(k_{1m}a)}{J_1(k_{1m}a)} - \frac{1}{k_{2m}} \frac{H_1^{(1)\prime}(k_{2m}a)}{k_{2m} H_1^{(1)}(k_{2m}a)} \right] \left[\frac{1}{k_{1m}} \frac{J_1'(k_{1m}a)}{J_1(k_{1m}a)} - \frac{1}{k_{2m}} \frac{H_1^{(1)\prime}(k_{2m}a)}{H_1^{(1)}(k_{2m}a)} \right] - \frac{\frac{\gamma^2}{a^2}\left(\frac{1}{k_{1m}^2} - \frac{1}{k_{2m}^2}\right)^2}{\omega^2 \mu_0 \epsilon_0} \right\} \qquad (20)$$

when $\triangle = 0$, the expression obtained is identical to the equation of the HE_{lm} mode propagating on an infinitely long source-free dielectric rod. Solving for b_m and c_m, we have

$$b_m = -\frac{1}{4}\left\{\frac{k_{2m}b}{2j} H_1^{(2)\prime}(k_{2m}b)\right.$$

$$-\frac{\dfrac{bk_{2m}}{2j} H_1^{(1)\prime}(k_{2m}b)\dfrac{H_1^{(2)}(k_{2m}a)}{H_1^{(1)}(k_{2m}a)}\left\{\left[\dfrac{1}{k_{1m}}\dfrac{J_1'(k_{1m}a)}{J_1(k_{1m}a)} - \dfrac{1}{k_{2m}}\dfrac{H_1^{(1)\prime}(k_{2m}a)}{H_1^{(1)}(k_{2m}a)}\right]\left[\dfrac{\bar{\epsilon}_1}{k_{1m}}\dfrac{J_1'(k_{1m}a)}{J_1(k_{1m}a)} - \dfrac{1}{k_{2m}}\dfrac{H_1^{(2)\prime}(k_{2m}a)}{H_1^{(2)}(k_{2m}a)}\right] - \dfrac{\gamma^2\left(\dfrac{1}{k_{1m}^2}-\dfrac{1}{k_{2m}^2}\right)^2}{a^2\omega^2\mu_0\epsilon_0}\right\}}{\left\{\left[\dfrac{1}{k_{1m}}\dfrac{J_1'(k_{1m}a)}{J_1(k_{1m}a)} - \dfrac{1}{k_{2m}}\dfrac{H_1^{(1)\prime}(k_{2m}a)}{H_1^{(1)}(k_{2m}a)}\right]\left[\dfrac{\bar{\epsilon}_1}{k_{1m}}\dfrac{J_1'(k_{1m}a)}{J_1(k_{1m}a)} - \dfrac{1}{k_{2m}}\dfrac{H_1^{(1)\prime}(k_{2m}a)}{H_1^{(1)}(k_{2m}a)}\right] - \dfrac{\gamma^2}{a^2}\dfrac{\left(\dfrac{1}{k_{1m}^2}-\dfrac{1}{k_{2m}^2}\right)^2}{\omega^2\mu_0\epsilon_0}\right\}}$$

$$\left.+\frac{\dfrac{2}{\pi}\dfrac{\gamma^2\left(\dfrac{1}{k_m^2}-\dfrac{1}{k_{2m}^2}\right)}{(k_{2m}a)^2\,\omega^2\mu_0\epsilon_0}\dfrac{H_1^{(1)\prime}(k_{2m}b)}{[H_1^{(1)}(k_{2m}b)]^2}}{\left\{\left[\dfrac{1}{k_{1m}}\dfrac{J_1'(k_{1m}a)}{J_1(k_{1m}a)} - \dfrac{1}{k_{2m}}\dfrac{H_1^{(1)\prime}(k_{2m}a)}{H_1^{(1)}(k_{2m}a)}\right]\left[\dfrac{\bar{\epsilon}_1}{k_{1m}}\dfrac{J_1'(k_{1m}a)}{J_1(k_{1m}a)} - \dfrac{1}{k_{2m}}\dfrac{H_1^{(1)\prime}(k_{2m}a)}{H_1^{(1)}(k_{2m}a)}\right] - \dfrac{\gamma^2}{a^2}\dfrac{\left(\dfrac{1}{k_{1m}^2}-\dfrac{1}{k_{2m}^2}\right)}{\omega^2\mu_0\epsilon_0}\right\}^2}\right\} \qquad (21)$$

$$
C_m = \frac{1}{4\omega\mu_0}\left\{\frac{1}{2j} H_1^{(2)}(k_{2m}b)\right.
$$

$$
- \frac{\dfrac{1}{2j} H_1^{(1)}(k_{2m}b)\dfrac{H_1^{(2)}(k_{2m}a)}{H_1^{(1)}(k_{2m}a)}\left\{\left[\dfrac{\bar{\epsilon}_1}{k_{1m}}\dfrac{J_1'(k_{1m}a)}{J_1(k_{1m}a)} - \dfrac{1}{k_{2m}}\dfrac{H^{(1)'}(k_{2m}a)}{H_1^{(1)}(k_{2m}a)}\right]\left[\dfrac{1}{k_{1m}}\dfrac{J_1'(k_{1m}a)}{J_1(k_{1m}a)} - \dfrac{1}{k_{2m}}\dfrac{H_1^{(2)'}(k_{2m}a)}{H_1^{(2)}(k_{2m}a)}\right] - \dfrac{\gamma^2\left(\dfrac{1}{k_{1m}^2} - \dfrac{1}{k_{2m}^2}\right)^2}{a^2\omega^2\mu_0\epsilon_0}\right\}}{\left\{\left[\dfrac{1}{k_{1m}}\dfrac{J_1'(k_{1m}a)}{J_1(k_{1m}a)} - \dfrac{1}{k_{2m}}\dfrac{H_1^{(1)'}(k_{2m}a)}{H_1^{(1)}(k_{2m}a)}\right]\left[\dfrac{\bar{\epsilon}_1}{k_{1m}}\dfrac{J_1'(k_{1m}a)}{J_1(k_{1m}a)} - \dfrac{1}{k_{2m}}\dfrac{H_1^{(1)'}(k_{2m}a)}{H_1^{(1)}(k_{2m}a)}\right] - \dfrac{\gamma^2}{a^2}\dfrac{\left(\dfrac{1}{k_{1m}^2} - \dfrac{1}{k_{2m}^2}\right)^2}{\omega^2\mu_0\epsilon_0}\right\}}
$$

$$
\left. + \frac{\dfrac{2}{\pi}\dfrac{k_{2m}b}{(k_{2m}a)^2}\left(\dfrac{1}{k_{1m}^2} - \dfrac{1}{k_{2m}^2}\right)\dfrac{H_1^{(1)'}(k_{2m}b)}{[H_1^{(1)}(k_{2m}a)]^2}}{\left\{\left[\dfrac{1}{k_{1m}}\dfrac{J_1'(k_{1m}a)}{J_1(k_{1m}a)} - \dfrac{1}{k_{2m}}\dfrac{H_1^{(1)'}(k_{2m}a)}{H_1^{(1)}(k_{2m}a)}\right]\left[\dfrac{\bar{\epsilon}_1}{k_{1m}}\dfrac{J_1'(k_{1m}a)}{J_1(k_{1m}a)} - \dfrac{1}{k_{2m}}\dfrac{H_1^{(1)'}(k_{2m}a)}{H_1^{(1)}(k_{2m}a)}\right] - \dfrac{\gamma^2}{a^2}\dfrac{\left(\dfrac{1}{k_{1m}^2} - \dfrac{1}{k_{2m}^2}\right)^2}{\omega^2\mu_0\epsilon_0}\right\}}\right\} \tag{22}
$$

To solve the partial differential eqns (12, 13) the method of integral transforms is used by letting,

$$E_{z3} = \int_{-\infty}^{\infty} E_{z31} e^{j\gamma z} d\gamma = \int_{-\infty}^{\infty} b_m H_1^{(1)} (k_{2m}\rho) e^{j\gamma z} d\gamma \tag{23}$$

$$H_{z3} = \int_{-\infty}^{+\infty} H_{z31} e^{j\gamma z} d\gamma = \int_{-\infty}^{\infty} c_m \gamma H_1^{(1)} (k_{2m}\rho) e^{j\gamma z} d\gamma \tag{24}$$

$$E_{\phi 3} = \int_{-\infty}^{\infty} E_{\phi 31} e^{j\gamma z} d\gamma = \int_{-\infty}^{\infty} \frac{j}{k_{2m}^2} \left[\frac{\gamma}{\rho} H_1^{(1)} (k_{2m}\rho) - \omega\mu_0 k_{2m} c_m H_1^{(1)\prime} (k_{2m}\rho) \right] e^{j\gamma z} d\gamma \tag{25}$$

$$H_{\phi 3} = \int_{-\infty}^{\infty} H_{\phi 31} e^{j\gamma z} d\gamma = \int_{-\infty}^{\infty} \frac{-j}{k_{2m}^2} \left[\frac{\gamma}{\rho^2} c_m H_1^{(1)} (k_{2m}\rho) - \omega\epsilon_0 k_{2m} b_m H_1^{(1)\prime} (k_{2m}\rho) \right] e^{j\gamma z} d\gamma \tag{26}$$

9. Evaluation of the infinite integrals

The above four real integral eqns. (23–26) can be evaluated by considering them to be contour integrals in the γ-plane and applying the Cauchy Residue Theorem

Since

$$k_{1m} = \pm \sqrt{\omega^2 \mu_0 \epsilon_1 - \gamma^2} \tag{27 a}$$

and

$$k_{2m} = \pm \sqrt{\omega^2 \mu_0 \epsilon_0 - \gamma^2} \tag{27 b}$$

k_{1m} is multiple valued at $\gamma = \pm \sqrt{\omega^2 \mu_0 \epsilon_1}$ and k_{2m} is multiple valued at $\gamma = \pm \sqrt{\omega^2 \mu_0 \epsilon_0}$. The integrands of the above equation are even functions of k_{1m} and hence the only branch points occur at

$$\gamma = \pm \gamma_b = \pm \sqrt{\omega^2 \mu_0 \epsilon_0}, \; k_2 = 0 \tag{28 a}$$

Let the branch cuts (Γ) be along the line

$$\pm \gamma_b + \text{Im}\, \gamma \tag{28 b}$$

The integrands of the eqns. (23–26) are determined by setting the denominator of b_m and c_m given by eqns (21) and (22) respectively equal to zero The poles at $\pm \gamma_b$ are determined by letting $\gamma = \gamma_p$, $k_{1m} = k_{1mp}$ and $k_{2m} = k_{2mp}$ in the resulting equations (27 a, b) where k_{1m} and k_{2m} have been already defined as transverse wave numbers at the poles. A path of integration in the complex γ-plane which assures convergence of each of the integrals is shown in Fig 2

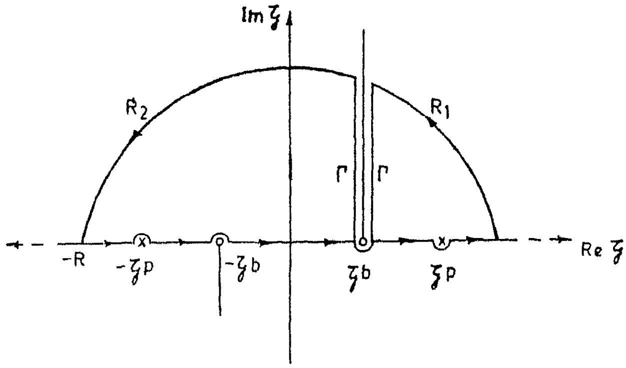


Fig 2 Contour of integration in the ζ-plane.

Only the branch cut from $+\gamma$ will be considered since it will be shown that it represents radiation away from the magnetic line source. Similarly only the pole at γ_p is included in the contour of integration since it represents surface wave propagation away from the source in the positive z-direction.

Consider the integration in the complex γ-plane around the closed curve shown in Fig. 2. Let the function $F(\gamma)$ be representative of the integrands in the eqns. (23–26). From Cauchy's Residue Theorem,

$$\int_{-R}^{R} F(\gamma)\,d\gamma + \int_{R_1} F(\gamma)\,d\gamma + \int_{R_2} F(\gamma)\,d\gamma + \int_{\Gamma} F(\gamma)\,d\gamma$$
$$= 2\pi j \text{ (sum of the residues)} \tag{29}$$

where the residues are taken at the poles of the integrand within the closed curve of integration As $R \to \infty$, it can be shown that the integrals on R_1 and R_2 vanish for any of the integrands for $z > 0$ and $k_2 = +\sqrt{\omega^2 \mu_0 \epsilon_0}$. Therefore, the eqn (29) becomes

$$\int_{-\infty}^{\infty} F(\gamma)\,d\gamma = -\int_{\Gamma} F(\gamma)\,d\gamma + 2\pi j \text{ (sum of the residues)} \tag{30}$$

10. Evaluation of the radiation field by SDP method

Instead of integrating along Γ, Γ is deformed into the path of steepest descent, C_s, which passes through the saddle point of the integrand. The path is directed in the reverse sense and as a result the eqn (30) becomes

$$\int_{-\infty}^{\infty} F(\gamma)\,d\gamma = \int_{C_s} F(\gamma)\,d\gamma + 2\pi j \text{ (sum of the residues).} \tag{31}$$

In order to calculate the radial power flow, expressions for the E_ϕ and H_ϕ components of the radiation field will be needed. Referring above to expressions (25, 26) for E_φ and H_ϕ we define

$$L^R_{\phi 3} = \int_{C_s} \frac{j}{k^2_{2m}} \left[\frac{\gamma}{\rho} b_m H^{(1)}_1 (k_{2m} \rho) - \omega\mu_0 k_{2m} \gamma c_m H^{(1)\prime}_1 (k_{2m} \rho) \right] e^{j\gamma_z} d\gamma \tag{32}$$

$$H^R_{\phi 3} = \int_{C_s} \frac{-j}{k^2_{2m}} \left[\frac{\gamma^2}{\rho} c_m H^{(1)}_1 (k_{2m} \rho) - \omega\epsilon_0 k_{2m} b_m H^{(1)\prime}_1 (k_{2m} \rho) \right] e^{j\gamma_z} d\gamma \tag{33}$$

The following transformations to the τ-plane (Fig. 3) are used by letting

$$\gamma = \gamma_b \sin \tau \text{ where } \tau = \sigma + j\eta \tag{34}$$

Therefore

$$\mathrm{Re}(\gamma) = \gamma_b \sin \sigma \cosh \eta \tag{35 a}$$

$$\mathrm{Im}(\gamma) = \gamma_b \cos \sigma \sinh \eta . \tag{35 b}$$

The branch cuts at $\mathrm{Re}(\gamma) = \pm \gamma_b$ are transformed to the curves

$$\sin \sigma \cosh \eta = \pm 1 \tag{36}$$

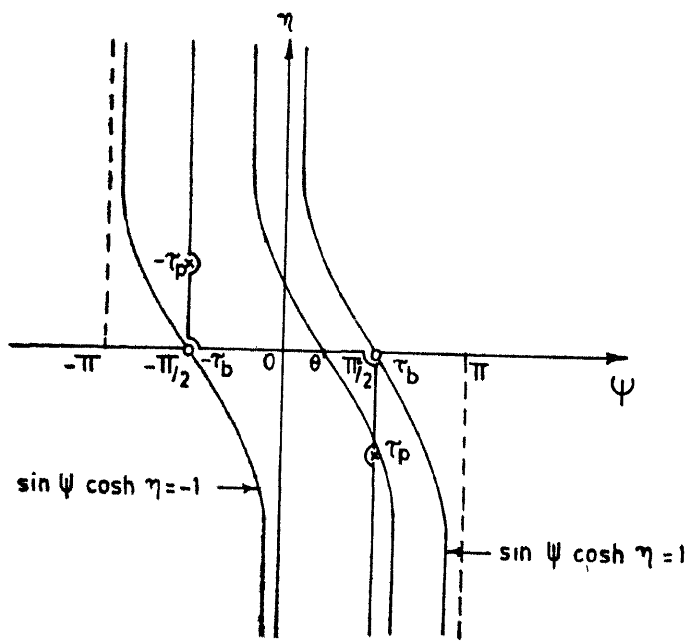


FIG. 3 Path of integration in the τ-plane

while the branch points at $\gamma = \pm \gamma_b$ transform to the points $\sigma = \pm \pi/2$, $\eta = 0$ For

$$\gamma = +\gamma_b, \quad \eta = 0, \sigma = \pi/2 \tag{37 a}$$

$$\gamma = -\gamma_b, \quad \eta = 0, \sigma = \pi/2. \tag{37 b}$$

From the eqn. (35 *a*, *b*) for Re (γ) and Im (γ), it can be seen that the path of integration along the real axis on the γ-plane transforms to the following three straight line segments on the τ-plane

$$\sigma = -\pi/2, \quad \eta > 0 \text{ for } \gamma < -\gamma_b \tag{38 a}$$

$$-\pi/2 < \sigma < \pi/2, \quad \eta = 0 \text{ for } -\gamma_b < \gamma < \gamma_b \tag{38 b}$$

$$\sigma = \pi/2, \quad \eta < 0 \text{ for } \gamma > \gamma_b \tag{38 c}$$

From the relations $k_{1m}^2 = \omega^2 \mu_0 \epsilon_1 - \gamma^2$, and

$k_{2m}^2 = \omega^2 \mu_0 \epsilon_0 - \gamma^2$ it can be shown that

$$k_{2m} = +\gamma_b \cos \tau \tag{39 a}$$

$$k_{1m} = \pm \gamma_b \left(\frac{\epsilon_1}{\epsilon_0} - \sin^2 \tau \right)^{1/2} \tag{39 b}$$

$$\gamma = \gamma_b \sin \tau \tag{39 c}$$

substituting these eqns. 39 (*a*–*c*) in the expression for $E^R_{\phi 3}$ (eqn. 32) and $H^R_{\phi 3}$ (eqn 33), we obtain

$$E^R_{\phi 3} = \int_{C_8} j \frac{\sin \tau}{\cos \tau} \Big[\frac{1}{\rho} b_m(\tau) H_1^{(1)} (\rho \gamma_b \cos \tau) - \omega \mu_0 \gamma_b \cos \tau \, c_m(\tau) H_1^{(1)\prime} (\gamma_b \rho \cos \tau) \Big] e^{j\gamma_b \sin \tau z} d\tau \tag{40}$$

$$H^R_{\phi 3} = \int_{C_8} - \frac{j}{\cos \tau} \Big[\frac{\gamma_b}{\rho} \sin^2 \tau \, c_m(\tau) H_1^{(1)} (\rho \gamma_b \cos \tau) - \omega \epsilon_0 \cos \tau \, b_m(\tau) H_1^{(1)\prime} (\rho \gamma_b \cos \tau) \Big] e^{j\gamma_b \cos \tau z} d\tau \tag{41}$$

Transform to the spherical co-ordinate system with the help of the transformation

$$\rho = r \cos \theta, \quad z = r \sin \theta. \tag{42 a, b}$$

For the radiation field components at large distances from the source, $H_1^{(1)}(k_{2m}\rho)$ and $H_1^{(1)\prime}(k_{2m}\rho)$ can be replaced by their asymptotic forms. As $r \to \infty$, the Hankel function $H_1^{(1)}$ and its derivative become

$$H_1^{(1)}(\rho\gamma_b \cos\tau) \simeq \sqrt{\frac{2}{\pi r\, \gamma_b \cos\tau \cos\theta}}\; e^{-j\pi/4}\, e^{jr\gamma_b \cos\tau\cos\theta} \tag{43 a}$$

$$H_1^{(1)\prime}(\rho\gamma_b \cos\tau) \simeq \sqrt{\frac{2}{\pi r\, \gamma_b \cos\tau \cos\theta}}\; e^{-j\pi/4}\, e^{jr\gamma_b \cos\tau\cos\theta} \tag{43 b}$$

and hence $E^R_{\phi_3}$ and $H^R_{\phi_3}$ become,

$$E^R_{\phi_3} = j\sqrt{\frac{2}{\pi r\cos\theta}} \int_{c_s} \frac{\sin\tau}{\cos\tau}\left[\frac{e^{-j\pi/4}}{r\cos\theta}\sqrt{\frac{1}{\gamma_b\cos\tau}}\; b_m(\tau)\right.$$

$$\left. - \omega\mu_0 e^{-j\pi/4}\sqrt{\gamma_b\cos\tau}\; c_m(\tau)\right] e^{j\gamma_b r\cos(\tau-\theta)}\, d\tau \tag{44}$$

$$H^R_{\phi_3} = -j\,\frac{2}{\pi r\cos\theta} \int_{c_s} \frac{1}{\gamma_b\cos\tau}\left[\frac{\gamma_b^2\sin^2\tau}{r\cos}\;\frac{e^{-j3\pi/4}}{\sqrt{\gamma_b\cos\tau}}\; c_m(\tau)\right.$$

$$\left. - \omega\epsilon_0 e^{-j\pi/4}\sqrt{\gamma_b\cos\tau}\; b_m(\tau)\right] e^{\gamma_b r\cos(\tau-\theta)}\, d\tau \tag{45}$$

The saddle point is defined for both integrals by

$$\frac{\partial}{\partial\tau}[\cos(\tau-\theta)] = 0 \tag{46}$$

therefore $\tau = \theta$ is the saddle point in each case The path of steepest descent, C_s, which passes across the saddle point ($\tau = \theta$) is one on which the imaginary part of the exponent remains constant, is given by the equation

$$\cos(\sigma-\theta)\;\cosh\eta = 1. \tag{47}$$

The following rule for saddle point integration is used, where $\tau = \theta$ is the saddle point and $d/d\tau\,(e^{ng(\tau)}) = 0$

$$\lim_{n\to\infty}\int_C F(\tau)\, e^{ng(\tau)}\, d\tau = F(\theta)\, e^{ng(\theta)}\sqrt{\frac{2\pi}{n}\;\frac{1}{e^{j\pi}g''(\theta)}}. \tag{48}$$

Provided that $g''(\theta) = 0$. Applying this rule to eqns. (44, 45) and simplifying

$$E^R_{\phi_3} = \frac{je^{j\gamma_b r}}{4r}\, g_1(\theta)\cos\phi \tag{49}$$

$$H^{R}_{\phi 3} = -j\sqrt{\frac{\epsilon_0}{\mu_0}}\,\frac{e^{i\gamma_b r}}{4r}\,g_2(\theta)\sin\phi \tag{50}$$

where

$$X_1 = \frac{2\pi a}{\lambda_0}\,(\bar{\epsilon}_1 - \sin^2\theta)$$

$$X_2 = \frac{2\pi a}{\lambda_0}\cos\theta$$

$$g_1(\theta) = G_1\frac{b}{a}H_0^{(1)}\left(\frac{b}{a}X_2\right) + G_2H_1^{(1)}\left(\frac{b}{a}X_2\right) + G_3H_1^{(2)}\left(\frac{b}{a}X_2\right) \tag{51 a}$$

and

$$g_2(\theta) = G_4\frac{b}{a}H_0^{(1)}\left(\frac{b}{a}X_2\right) + G_5H_1^{(1)}\left(\frac{b}{a}X_2\right) + G_6H_1^{(2)}\left(\frac{b}{a}X_2\right) \tag{51 b}$$

$$G_1 = -X_2\frac{P_3}{P_1}\tan\theta$$

$$G_2 = \left\{P_3 - P_2\frac{H_1^{(1)\prime}(X_2)}{H_1^{(1)\prime}(X_2)}\right\}\frac{\tan\theta}{P_1}$$

$$G_3 = P_1\tan\theta$$

$$G_4 = -\frac{X_2}{\cos\theta}\,\frac{H_1^{(2)}(X_2)}{H_1^{(1)}(X_2)}\,\frac{P_4}{P_1}$$

$$G_5 = \left[\frac{H_1^{(2)}(X_2)}{H_1^{(1)}(X_2)}\,P_1 - P_3\sin^2\theta\right]\Big/\,P_1\cos\theta$$

$$G_6 = \frac{X_2}{\cos\theta}$$

$$P_1 = \left[\frac{1}{X_1}\,\frac{J_1'(X_1)}{J_1(X_1)} - \frac{1}{X_2}\,\frac{H_1^{(1)\prime}(X_2)}{H_1^{(1)}(X_2)}\right]\left[\frac{\bar{\epsilon}_1}{X_1}\,\frac{J_1'(X_1)}{J_1(X_1)} - \frac{1}{X_2}\,\frac{H_1^{(1)\prime}(X_2)}{H_1^{(1)}(X_2)}\right] - \sin^2\theta\left(\frac{1}{X_1^2} - \frac{1}{X_2^2}\right)^2$$

$$P_2 = \left[\frac{\bar{\epsilon}_1}{X_1}\,\frac{J_1'(X_1)}{J_1(X_1)} - \frac{1}{X_2}\,\frac{H_1^{(1)\prime}(X_2)}{H_1^{(1)}(X_2)}\right]\left[\frac{1}{X_1}\,\frac{J_1'(X_1)}{J_1(X_1)} - \frac{1}{X_2}\,\frac{H_1^{(1)\prime}(X_2)}{H_1^{(1)}(X_2)}\right] - \sin^2\theta\,\frac{1}{X_1^2} - \frac{1}{X_2^2}\Bigg]^2$$

$$P_3 = \frac{4j}{\pi}\,\frac{X_2^2 - X_1^2}{X_1^4 \times X_2^4}$$

$$P_1 = \left[\frac{1}{X_1}\frac{J_1'(X_1)}{J_1(X_1)} - \frac{1}{X_2}\frac{H_1^{(1)\prime}(X_2)}{H_1^{(1)}(X_2)}\right]\left[\frac{\bar{\epsilon}_1}{X_1}\frac{J_1'(X_1)}{J_1(X_1)} - \frac{1}{X_2}\frac{H_1^{(2)\prime}(X_2)}{H_1^{(2)}(X_2)}\right]$$
$$- \sin^2\theta\left(\frac{1}{X_1^2} - \frac{1}{X_2^2}\right)^2.$$

11. Power flow

The field components which contribute the power flow in the radial direction are E_ϕ^R, H_ϕ^R, E_θ^R, H_θ^R. To find out the amount of power radiated outward, the radial component of Poynting's vector will be integrated over the sphere at infinity The radial component of Poynting's vector is

$$s_r = \tfrac{1}{2}\,\mathrm{Re}\,[E^R \times H^{R*}]_r = \tfrac{1}{2}\,\mathrm{Re}\,[E_\phi H_\theta^* - E_\theta H_\phi^*]$$
$$= \tfrac{1}{2}\left[\sqrt{\frac{\epsilon_0}{\mu_0}}\,|E_\phi|^2 + \sqrt{\frac{\mu_0}{\epsilon_0}}\,|H_\phi|^2\right]$$
$$= \frac{\sqrt{\epsilon_0/\mu_0}}{32r^2}\,[|g_1(\theta)|^2\cos^2\phi + |g_2(\theta)|^2\sin^2\phi]. \tag{52}$$

Physically, the magnetic ring source is realized by properly illuminating an annular slot in a conducting plane located at $z = 0$ plane, therefore, only the fields in the half space ($z > 0$) need be considered Integrating s over the half sphere in front of the waveguide ($0 < \theta < \pi/2$) the power flow in the radial direction becomes

$$P_r = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi/2} s_r\, r^2 \cos\theta\, d\theta\, d\phi$$
$$= \frac{\pi\sqrt{\epsilon_0/\mu_0}}{32}\int_0^{\pi/2}\cos\theta\, g(\theta)\, d\theta \tag{53}$$

where $g(\theta) = |g_1(\theta)|^2 + |g_2(\theta)|^2$ is given by eqns (51 *a*, *b*)

It should be noted that the function $g(\theta)$ is independent of the radial propagation constants and the guidewavelength They depend only on the b/a ratio and the angular variation, θ. Since the higher order modes are characterized by their propagation constants and therefore their amplitudes[14], the individual contribution of each mode cannot be determined However the term b/a defines the combined effects of the higher order modes and hence it can be concluded that the radiated power as obtained by eqn (53) for a particular rod diameter d and for a particular b/a ratio is due to all the modes supported by the rod since the radiated power is a continuous mode spectrum

12. Evaluation of the surface wave field (Pole residue)

The surface wave component of the total field as shown by eqn (30) is given by

$$H^{s}_{z3} = 2\pi j \text{ (sum of the residues of the integrand)}$$
$$(c_m H_1^{(1)}(k_{2m}\rho)\, e^{j\gamma z}) \tag{54}$$

The integrand can be written in the form of the ratio $f(\gamma)/g(\gamma)$ where

$$g(\gamma) = \left[\frac{1}{k_{1m}a}\frac{J_1'(k_{1m}a)}{J_1(k_{1m}a)} - \frac{1}{k_{2m}a}\frac{H_1^{(1)\prime}(k_{2m}a)}{H_1^{(1)}(k_{2m}a)}\right].$$
$$\left[\frac{\bar{\epsilon}_1}{k_{1m}a}\frac{J_1'(k_{1m}a)}{J_1(k_{1m}a)} - \frac{1}{k_{2m}a}\frac{H_1^{(1)\prime}(k_{2m}a)}{H_1^{(1)}(k_{2m}a)}\right] - \left(\frac{\bar{\epsilon}_1}{(k_{1m}a)^2} - \frac{1}{(k_{2m}a)^2}\right)$$
$$\times\left(\frac{1}{(k_{1m}a)^2} - \frac{1}{(k_{2m}a)^2}\right) \tag{55 a}$$

and

$$f(\gamma) = \left[\frac{\bar{\epsilon}_1}{k_{1m}a} - \frac{1}{k_{2m}a}\right]\left[\frac{1}{(k_{1m}a)^2} - \frac{1}{(k_{2m}a)^2}\right]$$
$$= \frac{\gamma^2}{\omega^2\mu_0\epsilon_0}\left[\frac{1}{(k_{1m}a)^2} - \frac{1}{(k_{2m}a)^2}\right]^2. \tag{55 b}$$

Below a critical radius to wavelength ratio, the dielectric rod will support only the HE_{11} mode. It will be assumed that the only pole within the contour of integration corresponds to this mode This pole is γ_p

Poles of the arc determined by solutions to $g(\gamma) = 0$, which is identical to the characteristic equation of the $HE_{.m}$ mode in the source free case As the diameter is increased higher order modes appear and the corresponding poles can be evaluated by the same expression $g(\gamma) = 0$ In the following analysis the surface wave field due to the lowest mode, HE_{11}, will then be calculated Assumption of the surface wave power of all the modes will then give the total surface wave field of the overmoded guide

If $g(\gamma) = 0$ gives the poles, the value of the residue at $\gamma = \gamma_p$ is $\dfrac{f(\gamma_p)}{g(\gamma_p)}$, since it can be shown that γ_p is a first order pole, that is $g'(\gamma_p) = 0$.

With this assumption, defining $(H_{z3})^s_m$ and the surface wave field of the m^{th} mode in Med. III, we have

$$(H_{z3})^s_m = 2\pi j\frac{f(\gamma_p)}{g(\gamma_p)}\cos$$

$$= -j\frac{4f_4}{\pi}\frac{F_1}{F_2}\left[F_3 H_1^{(1)}\left(\frac{b}{a}x_{2m}\right) + \left(\frac{b}{a}x_{2m}\right)\right.$$
$$\left. H_0^{(1)}\left(\frac{b}{a}x_{2m}\right)\right] H_1^{(1)}(k_{2m}\rho)\cos\phi\, e^{j\gamma_p z} \quad (56)$$

where, $x_{1m} = k_{1m}a, \quad x_{2m} = k_{2m}a$

$$F_1 = \sqrt{\frac{\epsilon_0}{\mu_0}}\frac{\pi}{4}\frac{1}{x_{1m}^4 [H_1^{(1)}(x_{2m})]^2}$$

$$F_2 = f_3 f_5(\bar{\epsilon}_1 + f_1^2) + f_3 f_6(1 + f_4)^2 + \frac{2f_4}{x_{1m}^2}(\bar{\epsilon}_1 + f_3^2) - \frac{2f_1}{x_{2m}^2}(1 + f_4^2)$$

$$F_3 = \frac{1}{x_{1m}^2 - x_{2m}^2}\left[x_{1m}x_{2m}^2\bar{\epsilon}_1\frac{J_1'(x_{1m})}{J_1(x_{1m})} - \frac{1}{x_{2m}}\frac{H_1^{(1)\prime}(x_{2m})}{H_1^{(1)}(x_{2m})} - 1\right]$$

$$f_1 = \frac{1}{x_{1m}}\frac{J_1'(x_{1m})}{J_1(x_{1m})}$$

$$f_2 = \frac{1}{x_{2m}}\frac{H_1^{(1)\prime}(x_{2m})}{H_1^{(1)}(x_{2m})}$$

$$f_3 = \left(\frac{x_{1m}^2 - \bar{\epsilon}_1 x_{2m}^2}{x_{1m}^2 x_{2m}^2}\right)^{1/2}$$

$$f_4 = \frac{\epsilon_1 f_1 - f_2}{f_1 - f_2}$$

$$f_5 = f_1^2 + \frac{2f_1 + 1}{x_{1m}^2} - \frac{1}{x_{1m}^4}$$

$$f_6 = \frac{1}{x_{2m}^4} - f_2^2 + \frac{2f_2 + 1}{x_{2m}^2}. \quad (57)$$

$(H_{z3})_m^s$ can be calculated for each of the higher order modes

13. Surface wave power flow

The total power flow in the surface wave (P_z) can be obtained by making use of the source free analysis of the HE_{11} mode propagating on a circular dielectric rod. At large axial distances from the plane of the ring source there is no distinction between regions II and III. Power flow for the source free case

$$P_{zm} = \frac{1}{2}\left(\frac{\pi}{2}\right)\left(\frac{2\pi a}{\lambda_0}\right)^2\sqrt{\frac{\mu_0}{\epsilon_0}}[H_1^{(1)}(x_{2m})]^2\frac{|a\,c_m|^2}{f_4^2}F_2 \quad (58)$$

where D_m is an arbitrary constant contained in the expression for H_z outside the rod

$$H_z = c_m H_1^{(1)}(k_{2m}\rho)\cos\phi\, e^{j\gamma_p z}, \quad (59)$$

Comparing eqn (59) with that obtained for $(H_{z3})^s_m$ eqn (56),

$$c_m = -j\,\frac{4f_1}{\pi}\,\frac{F_1}{F_2}\left[F_3\,H_1^{(1)}\left(\frac{b}{2}\,x_{2m}\right)+\left(\frac{b}{a}\,x_{2m}\right)\;H_0^{(1)}\left(\frac{b}{a}\,x_{2m}\right)\right]$$

Substituting in eqn (54),

$$P_{zm} = \frac{F_1}{F_2}\left|\,F_3H_1^{(1)}\left(\frac{b}{a}\,x_{2m}\right)+\left(\frac{b}{a}\,x_{2m}\right)H_0^{(1)}\left(\frac{b}{a}\,x_{2m}\right)\right|^2$$

The total surface wave field due to all the higher order modes is

$P_z = \sum_m P_{zm}$, since the modes are orthogonal, the power is additive.

14. Launching efficiency

The launching efficiency is defined by

$$\eta = \frac{p_z}{p_z + p_r}\,.$$

15. Numerical evaluation

The launching efficiency η, the total surface wave power P_z, and the total radiated power have been calculated for different degrees of overmoding Fig. 4 (*a–f*) shows the variation of radiation field with θ for different diameters and different values of b/a. Figs *c–e* are the overmoded guides supporting higher order modes as indicated Fig 5 shows the variation of the total surface wave power with b/a both for single moded (—) and overmoded rods (— —). The variation of launching efficiency with b/a is given in Fig. 6

16. Discussion

The following observations can be made from Figs 4–6.

(i) Since the radiation field [eqn (53), Fig 4] is independent of the propagation constants k_{1m} and k_{2m} and the mode amplitudes, it is independent of the higher order modes.

(ii) The surface wave field however depends on the higher order modes (Fig 5) which appear as the rod diameter $(2\,a)$ is increased Decrease of radial field spread with increase in $(2\,a)$ result in the wave being more tightly bound to the rod

(iii) For single moded rods (Fig. 6) η decreases with increase in $(2\,a)$ whereas η increases with $(2\,a)$ for overmoded rods

(iv) Similar observations as the characteristic of η have been observed regarding the gain for overmoded rods.

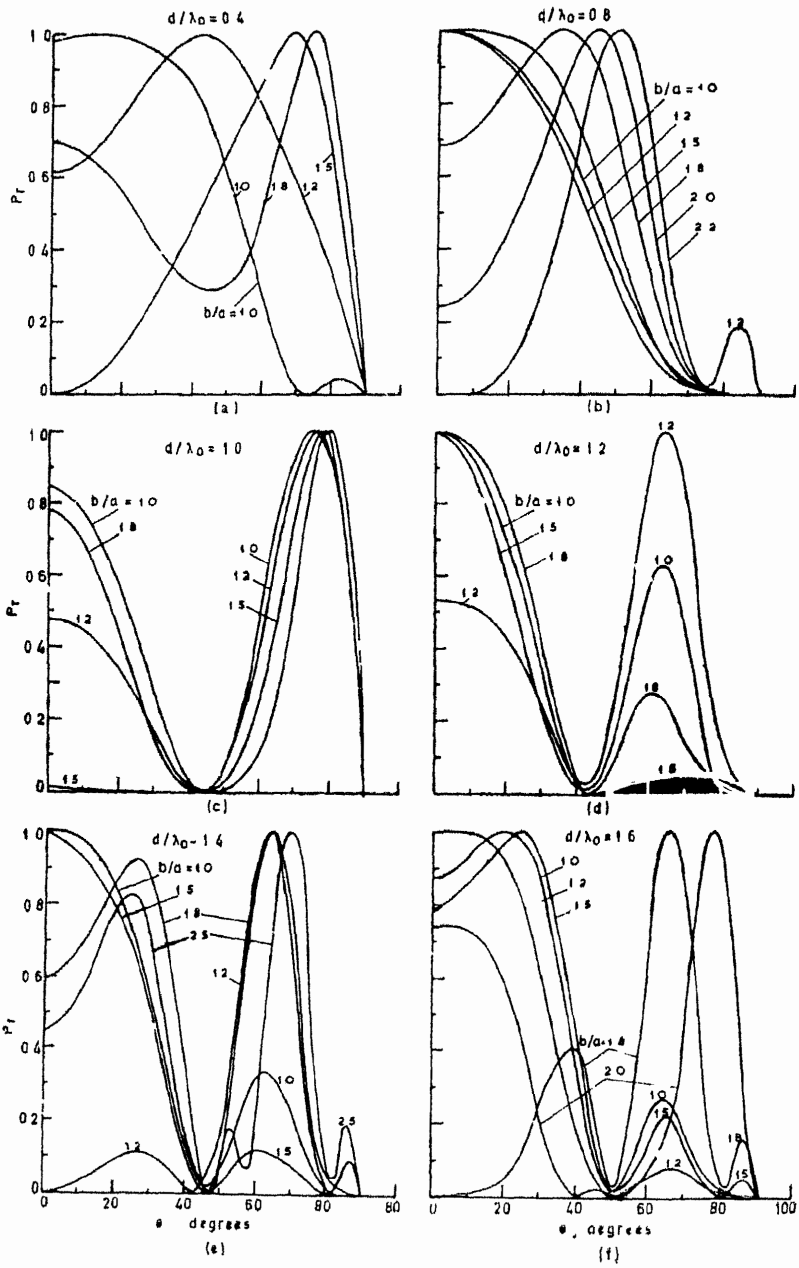


FIG. 4. Radiation field *vs.* θ

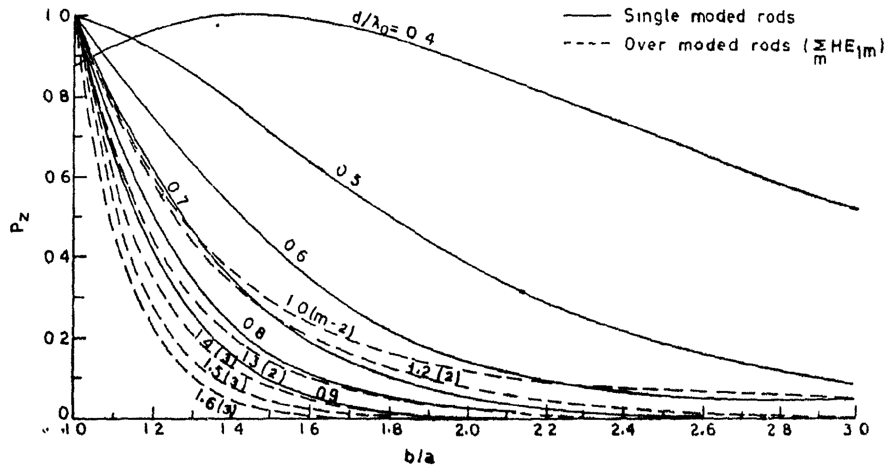


FIG 5 Surface wave power P_z vs. b/a

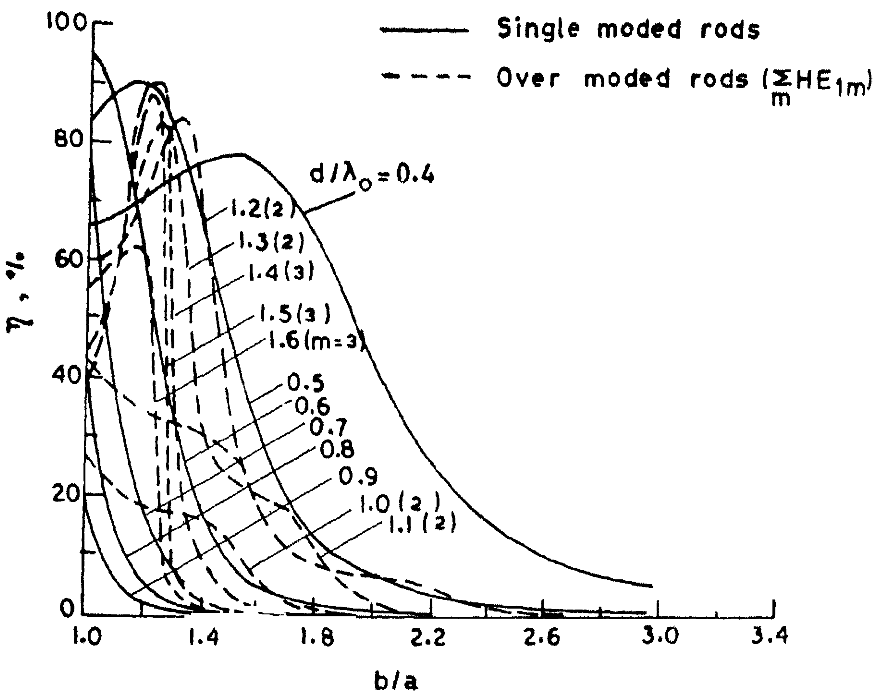


FIG 6. Launching efficiency η vs. b/a.

17. Acknowledgements

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