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A note on the first order chromatic aberrations of a hyperchromatic lens

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Abstract

The expressions for the two first order chromatic aberration coefficients of a hyperchromatic lens are presented and discussed with a view to remove certain inconsistencies which appeared in a paper published by Singh[1]. A numerical example is included to illustrate the significance of hyperchromatism of these special lenses. A list of pairs of glasses for use in the design and construction of a hyperchromatic lens or plate is also given for ready reference.

Key words : Chromatic aberration, cemented doublet lenses, thick cemented glass plates.

1. Introduction

A hyperchromatic lens is a cemented doublet consisting of positive and negative lenses made of glasses of equal refractive index and different dispersions and having equal and opposite powers and shapes. The plano-convex and plano-concave lens combination cemented at their common radius of curvature has the appearance of a plate and exhibits all the properties of a plate excepting the large contribution to the two chromatic aberrations.

Andreyev[2] had derived differential expressions for the two first order chromatic aberrations of a hyperchromatic lens. He had briefly explained the construction, properties and applications of a hyperchromat. Singh[1] had also described the hyperchromat in detail and presented equations for the two chromatic defects, expressing them in a more popular notation. These equations, however, are not correctly written and therefore, lead to wrong results and erroneous conclusions.

In the present paper an attempt is made to present correct expressions for the two chromatic coefficients. The numerical values of these coefficients calculated from the equations are finally checked against the values obtained from an actual ray trace for a simple hyperchromatic lens, so as to ensure that all terms are taken into account while calculating using the concerned equations. Similar equations for a thick hyperchromatic plate are also presented.

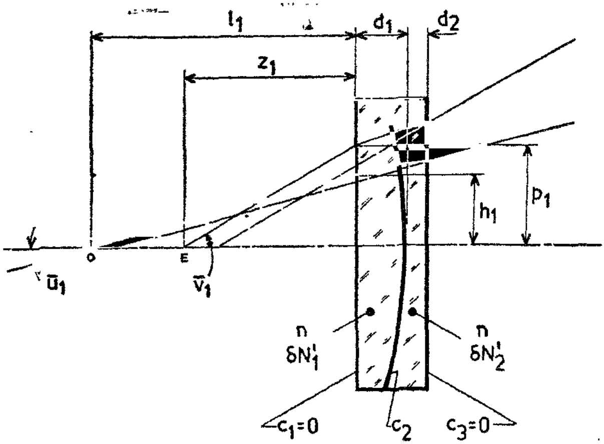

FIG. 1. Hyperchromatic doublet (System parameters and initial ray data).

In what follows, a cemented doublet consisting of two thick lenses with plano-convex and plano-concave shapes is considered. The related optical parameters and the concerned ray paths are shown in Fig. 1. The notation and symbols used in this paper are explained in the nomenclature given at the end. It is assumed that the ray trace data of the two paraxial rays, namely, the paraxial aperture ray starting at the axial object point 0 and the paraxial principal ray starting at the stop located at E are available from the traces of these rays through the doublet lens consisting of three surfaces separated by glass thicknesses d_1 and d_2. If L and T are the coefficients of the first order longitudinal chromatic aberration and transverse chromatic aberration respectively, then

$$\left.\begin{aligned} L &= \sum_{j=1}^{3} L_j \\ T &= \sum_{j=1}^{3} T_j \end{aligned}\right] \quad (1)$$

where

$$\left.\begin{aligned} L_j &= h_j\,(Ni)_j \left(\triangle \frac{\delta N}{N}\right)_j \\ T_j &= h_j\,(N\bar{i})_j \left(\triangle \frac{\delta N}{N}\right)_j \end{aligned}\right] \quad (2)$$

and $j = 1, 2, 3$ refer to the surfaces. Equation (2) can be simplified using the ray trace data and the well known relations between these data and the surface parameters at each surface. The expressions for L and T may now be written in terms of the initial data of the two paraxial rays and the system parameters as

$$L = c_2 (\delta N_2' - \delta N_1') \left(h_1 - \frac{d_1}{n} u_1 \right)^2 - \frac{u_1^2}{n^2} (d_1 \delta N_1' + d_2 \delta N_2') \tag{3}$$

$$T = c_2 (\delta N_2' - \delta N_1') \left[\frac{d_1^2 u_1 v_1}{n^2} + h_1 p_1 - \frac{d_1}{n} (p_1 u_1 + h_1 v_1) \right] - \frac{u_1 v_1}{n^2} (d_1 \delta N_1' + d_2 \delta N_2'). \tag{4}$$

Equations (3) and (4) reduce to the following when $d_1 = d_2 = 0$ is substituted

$$L_t = h_1^2 c_2 (\delta N_2' - \delta N_1') \tag{5}$$

$$T_t = h_1 p_1 c_2 (\delta N_2' - \delta N_1'). \tag{6}$$

These expressions for L_t and T_t are the coefficients for the longitudinal and transverse chromatic aberrations of *a thin* doublet hyperchromatic lens.

The following conclusions can be drawn from eqns. (5) and (6) :

$L_t = T_t = 0$. if

(1) $\delta N_2^1 = \delta N_1^1$ (Dispersions of the two glasses are equal)

(2) $h_1 = 0$ (The obejct is located at the lens)

(3) $c_2 = 0$ (The thin lens reduces to a thin plate consisting of two component plates).

In addition, $T_t = 0$, if $p_1 = 0$ indicating that the stop is at the lens.

It may be noted that Singh[1] expects finite values for L_t and T_t when $c_2 = 0$, which is contrary to the conclusion drawn above.

Alternatively, the same conclusions as given above can also be arrived at by a more direct approach adopting thin lens aproximation. The following equations hold for a thin cemented doublet,

$$\left.\begin{aligned} K_1 + K_2 &= K \\ h_1^2 \left(\frac{K_1}{V_1} + \frac{K_2}{V_2} \right) &= L_t \\ p_1 h_1 \left(\frac{K_1}{V_1} + \frac{K_2}{V_2} \right) &= T_t \end{aligned}\right\} \tag{7}$$

where V_1 and V_2 are Abbe Numbers.

Noting that $K_1 = -K_2$, $n_1 = n_2 = n$ for the two component lenses of the hyperchromatic doublet by assumption, and writing

$$V_1 = \frac{n-1}{\delta N_1'} \; ; \quad V_2 = \frac{n-1}{\delta N_2'} \quad \text{and} \quad K_1 = (n-1)(-c_2)$$

for plano-convex and plano-concave shapes of the lenses, eqns. (7) can be reduced to eqns. (5) and (6).

It is useful to give here for comparison purposes, the chromatic aberration coefficients of a hyperchromatic plate consisting of two thick plates cemented together. It may be noted that a thick doublet hyperchromatic lens reduces to the form of a thick plate when the cemented curved surface is flattened so that $c_2 = 0$. When this value of $c_2 = 0$ is substituted in eqns. (3) and (4) one gets,

$$\left.\begin{aligned} L_{pl} &= -\frac{u_1^2}{n^2}(d_1\delta N_1' + d_2\delta N_2') \\ T_{pl} &= -\frac{u_1 v_1}{n^2}(d_1\delta N_1' + d_2\delta N_2') \end{aligned}\right] \tag{8}$$

While L_{pl} is a small negative value, the sign of T_{pl} depends on the sign of the product $u_1 v_1$. It is clear from the above that $L_{pl} = T_{pl} = 0$ if (i) $d_1 = d_2 = 0$ and (ii) if $u_1 = 0$ or $v_1 = 0$.

A list of a few pairs of glasses is given in Table I for ready reference for design calculations and construction of hyperchromatic doublet lenses or plates. Since suitable glass pairs are not available in the country, the glasses have been selected from Chance-Pilkington glass catalogue. The refractive indices of each of the first three pairs agree with each other up to the third decimal place and in the rest of the pairs the difference is 2 to 3 units in the third decimal place. It is expected that these differences may not substantially effect the calculations and conclusions made in the foregoing paragraphs.

2. Illustration

In the following an example is worked out to illustrate the variation of L and T with c_2 and therefore the significance of hyperchromatism in a doublet lens using eqns. (3) and (4). These values are checked for their validity by actual numerical ray traces.

A doublet lens consisting of positive and negative lenses of equal and opposite powers and having plano-convex and plano-concave shapes, is chosen with the system parametry and initial ray data as shown in Fig. 1. The glass pair comprising DBC ($n_d = 1 \cdot 613230$; $\delta N^1 = 0 \cdot 010350$) and DF ($n_d = 1 \cdot 613230$; $\delta N^1 = 0 \cdot 016610$) is selected from Table I. The two component lenses have the same refractive index but different dispersions. Accordingly, two configurations of the doublet with the same positive lens leading can be formed by changing only the dispersion of the positive lens. In Tables II and III are presented the values of L and T calculated for different values of c_2 using eqns. (3) and (4), for these two configurations of the hyperchromatic doublet.

Table I

Pair No.	Glass type	n_d	$V = \dfrac{n_d - 1}{n_F - n_c}$	$\delta N_{F(-C)}$
1	BF	1·62284	39·56	0·01574
	BBC	1·62253	60·32	0·01032
2	DF	1·61323	36·92	0·01661
	DBC	1·61342	59·27	0·01035
3	DF	1·61676	36·58	0·01686
	DBC	1·61700	53·98	0·01143
4	BF	1·65301	46·15	0·01405
	SBC	1·65100	58·60	0·01111
5	BF	1·72300	37·99	0·01903
	SBC	1·72000	50·31	0·01431

Table II

c_2	L	T
0·0	−0·000057	−0·000287
−0·5	−0·008320	−0·016175
−1·0	−0·016583	−0·032062
−1·5	−0·024846	−0·047950
−2·0	−0·033109	−0·063837

Table III

c_2	L	T
0·0	−0·000067	−0·000335
−0·5	0·008196	0·015553
−1·0	0·016459	0·031440
−1·5	0·024722	0·047328
−2·0	0·032985	0·063215

The following initial ray data and the system parameters are arbitrarily chosen for the calculations :

$$l_1 = -8\cdot0 \qquad h_1 = 1\cdot6$$

$$u_1 = -0\cdot2 \qquad p_1 = 3\cdot0$$

$$z_1 = -3\cdot0 \qquad d_1 = 0\cdot2$$

$$v_1 = -1\cdot0 \qquad d_2 = 0\cdot1$$

2.1. *First configuration*

$$n_1 = n_2 = n = 1\cdot613230$$

$$\delta N_1^1 = 0\cdot010350$$

$$\delta N_2^1 = 0\cdot016610$$

Equations (3) and (4) can now be written as

$$L = 0\cdot016526c_2 - 0\cdot000057$$

$$T = 0\cdot031775c_2 - 0\cdot000287$$

Table II shows the variation of L and T for different values of c_2.

2.2. *Second configuration*

$$n_1 = n_2 = n = 1\cdot613230$$

$$\delta N_1^1 = 0\cdot016610$$

$$\delta N_2^1 = 0\cdot010350,$$

Equations (3) and (4), now take the form

$$L = -0\cdot016526c_2 - 0\cdot000067$$

$$T = -0\cdot031775c_2 - 0\cdot000335.$$

Table III shows the variation of L and T for different values of c_2.

The variations of the values of L and T for both the configurations are linear with c_2, as shown in Fig. 2. One feature of the second configuration of the thick doublet hyperchromatic lens is that both L and T have zero values for some curvatures, c_2 lying between 0 and $-0\cdot5$ which can be calculated from the respective equations. The first configuration also exhibits such a nature but only for positive values of c_2, which make the first lens a negative and, therefore, are not relevant to the positive lens leading configurations considered in this paper.

The values of L and T for $c_2 = 0$ in Tables II and III are the two chromatic aberration coefficients for a thick plate belonging to the concerned configuration.

3. Conclusion

A wide range of values for L and T, both positive and negative, can be designed into a thick hyperchromatic doublet. This property makes this optical element a useful device for correcting or balancing the chromatic aberrations of any optical system.

4. Nomenclature

$h_{1,2,3}$	are the heights of the paraxial aperture ray at the three surfaces of the lens respectively.
$p_{1,2,3}$	are the heights of the paraxial principal ray at the three surfaces of the lens respectively.
l_1	is the distance of the object from the first surface of the lens.
z_1	is the distance of the stop (entrance papil) from the first surface of the lens.
n	is the refractive index of the glass for the d-line chosen as the mean wavelength.
$\delta N^1_{1,2}$	are the dispersions of the glasses used for the first and second lenses of the doublet, *i.e.*, $(n_F - n_c)$ for each lens.
d_1	is the thickness of the first (positive) lens of the doublet.
d_2	is the thickness of the second (negative) lens of the doublet.
c_2	is the curvature of the second (cemented) surface of the doublet.

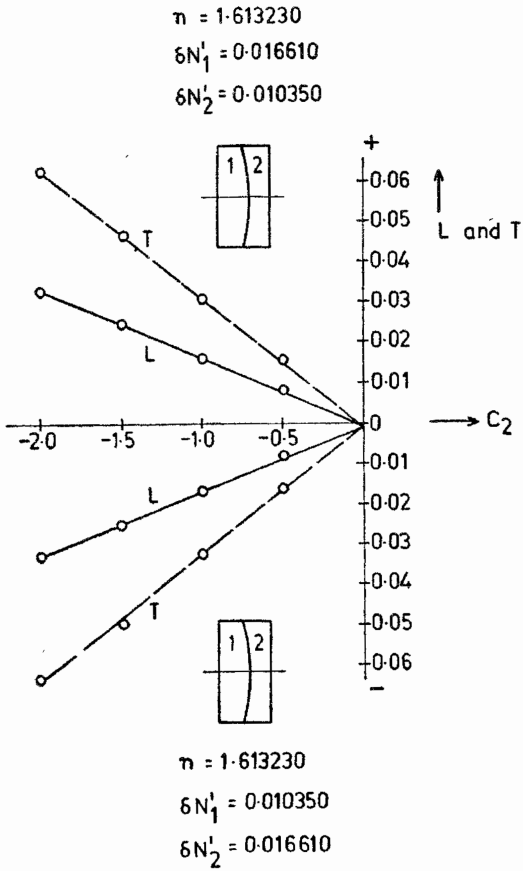

FIG. 2. Variation of L and T with C_2 for the two configurations.

u_1 is the angle which the paraxial aperture ray makes with the axis in the object space of the lens.

v_1 is the angle which the paraxial principal ray makes with the axis in the object space of the lens.

$N^1_{1,2,3}$ are the refractive indices of the media lying to the right of the three surfaces of the lens respectively.

$i_{1,2,3}$ are the angles of incidence of the paraxial aperture ray at the three surfaces of the lens respectively.

$\bar{i}_{1,2,3}$ are the angles of incidence of the paraxial principal ray at the three surfaces of the lens respectively.

$\left(\triangle \frac{\delta N}{N}\right)_{1,2,3}$ define the differences such as

$$\left(\frac{\delta N_1^1}{N_1^1} - \frac{\delta N_1}{N_1}\right), \quad \left(\frac{\delta N_2^1}{N_2^1} - \frac{\delta N_1^1}{N_1^1}\right), \quad \left(\frac{\delta N_3^1}{N_3^1} - \frac{\delta N_2^1}{N_2^1}\right)$$

at the three surfaces respectively. Note that $\delta N_1 = \delta N_3^1 = 0$ and $N_1 = N_3^1 = 1 \cdot 0$ for air.

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