J. Indian Inst. Sci. 62 (A), Mar 1980, Pp. 57–69

Effect of wave parameters on flood wave subsidence

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Received on March 5, 1980.

Abstract

A detailed analysis on the propagation of a sinusoidal flood wave in a wide prismatic open channel has been made by numerically integrating the governing nondimensional equations of unsteady flow in an open channel. Emphasis has been laid on the effect of wave parameters on the propagation of the sinusoidal wave. Results show that the amount of subsidence is more in the case of small wave amplitude and wave duration cases. Further, wave duration has been noticed to have a relatively higher influence on subsidence than wave amplitude. The speed at which the peak of the wave moves is observed to be a function of only the wave amplitude.

Key words : Floods, hydraulics, open channel flows, wave subsidence.

1. Introduction

A parametric study on the effect of different governing parameters on the propagation of a flood wave is scarce. Importance has been generally given only to the methods of solution of the flood wave problem[1]. The effects of bed slope, roughness and wave amplitude on flood wave subsidence was studied by Mozayeny and Song[2] for a specific case. They have studied the effect of wave amplitude and bed slope on the propagation of a sinusoidal flood wave in a long prismatic open channel of 0·3 m (1 ft) width in which the initial flow was uniform with a depth of 0·09 m (0·3 ft). The wave duration of the sinusoidal wave was 30 secs with wave amplitude varying from 0·06 cm (0·002 ft) to 0·6 cm (0·02 ft). While these results give an indication of the damping of a flood wave in prismatic channels, the studies are confined to a specific channel and specific initial flow which makes it impossible to interpret the results more generally. Further the wave amplitudes considered by them are very small relative to the initial uniform flow depth. By making a nondimensional parametric study, a generalised picture of damping of a flood wave may be obtained. A somewhat similar study has been reported by Chin-lien Yen[3] in a situation where the storage effect is the dominant factor. This paper presents a parametric study of subsidence where storage is not

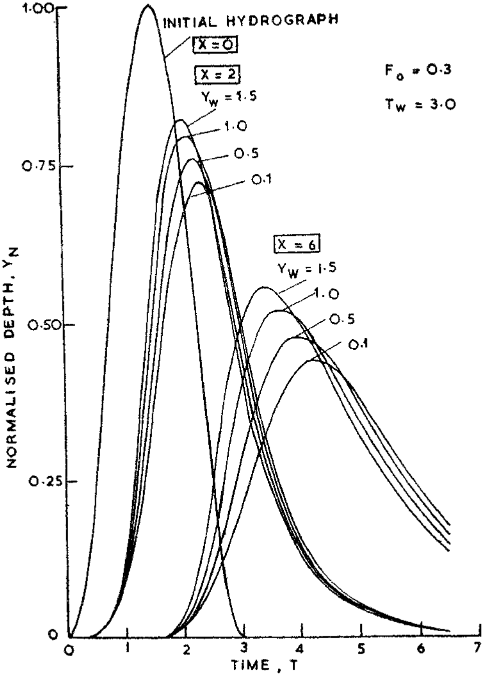

FIG. 1. Effect of wave amplitude on stage hydrograph.

the dominant factor as in a wide prismatic channel. Attention is focussed on the effect of wave parameters on subsidence. Effect of wave parameters, namely, wave amplitude and wave duration, on the propagation of a sinusoidal flood wave in a wide channel in which the initial flow is uniform has been studied.

2. Governing equations and formulation of the problem

The governing equations of unsteady flow in wide open channels are given by

$$v \frac{\partial y}{\partial x} + y \frac{\partial v}{\partial x} + \frac{\partial y}{\partial t} = 0 \tag{1}$$

$$v \frac{\partial v}{\partial x} + g \frac{\partial y}{\partial x} + \frac{\partial v}{\partial t} = g(S_0 - S_f) \tag{2}$$

where x is the distance along the channel; t is the time; y is the depth of flow; v is the velocity of flow; g is the acceleration due to gravity; S_0 is the bed slope and S_f is the friction slope.

The governing equations of the flow are nondimensionalised to enable a generalised parametric study. The nondimensional depth, velocity and time are defined as,

$$Y = \frac{y}{y_0}; \quad V = \frac{v}{v_0}$$

$$X = \frac{x}{l_0} = \frac{xS_0}{y_0}; \quad T = \frac{t}{t_0} = \frac{tv_0}{l_0} \tag{3}$$

where y_0 is the initial uniform flow depth and v_0 is the uniform velocity. Using eqns. (3) and Manning's formula, eqns. (1) and (2) become,

$$V\frac{\partial Y}{\partial X} + Y\frac{\partial V}{\partial X} + \frac{\partial Y}{\partial T} = 0 \tag{4}$$

$$\frac{\partial Y}{\partial X} + F_0^2\, V\frac{\partial V}{\partial X} + F_0^2\,\frac{\partial V}{\partial T} = 1 - V^2\, Y^{-4/3} \tag{5}$$

where

$$F_0 = \frac{v_0}{\sqrt{gy_0}} \tag{6}$$

is the initial uniform flow Froude number.

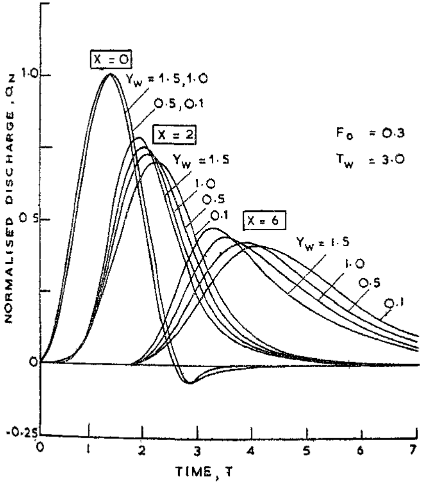

FIG. 2. Effect of wave amplitude on discharge hydrograph.

A sinusoidal wave is now introduced at the left boundary and takes the nondimensional form

$$Y = 1 + \frac{Y_w}{2}\left[1 - \cos\left(\frac{2\pi T}{T_w}\right)\right]\ ; \quad 0 \leqslant T \leqslant T_w$$

$$Y = 1\ ; \quad T > T_w \tag{7}$$

where

$$Y_w = \frac{y_w}{y_0}\ ; \quad T_w = \frac{t_w}{t_0} \tag{8}$$

are the nondimensional wave amplitude and wave duration respectively. A uniform flow boundary condition is imposed on the right boundary, that is, the channel is considered long.

The problem of subsidence of sinusoidal wave is governed by eqns. (4), (5) and (7). The parameters governing this problem are, initial uniform flow Froude number F_0, wave amplitude Y_w and wave duration T_w. In the present study, the effects of variations in wave amplitude and wave duration on subsidence is studied. The nondimensional wave amplitude Y_w is varied from 0·1 to 1·5 and the nondimensional wave duration is varied from 0·5 to 3·0. The third parameter of the problem, namely, initial uniform flow Froude number F_0 is chosen at 0·3.

The problem is solved numerically by using the direct explicit finite difference method on a staggered rectangular grid[4].

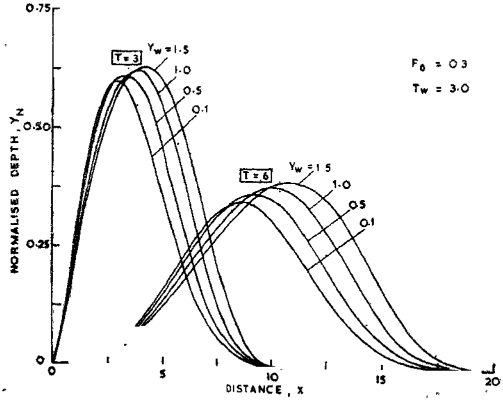

FIG. 3. Effect of wave amplitude on wave front.

3. Effect of wave amplitude

3.1. *Modification of hydrographs*

3.1.1. *Stage hydrograph*

Superimposed normalised hydrographs for different wave amplitudes are presented in Fig. 1. for the case with $F_0 = 0{\cdot}3$ and $T_w = 3{\cdot}0$. The nondimensional normalised depth Y_N is defined as

$$Y_N = \frac{Y - 1}{Y_W} . \tag{9}$$

This normalisation reduces the hydrograph at $X = 0$ to the same shape for all Y_w values, enabling a study of the relative subsidence of the wave form for diffrent wave amplitudes. It is clear from the figure that initial wave disturbance is felt practically at the same time at a given location for all Y_w values confirming that the speed of the propagation of the initial disturbance is dependent on the initial flow Froude number only. However, it is seen that the speed of propagation of the wave peak is clearly dependent on the wave amplitude. The wave peak arrives earlier at a particular section for higher wave amplitudes confirming that the higher amplitude wave travels faster (celerity of the gravity wave is directly proportional to the square root of depth). The difference in the time of arrival of the peak for different wave amplitudes is seen conspicuously in the hydrograph at $X = 6$.

The wave amplitude Y_w has a noticeable effect on the relative damping. The damping is relatively less for higher wave amplitudes and hence a linearising assumption (such as used in the unit hydrograph theory) does not strictly hold good. This was also indicated by the results of Mozayeny and Song[2].

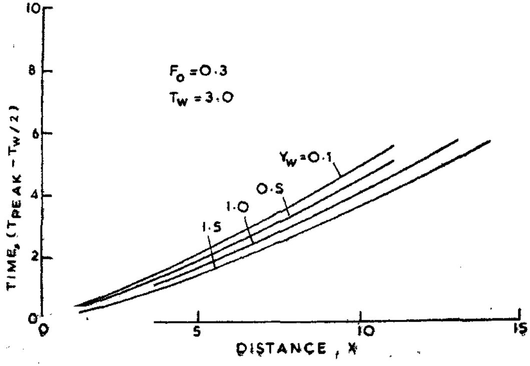

Fig. 4. Speed of travel of wave peak for different wave amplitudes.

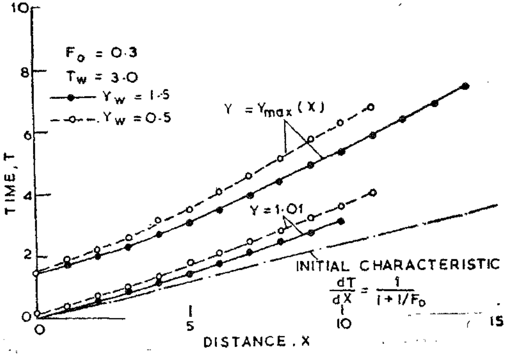

FIG. 5. Path of wave in XT plane.

3.1.2. *Discharge hydrograph*

Figure 2 presents a superposed picture of the discharge hydrographs for different wave amplitudes. Q_N, the normalised discharge rise, which is analogous to the normalised depth rise Y_N, is defined as

$$Q_N = \frac{Q(X) - 1}{Q(0) - 1}. \tag{10}$$

In Fig. 2. Q_N is plotted against time T at $X = 0$, 2 and 6 for different wave amplitudes. The normalised hydrograph at $X = 0$ is practically the same for all Y_N values and the tail end of the hydrograph dips below the uniform flow value. Such a result could be expected because in the recession stage of a flood, the same stage yields lower discharge due to the nature of the water surface elevation along the channel. Maximum dip below the uniform flow value is about 6 per cent of the initial increase in discharge, with the effect vanising with distance. The relative damping is less and speed of the peak is more for higher wave amplitudes. A comparison of Figs. 1 and 2 shows that the nonlinearity effects of Y_w are more pronounced on stage hydrographs than on discharge hydrographs and this is a significant result in the application of unit hydrograph theory.

3.2. *Modification of wave front*

The wave fronts at $T = 3{\cdot}0$ and $6{\cdot}0$ for different wave amplitudes are presented in Fig. 3. Even though the wave starts at the same time for all Y_w cases, it spreads over a larger distance for higher Y_w cases because of its higher speed.

3.3. *Speed of travel of wave peak*

Figure 4 gives the time at which the peak occurs at a particular section for different Y_w values. Wave amplitude has a significant effect on the speed of movement of the

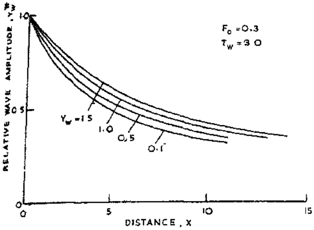

FIG. 6. Subsidence of relative wave amplitude—effect of wave amplitude.

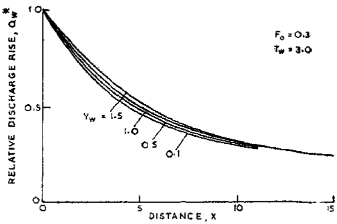

FIG. 7. Subsidence of relative discharge rise—effect of wave amplitude.

wave peak. The wave with higher amplitude moves faster because the celerity of a gravity wave is directly proportional to the square root of depth. If we consider the average speed of travel up to $X = 5$, the wave peak for $Y_w = 1 \cdot 5$ moves $1 \cdot 46$ times faster than for $Y_w = 0 \cdot 1$. As the nondimensional wave speed has been found to be practically independent of F_o and T_w, Fig. 4 can also be used to estimate in a wide channel, the time at which the wave peak would occur at a particular section for any wave amplitude Y_w between $0 \cdot 1$ and $1 \cdot 5$.

The path of the peak of the wave, the path along which depth is 1 per cent above the normal depth and the path of the initial characteristic are presented in Fig. 5 for $Y_w = 0 \cdot 1$ and $1 \cdot 5$. The line along which the depth is 1 per cent above the normal depth moves closer to the initial characteristic for higher Y_w value. It must be noted that 1 per cent rise above the normal depth represents a smaller fraction of the wave amplitude for $Y_w = 1 \cdot 5$ than for $Y_w = 0 \cdot 1$.

3.4. *Subsidence of wave amplitude*

3.4.1. *Subsidence of stage*

Variation of relative wave amplitude Y_w^* with X for different Y_w values is shown in Fig. 6. Y_w^* is defined by

$$Y_w^* = \frac{Y_{\max}(X) - 1}{Y_w} \qquad (11)$$

Wave amplitude has some effect on the relative damping rate confirming the non-linearity of the phenomenon. The relatively lesser damping for higher wave amplitudes might be partly associated with lesser resistance effects at higher flow depths. The relative wave amplitude Y_w^* is found to vary exponentially with distance X, but the exponent is not a constant for the whole channel reach in contrast to the claim made by Mozayeny and Song[2],

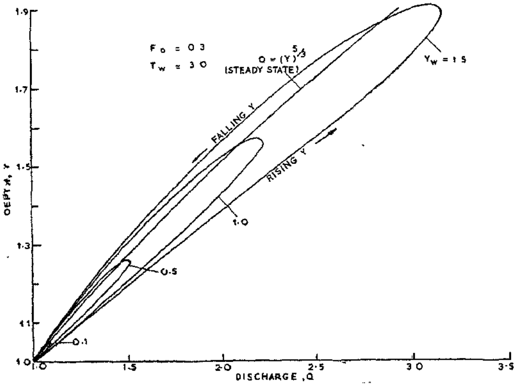

FIG 8 Rating curves for different wave amplitudes

3.4.2. *Subsidence of discharge*

Relative discharge rise Q_w^* is defined as,

$$Q_w^* = \frac{Q_{\max}(X) - 1}{Q_{\max}(O) - 1}. \tag{12}$$

It is seen that at large distances, the effect of Y_w on the relative damping rate is not particularly significant (Fig. 7). In fact, the nonlinearity in the discharge peak is clearly lesser than that in the stage peak as already indicated.

3.5. *Rating curve from computational results*

Figure 8 presents the rating curve for different wave amplitudes as obtained from the computational results at $X = 5$. Greater the wave amplitude greater the difference between the rising and falling stage flood and the results show that there can be very significant difference between the two values. The steady state curve lies between the rising and falling stage values but is closer to the falling stage.

4. Effect of wave duration

4.1. *Modification of hydrographs*

4.1.1. *Stage hydrograph*

Results of hydrographs for different T_w values for $Y_w = 0 \cdot 5$ and $F_o = 0 \cdot 3$ are presented in Fig. 9. In order to facilitate a comparison of the hydrographs for different wave durations the time T is normalised with respect to the wave period T_w by defining

$$T_N = \frac{T}{T_w}. \tag{13}$$

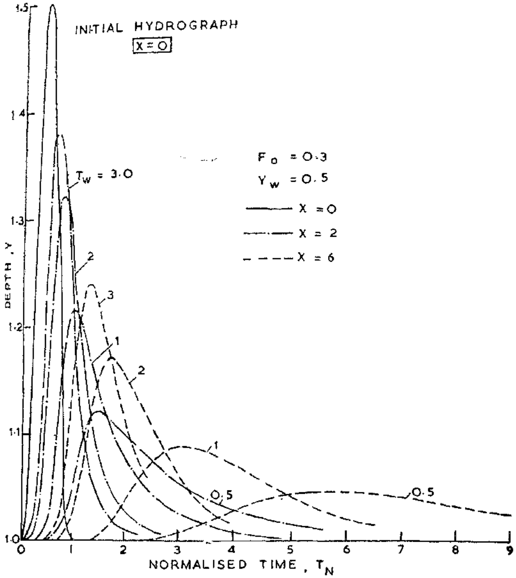

FIG. 9. Effect of wave duration on stage hydrograph.

This makes the initial hydrograph at $X = 0$ common for all T_w values. It is seen from Fig. 9 that the rate of subsidence is very significantly affected by T_w value and the wave with smaller T_w values subsides rapidly. In conjunction with the previous discussion, we see that as the bulk of the wave form reduces, either through reduction of Y_w or T_w, the subsidence rate increases. But a variation in T_w has a much stronger influence than a variation in Y_w.

Larger spread of the wave for lower T_w values is only a scale effect arising from the normalisation of time co-ordinate. The hydrograph for larger T_w values spreads over a larger base time corresponding to the larger base time of the initial hydrograph. Figure 9 clearly shows that the relative rate of spread of the base of the hydrograph is more for smaller T_w values. Hydrograph starts late for lower T_w cases. This is only a scale effect as the speed of the initial disturbance is independent of wave period.

4.1.2. *Discharge hydrograph*

Effect of wave duration T_w on discharge hydrograph is given in Fig. 10. Again the time scale is normalised (eqn. 13) to enable a comparison. There is a significant

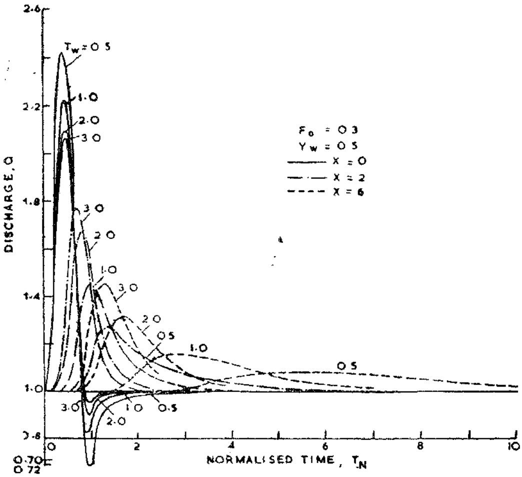

FIG. 10. Effect of wave duration on discharge hydrograph.

variation in the peak of the hydrograph at $X=0$ for different T_w values. The discharge peak is maximum for lowest T_w value. This might be due to the more rapid change in depth corresponding to smaller T_w values. However, the hydrographs at $X = 2$ and 6 show that the peak for smaller T_w values are clearly smaller. This corresponds to the much greater subsidence rate for smaller T_w values as already observed with respect to stage hydrographs (Fig. 9).

4.2. *Modification of wave front*

Figure 11 gives a superimposed picture of the wave front for different wave durations. Wave fronts at $T = 3T_w$ are presented. Thus the instantaneous wave profiles at $T = 1 \cdot 5$, $3 \cdot 0$ and $6 \cdot 0$ are presented for $T_w = 0 \cdot 5$, $1 \cdot 0$ and $2 \cdot 0$ respectively. It is seen from the figure that the wave peak has subsided to a greater extent at $T = 1 \cdot 5$ for $T_w = 0 \cdot 5$ case than at $T = 6 \cdot 0$ for $T_w = 2 \cdot 0$ case. Wave fronts for higher T_w cases have moved a larger distance because of the greater absolute time that has elapsed.

4.3. *Speed of travel of wave peak*

Figure 12 gives the time of arrival of the wave peak at any location. All the computational results for different wave durations lie practically on a single curve indicating that the speed of the wave peak is practically independent of the wave duration. Strictly

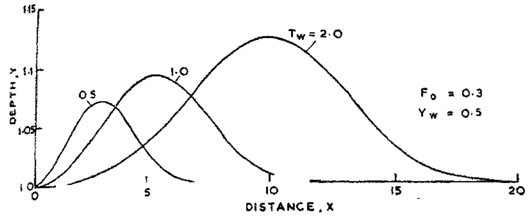

FIG. 11. Effect of wave duration on wave front ($T = 3T_N$).

the speed is seen to be slightly larger for higher T_w values (corresponding to lesser subsidence), but the differences are not significant. It might be noted that Y_w had a fairly significant influence on the wave speed (Fig. 4) in view of larger variation in the flow depth.

4.4. *Subsidence of wave amplitude*

4.4.1. *Subsidence of stage*

Figure 13 presents the variation of relative wave amplitude with distance for different wave durations. The pronounced effect of T_w is clearly brought out by this figure. The rate of subsidence is very high in the initial reaches for low T_w values and the subsidence rate comes down only after the base of the hydrograph has spread significantly at a sufficiently downstream location increasing the local wave duration. Thus we might conclude that a step like flood of small duration existing in isolation subsides rapidly.

4.4.2. *Subsidence of discharge*

Rapid subsidence of the relative discharge rise for low wave durations is clearly seen in Fig. 14. These results also confirm the significant influence of T_w as revealed by Fig. 13.

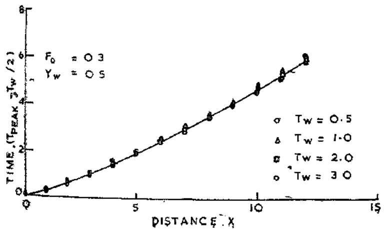

FIG. 12. Speed of travel of wave peak for different wave durations.

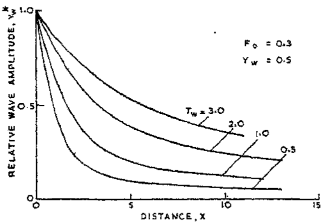

FIG. 13. Subsidence of relative wave amplitude—effect of wave duration.

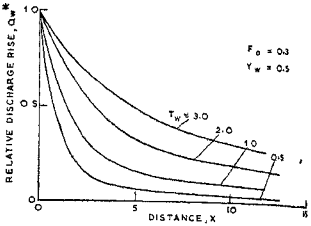

FIG. 14. Subsidence of relative discharge rise—effect of wave duration

4.5. *Rating curve from computational results*

Figure 15 presents the rating curve at $X = 5$ for different wave durations as obtained from computational results. As in the earlier case larger waves yield greater difference between the rising and falling stages and further the steady state curve lies between the rising and falling limbs being closer to the fallng limb of the rating curve.

5. Conclusions

Studies are made on the effect of wave parameters (wave amplitude and wave duration) on the propagation of a sinusoidal flood wave in a prismatic wide open channel in which the initial flow is uniform. Aspects studied include modifications of stage hydrographs, discharge hydrographs, wave fronts, speed of travel of the wave peak, subsidence of relative wave amplitude and relative discharge rise and generation of rating curves.

Initial wave amplitude Y_w has some influence on subsidence with lower Y_w values giving slightly higher subsidence. This nonlinearity is found to be slightly lesser in discharge results. Speed of travel of the wave peak is significantly affected by the change in Y_w value with higher Y_w values moving faster.

Wave duration T_w has a pronounced effect on subsidence with subsidence being significantly more for lower T_w values. Further, the rate of subsidence in the initial reaches is very high for lower T_w values suggesting that a step like flood of small duration existing in isolation subsides rapidly. While a moderate variation in T_w affects the rate of subsidence very significantly, it has only a small influence on the speed of the wave peak, with the speed being slightly larger for higher T_w values corresponding to lesser subsidence.

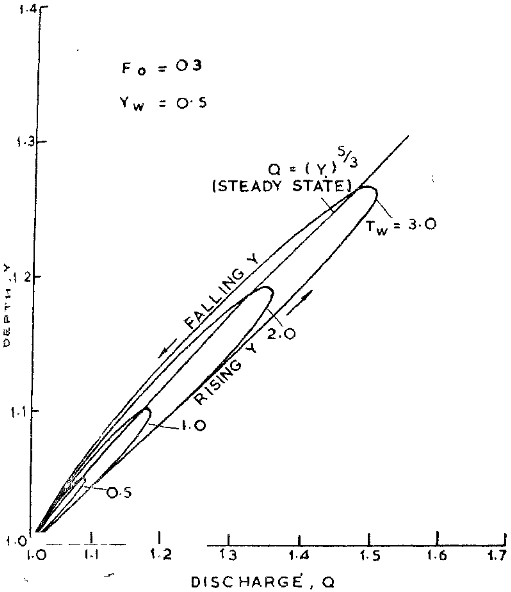

FIG. 15. Rating curves for different wave durations.

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