

Imbibition in the flow of two immiscible liquids through porous media due to differential wettability

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Abstract

A study of the phenomenon of imbibition which arises in the flow of two immiscible liquids through porous media due to the difference of the wetting abilities of the liquids has been made in this paper. Two special cases have been discussed in detail.

Key words: Imbibition, porous medium, wettability, perturbation, similarity, saturation.

1. Introduction

When a porous medium filled with some fluid is brought in contact with other fluid, which preferentially wets the solid, the wetting fluid will flow spontaneously along the solid walls of the pores into the medium and some of the residual fluid will be expelled. This well-known phenomena is known as imbibition. In every day life one can see blotter soaking up ink and expelling air from its interstices and dry bricks on houses soaking up rain water and expelling air.

This phenomenon of imbibition in porous media has attracted many research workers recently and has been discussed by many authors. Brownscombe and Dyes¹, Enright², Graham and Richardson³, Rijik⁴, Scheidegger¹⁰ made both theoretical and experimental studies to develop an understanding of the imbibition process. Verma^{11,12}, Mishra and Verma³, Mehta and Verma⁹ have obtained some perturbation solutions of this problem.

Though no systematic analysis has been presented so far, the imbibition tests which consist of immersing a fluid-filled porous medium into another fluid are reported.

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Since these are very simple, it is interesting to examine theoretically the effects of difference of the wettabilities.

In this paper we have obtained the analytical solution of the flow of two immiscible fluids through porous media by using Birkhof's similarity technique and perturbation method (Nayfeh⁶).

In the first case the saturation distribution of the displacing (wetting) liquid in terms of the saturations at the ends of the porous matrix has been obtained, which is the composite solution describing the displacing phase saturation by applying the matching principle of inner and outer expansions.

In the second case the saturation distribution is obtained in terms of ξ and θ by applying similarity solution (Hansen⁴). The results obtained here are in perfect agreement with the physical situation. This can be realised by conducting an experiment with the help of a capillary filled with oil. If water is poured into the capillary it will always find ways in the neighbourhood of the walls. This can be verified from the expression obtained for saturation. It is of great significance in oil recovery, where it can be responsible to increase oil production up to 35% in some cases.

We consider here a cylindrical piece (linear co-ordinate x) of homogeneous porous matrix of length L , which is completely filled with an original liquid (n) surrounded by an impermeable surface except for one end of the cylinder which is designated as the imbibition face and this end is exposed to an adjacent formation of the injected liquid (i). It is assumed that the liquid (i) is wetting phase and the liquid (n) is a native liquid and so this arrangement gives rise to the phenomenon of imbibition.

2. Formulation of the problem and its solution

Assuming that both the flow of wetting liquid (i) and the counter flow of original liquid (n) are governed by Darcy's law, we may write the basic flow equations for imbibition phenomenon as

$$v_i = -\frac{K_i}{\delta_i} k \frac{\partial p_i}{\partial x} \quad (2.1)$$

$$v_n = -\frac{k_n}{\delta_n} k \frac{\partial p_n}{\partial x} \quad (2.2)$$

$$v_i = -v_n \quad (2.3)$$

$$p_* = p_n - p_i = f(s_i) \text{ say} \quad (2.4)$$

and

$$\rho \frac{\partial s_i}{\partial t} + \bar{c} \frac{\partial v_i}{\partial x} = 0 \quad (2.5)$$

where v_i and v_n are the velocities, k_i and k_n the relative permeabilities, δ_i and δ_n the viscosities, p_i and p_n the pressures of the wetting and non-wetting liquids respectively. ρ and K are the porosity and permeability of the homogeneous medium. s_i is the saturation of wetting liquid, p_c is capillary pressure and t is the time.

2.1. Case I

Combining equations (2.1)-(2.5) and using relation for capillary pressure as

$$p_c = \beta (s_i^{-1} - c)$$

(Jones⁵, Mishra and Verma⁷) together with

$$D(s_i) = \frac{k_i k_n}{\delta_n k_i + \delta_i k_n} k = \bar{D} s_i^2, \quad (2.6)$$

we get

$$\rho \frac{\partial s_i}{\partial t} - 2 \bar{D} \beta K \frac{\partial}{\partial x} \left(\frac{\partial s_i}{\partial x} \right) = 0 \quad (2.7)$$

where β is very small capillary coefficient, C and \bar{D} are constants.

A set of boundary conditions can be assumed as

$$s_i(0, t) = s_{i0}, \quad s_i(L, t) = s_{i1} \quad (2.8)$$

where s_{i0} and s_{i1} are the saturation at the imbibition face and at the end $x = L$ respectively.

Using the transformation

$$\xi = \frac{x}{L}, \quad \theta = \frac{2K\bar{D}}{\rho L^2} t. \quad (2.9)$$

Equations (2.7) and (2.8) are transferred as

$$\frac{\partial s_i}{\partial \theta} - \beta \frac{\partial^2 s_i}{\partial \xi^2} = 0 \quad (2.10)$$

and

$$s_i(0, \theta) = s_{i0} \quad \text{and} \quad s_i(1, \theta) = s_{i1} \quad (2.11)$$

(2.10) is the required equation to be solved under boundary conditions given by (2.11).

2.2. Solution of case I

To solve equation (2.10) we use Birkhoff's technique of one parameter group transformation. Let a group T_1 consisting of a set of transformations be defined as

$$T_1 : \bar{\xi} = a^\theta \xi, \quad \bar{\theta} = a^\theta \theta, \quad \bar{s}_i = a^\theta s_i \quad (2.12)$$

where the parameter $a \neq 0$, and p, r, s are real numbers to be determined.

Substituting these values in equation (2.10) and applying condition of absolute conformal invariant under T_1 , the invariants of the group T_1 are given by

$$\eta = \frac{\xi}{\sqrt{D\theta}}, \quad F(\eta) = \frac{s_1(\xi, \theta)}{\theta^A} \quad (2.13)$$

where A is an arbitrary constant. Putting these values in (2.10) and substituting

$$F(\eta) = u(z), \quad 2z = -a\eta^2, \quad \left(a = \frac{1}{4D}\right) \quad (2.14)$$

we obtain

$$\beta u''(z) + a(z)u'(z) + b(z)u(z) = 0 \quad (2.15)$$

(Mehta and Verma⁶)

where

$$u(0) = s_0, \quad u\left(-\frac{a}{2\theta}\right) = s_1$$

and

$$a(z) = \frac{1}{z}\left(\frac{\beta}{2} - z\right), \quad b(z) = -\frac{A}{z}$$

By taking $\beta = 0$ and using second boundary condition of (2.15) we obtain the outer expansion (valid near $x = L$) as

$$u^{\circ} = s_1 \exp\left[-\int_{-a/2\theta}^z \frac{b(v)}{a(v)} dv\right] \quad (2.16)$$

The first boundary condition of (2.15) has been dropped because $a(z) > 0$ in $(0, -a/2\theta)$ (Nayfeh⁸).

Now using the stretching transformation $y = z/\beta$ and the first boundary condition of (2.15) we obtain the inner expansion, valid near origin (Nayfeh⁸) as

$$u^{\circ} = s_0 + B e^{e^{(1)}/\beta} \quad (2.17)$$

Now applying the matching principle of u° and u° and using (2.13) and (2.14) (Nayfeh⁸) and Mehta and Verma⁶ we finally get

$$\begin{aligned} s_1(\xi, \theta) &= \theta^A \left(\frac{\beta/2 + 1/8D\theta}{\beta/2 + \xi/8D\theta}\right)^{-A} + \theta^A \left\{s_0 - s_1 \left(\frac{\beta/2 + 1/8D\theta}{\beta/2}\right)\right\} \\ &\quad \times \exp\left(\frac{1}{2} - \frac{\xi}{8D\theta}\right) + O(\beta) \end{aligned} \quad (2.18)$$

which gives the required saturation distribution of the displacing liquid in terms of the saturation at the two ends of the porous matrix.

2.3. Case II

Combining equations (2.1)–(2.5) and using relation for capillary pressure as

$$p_c = \beta s_i^2$$

together with

$$D(s_i) = \frac{K_n K_i}{\delta_n K_i + \delta_i K_n}$$

we get

$$\rho \frac{\partial s_i}{\partial t} + 2\beta \frac{\partial}{\partial x} \left\{ D(s_i) s_i K \frac{\partial s_i}{\partial x} \right\} = 0 \quad (2.19)$$

where β is small capillary pressure coefficient. Assuming $\bar{D}(s_i)$ is an average value of $D(s_i)$ (Mehta and Verma⁶) and using the transformation

$$\xi = \frac{x}{L}, \quad \theta = 2\beta \bar{D}(s_i) K t / \rho L^2 \quad (2.20)$$

the equation (2.19) can be put in a dimensionless form as

$$\frac{\partial s_i}{\partial \theta} + \left(\frac{\partial s_i}{\partial \xi} \right)^2 + s_i \frac{\partial^2 s_i}{\partial \xi^2} = 0. \quad (2.21)$$

2.4. Solution of case II

To solve equation (2.21) we again apply Birkhoff's technique of one parameter group transformation. Let a group T_1 consisting of a set of transformations be defined by

$$T_1 : \bar{\xi} = a^q \xi, \quad \bar{\theta} = a^r \theta, \quad \bar{s}_i = a^s s_i \quad (2.22)$$

where the parameter $a \neq 0$ and q, r, s are real numbers to be determined.

Plugging these values in equation (2.21) and applying condition of absolute conformal invariants under T_1 the invariants of the group T_1 are given by

$$\eta = \frac{\xi}{\theta^{B/A}}, \quad F(\eta) = \frac{s_i(\xi, \theta)}{\theta^A} \quad (2.23)$$

where $B - A = 1$ and A is an arbitrary chosen constant. The equation (2.23) in terms of new variable η and $F(\eta)$ now becomes

$$A F(\eta) - \frac{B}{2} \eta F'(\eta) + F'^2(\eta) + F(\eta) F''(\eta) = 0 \quad (2.24)$$

$$\theta^{A-1} \neq 0$$

which is non-linear ordinary differential equation of second order.

By considering the substitution

$$F(\eta) = \eta^2 u(z), \quad z = \log \eta$$

and $u'(z) = p$, the equation (2.24) can be reduced to the form

$$u p p'(u) + p^2 + \left(7u - \frac{B}{2}\right)p + (6u + 1) = 0 \quad (2.25)$$

which is Abel's equation of second kind from where we can easily visualize the solution of equation (2.24) to be

$$F(\eta) = \frac{\eta^2}{6}$$

for every value of A and B . Thus s_1 , the saturation of flowing (wetting) fluid is given by

$$s_1 = \frac{\xi^2}{6\theta}. \quad (2.26)$$

Equation (2.26) gives the formal solution of the equation (2.21) which is the required saturation distribution of the setting (displacing) liquid in terms of ξ and θ .

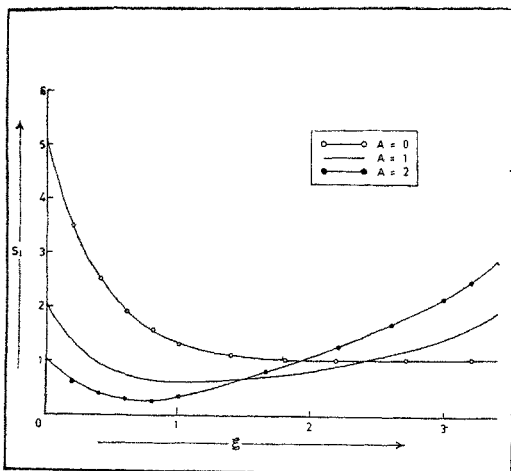


FIG. 1. Saturation distribution when $\theta = 0.5$ and $p_g = \beta (s_1^{-1} - c)$.

3. Conclusions

3.1. Case I

The graphs of saturation distribution have been drawn for various values of parameter A . In Fig. 1 the saturation distribution for $\theta = 0.5$ is obtained. We observe that for small values of ξ saturation decreases for every A , for greater values of ξ saturation increases when $A = 1$ and $A = 2$, while it approaches to unity when $A = 0$. It is evident from the graph that saturation is same at $\xi = 2.5$, $\xi = 1.9$ and $\xi = 1.4$ in the cases when $A = 0$, $A = 1$; $A = 0$, $A = 2$ and $A = 1$, $A = 2$ respectively. In Fig. 2,

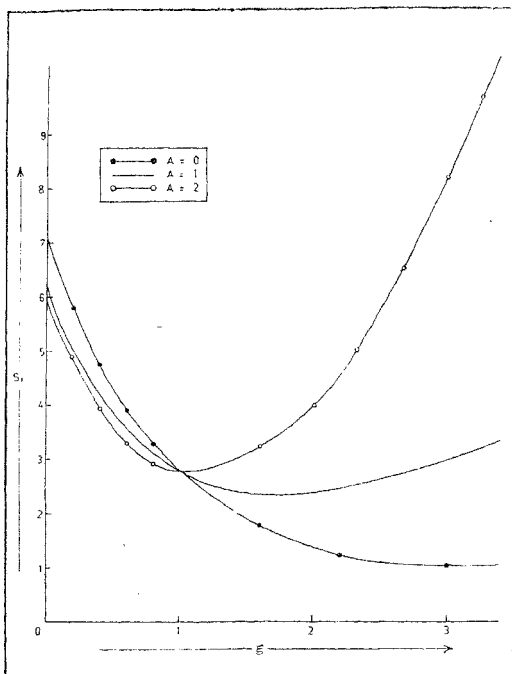


Fig. 2. Saturation distribution when $Q = 1$ and $p_c = \beta (s_1^{-1} - c)$.

when $\theta = 1$, the saturation is the same for every case at $\xi = 1$, but increases very rapidly when $A = 2$ in comparison to that when $A = 1$. For $A = 0$ there is a continuous decrease reaching to 1 ultimately. In all the three cases it decreases rapidly for all values of ξ less than 1. For $\theta = 0.5$ and $\theta = 1$ the fall in the saturation is ultimately the same when $A = 0$, but for $A = 1$ saturation first decreases and then it increases slowly and the curve is almost linear for greater values of ξ . When $A = 2$ and $\theta = 1$, the increase in saturation beyond $\xi = 1$ is extremely sharp.

3.2. Case II

The graphs for $\theta = 0.5$ and $\theta = 1$ against various values of ξ are drawn in Fig. 3. We observe that the saturation increases as ξ increases and the increase is significant beyond $\xi = 1$. It is also worth mentioning that the concavity upwards increases with decrease in θ .

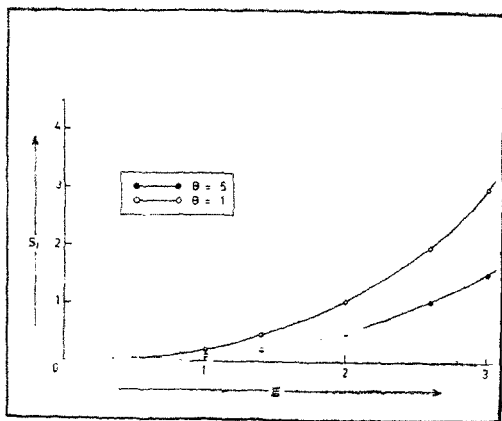


FIG. 3. Saturation distribution when $\theta = 0.5$, $\theta = 1$ and $\rho_r = \beta s_r^2$.

In Figs. 4 and 5 the graphs for saturation distribution have been drawn assuming capillary pressure proportion as s_r (Mehta and Verma⁵).

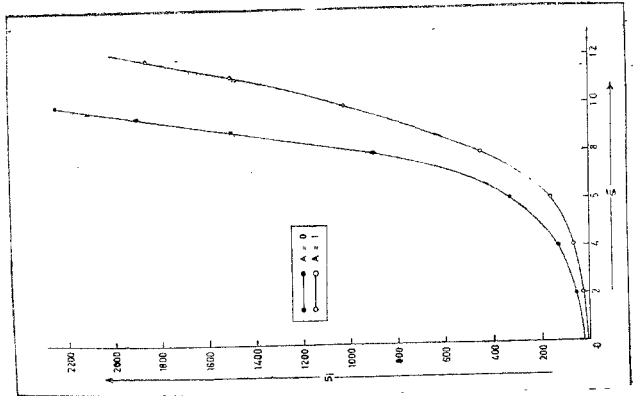


FIG. 4. Saturation distribution of wetting fluid when $\theta = 0.5$, $\bar{D} = 0.1$, $\beta = 0.2$ and $p_e = \beta \xi$.

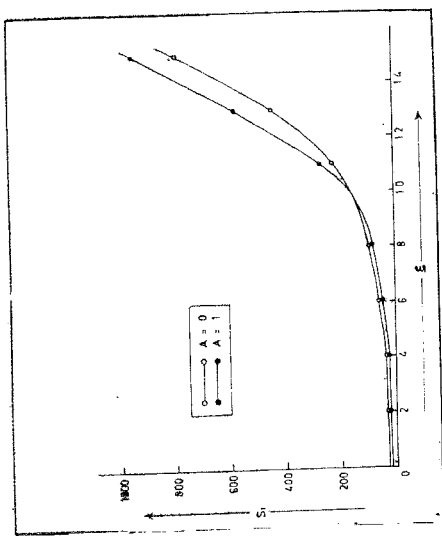


FIG. 5. Saturation distribution of wetting fluid when $\theta = 1$, $\bar{D} = 0.1$, $\beta = 0.2$ and $p_e = \beta \xi$.

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