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Unsteady laminar flow in a channel with porous bed

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Abstract

The unsteady flow of a viscous incompressible fluid in a fixed parallel-plate channel, one of whose bounding walls is a porous medium is considered in the context of the matching flow criterion of Beavers and Joseph. Flow due to time-dependent pressure-gradient of exponentially decaying and periodic types has been studied. The slip velocity has been calculated in particular cases.

Key words : Unsteady flow, parallel channel, porous medium, slip-flow.

1. Introduction

The rectilinear flow of a viscous incompressible fluid through a two-dimensional channel formed by a porous wall and a solid wall has been analysed by several investigators[1–3]. Such flows are of importance in industrial, bio-physical and hydrodynamic problems.

In the study of the flow over porous bed, use is made of the boundary condition suggested by Beavers and Joseph[1], for the permeable surface. This fluid motion is a coupled one satisfying Navier-Stokes equations in the free fluid and Darcy's law in the permeable material and matching conditions at the nominal surface. The Poiseuille flow has also been extended to the consideration of the flow of stratified fluid of variable density and viscosity[4, 5]. All the above investigations relate to steady flow.

Unsteady flow of viscous fluid over permeable bed has been treated by Hunt[6]. Prakash and Rajbanshi[7] obtained the solution of the fluctuating flow of a viscous fluid induced by a uniform free stream. The present study is concerned with the unsteady flow of a viscous fluid in the channel under time dependent pressure–gradients of the exponentially decaying and periodic types. The velocity component due to the flow over the permeable bed has been calculated and compared with the flow over the solid bed in the case of exponentially decreasing pressure gradient. The slip-velocity has been found for the flow due to periodic pressure-gradient.

2. Formulation of the boundary-value problem

We consider unsteady rectilinear flow of a viscous incompressible fluid through a two-dimensional parallel channel whose lower boundary is a porous wall and the upper boundary a solid wall. The porous medium comprising the lower boundary is taken to be homogeneous, isotropic and completely saturated. The coordinates along and perpendicular to the channel are denoted by x and y respectively. Laminar unidirectional flow is assumed to be set up by time-varying longitudinal pressure-gradient in the channel and in the porous medium. The flow in the channel is governed by the usual Navier-Stokes equation. In the porous medium, the flow is governed by Darcy's law. Walls of the channel are horizontal and infinitely long to allow the physical quantities to be independent of the axial coordinate. Velocity-field is assumed to have only one component in the direction of x-axis. The velocity u and pressure p are functions of x, y and t, t denoting the time. Due to equation of continuity,

$$u = u(y, t)$$

$$p = p(y, t).$$

The velocity-field u in the channel satisfies the differential equation

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \frac{\mu}{\rho}\frac{\partial^2 u}{\partial y^2} \tag{1}$$

where ρ is the fluid density and μ is the dynamic viscosity. When the external forces are absent, the filtration velocity is given by the relation[8] (p. 170)

$$V = -\frac{k}{\mu}\frac{dp}{dx} \tag{2}$$

in which k is the intrinsic permeability of the medium having the dimension of length squared. Thus, V is determined as soon as the pressure-gradient is given.

Following Beavers and Joseph[1], we specify the boundary condition at the surface of the porous medium (which is the nominal surface) by the relation

$$\frac{du}{dy} = \frac{\alpha}{\sqrt{k}}(u - V), \quad y = 0 \tag{3}$$

where α is a constant depending only on the porous material and not on its structure. Beavers and Joseph tabulated values of α for different materials (foametal and aloxite) of various permeabilities.

The second boundary condition is provided by the no-slip condition at the upper bounding wall, *i.e.*,

$$u = 0 \text{ when } y = h \tag{4}$$

Let us now make eqn. (1) non-dimensional by using the dimensionless quantities

$$x' = \frac{x}{h}, \quad \eta = \frac{y}{h}, \quad u' = \frac{u}{U}, \quad t' = \frac{t}{\frac{h}{U}}$$

$$p' = \frac{p}{\rho U^2}, \quad k' = \frac{k}{h^2} \tag{5}$$

where U is some characteristic velocity (*e.g.*, the mean velocity).

Equation (1) now reduces to

$$\frac{\partial u'}{\partial t'} = -\frac{\partial p'}{\partial x'} + \gamma \frac{\partial^2 u'}{\partial \eta^2} \tag{6}$$

where $1/\gamma$ represents the Reynolds number Uh/ν. It is evident that $\partial p/\partial x'$ is a function of t'.

The boundary conditions, in terms of the non-dimensional quantities become,

$$\frac{\partial u'}{\partial \eta} = \frac{\alpha}{\sqrt{k'}}\left[u' + \frac{k'}{\gamma}\frac{dp'}{dx'}\right] \text{for } \eta = 0 \tag{7}$$

$$u = 0 \text{ for } \eta = 1. \tag{8}$$

4. Flow due to exponentially decaying pressure-gradient

Let us now assume that the flow is driven by an unsteady pressure-gradient given by

$$-\frac{dp'}{dx'} = A \exp(-\lambda^2 t') \tag{9}$$

where A and λ^2 are known real constants. We assume that the velocity is given by

$$u' = A\, f(\eta) \exp(-\lambda^2 t'). \tag{10}$$

Equation (6) now becomes

$$\frac{d^2 f}{d\eta^2} + \frac{\lambda^2}{\gamma} f = -\frac{1}{\gamma} \tag{11}$$

leading to the solution

$$f = A' \cos\frac{\lambda\eta}{\sqrt{\gamma}} + B' \sin\frac{\lambda\eta}{\sqrt{\gamma}} - \frac{1}{\lambda^2} \tag{12}$$

where A', B' are functions independent of η.

Boundary conditions for f follow from (7) and (8). These are

$$\frac{df}{d\eta} = \frac{\alpha}{\sqrt{k'}}\left[f - \frac{k'}{\gamma}\right]$$

$$= \alpha\sigma\left[f - \frac{1}{\gamma\sigma^2}\right] \text{ for } \eta = 0 \tag{13}$$

where σ is written for $\frac{1}{\sqrt{k'}}$

and

$$f = 0 \text{ for } \eta = 1. \tag{14}$$

Determining A', B', by means of (13) and (14) and substituting in (12) we obtain

$$f(\eta) = \frac{1}{\lambda^2}\left[\frac{\frac{\lambda}{\sqrt{\gamma}}\cos\frac{\lambda\eta}{\sqrt{\gamma}} + \alpha\sigma\sin\frac{\lambda\eta}{\sqrt{\gamma}} + \alpha\sigma\left(1 + \frac{\lambda^2}{\gamma\sigma^2}\right)\sin\frac{\lambda}{\sqrt{\gamma}}(1-\eta)}{\frac{\lambda}{\sqrt{\gamma}}\cos\frac{\lambda}{\sqrt{\gamma}} + \alpha\sigma\sin\frac{\lambda}{\sqrt{\gamma}}} - 1\right]. \tag{15}$$

Then,

$$u' = Af(\eta)\exp(-\lambda^2 t'). \tag{16}$$

The slip-velocity is given by

$$u'_s = Af_s\exp(-\lambda^2 t') \tag{17}$$

where

$$f_s = \frac{1}{\lambda^2}\left[\frac{\frac{\lambda}{\sqrt{\gamma}} + \alpha\sigma\left(1 + \frac{\lambda^2}{\gamma\sigma^2}\right)\sin\frac{\lambda}{\sqrt{\gamma}}}{\frac{\lambda}{\sqrt{\gamma}}\cos\frac{\lambda}{\sqrt{\gamma}} + \alpha\sigma\sin\frac{\lambda}{\sqrt{\gamma}}} - 1\right].$$

If the bed of the channel had been impermeable the velocity profile would be given by

$$u'^* = Af^*(\eta)\exp(-\lambda^2 t') \tag{18}$$

with

$$f^*(\eta) = \frac{1}{\lambda^2}\left[\frac{\sin\frac{\lambda\eta}{\sqrt{\gamma}} + \sin\frac{\lambda}{\sqrt{\gamma}}(1-\eta)}{\sin\frac{\lambda}{\sqrt{\gamma}}} - 1\right]$$

satisfying the conditions $u'^* = 0$ on $\eta = 0$ and $\eta = 1$.

The above result (18) follows from (15) when we make $\sigma \to \infty$; which means $k' \to 0$. In this case, the slip velocity indicated by (17) tends to zero.

The skin-friction at the surface of the porous medium is given by

$$\tau_0 = \left(\frac{\partial u'}{\partial \eta}\right)_{\eta=0} = A\left(\frac{df}{d\eta}\right)_{\eta=0} \exp\left(-\lambda^2 t'\right)$$

where

$$\left(\frac{df}{d\eta}\right)_{\eta=0} = \frac{1}{\lambda^2}\,\frac{\frac{\alpha\sigma\lambda}{\sqrt{\gamma}}\left[1-\left(1+\frac{\lambda^2}{\gamma\sigma^2}\right)\cos\frac{\lambda}{\sqrt{\gamma}}\right]}{\frac{\lambda}{\sqrt{\gamma}}\cos\frac{\lambda}{\sqrt{\gamma}}+\alpha\sigma\sin\frac{\lambda}{\sqrt{\gamma}}}. \tag{19}$$

In the case of flow over an impermeable solid bed

$$\tau_0^* = A\left(\frac{df^*}{d\eta}\right)_{\eta=0} \exp\left(-\lambda^2 t'\right)$$

where

$$\left(\frac{df^*}{d\eta}\right)_{\eta=0} = \frac{1}{\lambda^2}\,\frac{\frac{\lambda}{\sqrt{\gamma}}\left(1-\cos\frac{\lambda}{\sqrt{\gamma}}\right)}{\sin\frac{\lambda}{\sqrt{\gamma}}}. \tag{20}$$

From (20) it follows that the skin-friction is positive on the solid wall.

From (19) we find that the skin-friction is positive if

$$1-\left(1+\frac{\lambda^2}{\gamma\sigma^2}\cos\frac{\lambda}{\sqrt{\gamma}}\right) > 0 \tag{21}$$

Assuming that the Reynolds number $1/\gamma$ is small, we expand $\cos\lambda/\sqrt{\gamma}$ in which we neglect terms containing $1/\gamma^2$ (21), therefore, implies that $\sigma^2 > 2$ for laminar flow.

5. Flow comparison for different values of η

To study the flow in a channel with permeable bed and that in a channel with solid bed we calculate u' and u'^* given by (16) and (18). We take $\lambda^2 = 1{\cdot}44$, $\alpha = {\cdot}1$, $\sigma = 10$ and $\gamma = 100$. Table I gives the values of u' and u'^* for different values of η at $t' = 0$.

From the results of the table we find that the effect of permeability is manifest in giving rise to a slip-flow on the bed which gradually diminishes as we move away from the bed.

Table I

η	u'/A	u'^*/A
0	·007521	0
·1	·007222	·004501
·2	·006823	·008011
·3	·006322	·001051
·4	·005721	·001201
·5	·005019	·001252
·6	·004216	·001201
·7	·003312	·001051
·8	·002308	·008011
·9	·001204	·004501
1	0	0

In the case of flow over a solid wall, the velocity profile follows the parabolic law; the velocity increases from zero to its maximum value at the middle of the channel and then decreases.

The magnitude of the flow-velocity in the case of flow over permeable bed is greater than that of flow over solid bed.

6. Flow due to periodic pressure-gradient

We assume that the driving pressure-gradient is oscillatory and write

$$-\frac{dp}{dx} = c \cos \omega t \tag{22}$$

where ω is the frequency of the oscillation and c is a known real constant.

It is convenient to use the complex notation and to put

$$-\frac{dp}{dx} = ce^{i\omega t} \tag{23}$$

attributing physical significance only to the real part

Equation of motion (1) then becomes

$$\frac{\partial u}{\partial t} - \nu \frac{\partial^2 u}{\partial y^2} = \frac{c}{p} e^{i\omega t} \tag{24}$$

where ν is the kinematic coefficient of viscosity.

We make equation (24) non-dimensional by taking

$$\eta = \frac{y}{h},\ t' = \omega t,\ u' = \frac{u}{h\omega}. \tag{25}$$

Equation (24) becomes equivalent to

$$\frac{\partial u'}{\partial t'} - \gamma \cdot \frac{\partial^2 u}{\partial \eta^2} = c' e^{i\omega t'} \tag{26}$$

where

$$\gamma = \frac{\nu}{h^2 \omega} \tag{27}$$

$$c' = \frac{c}{ph\omega^2}. \tag{28}$$

From (27) it follows that $1/\gamma$ is the Reynolds number of the flow.

We take the velocity in the form

$$u'(\eta, t') = C' F(\eta) e^{it'}. \tag{29}$$

Substituting in (26), we get the differential equation

$$\frac{d^2 F}{d\eta^2} - \frac{iF}{\gamma} = -\frac{1}{\gamma}. \tag{30}$$

Solution of the above equation is

$$F = D\, ch (1+i) \frac{\eta}{\sqrt{2\gamma}} + E\, sh (1+i) \frac{\eta}{\sqrt{2\gamma}} - i. \tag{31}$$

Boundary conditions are

$$\frac{dF}{d\eta} = \alpha\sigma \left[F - \frac{1}{\gamma\sigma^2} \right] \text{ when } \eta = 0 \tag{32}$$

$$F = 0 \qquad \text{when } \eta = 1. \tag{33}$$

These boundary conditions determine F and give

$$u' = c' \left[-i + \frac{\frac{i(1+i)}{\sqrt{2\gamma}} ch\,(1+i)\frac{\eta}{\sqrt{2\gamma}} + i\alpha\sigma\, sh\,(1+i)\frac{\eta}{\sqrt{2\gamma}} + \alpha\sigma\left(1+\frac{1}{\gamma\sigma^2}\right) sh\,\frac{1+i}{\sqrt{2\gamma}}(1-\eta)}{\frac{1+i}{\sqrt{2\gamma}}\, ch\,\frac{(1+i)}{\sqrt{2\gamma}} + \alpha\sigma\, sh\,\frac{1+i}{\sqrt{2\gamma}}} \right] e^{it'} \tag{34}$$

Taking real part of the above expression, we obtain

$$u' = \left(1 - \frac{\chi}{N}\right)\sin t' + \frac{\psi}{N}\cos t' \tag{35}$$

(cf. Ref. 9)

where

$$\begin{aligned}
\chi = {} & \frac{1}{\gamma}\left[ch\,\frac{\eta}{\sqrt{2\gamma}}\cos\frac{\eta}{\sqrt{2\gamma}}\, ch\,\frac{1}{\sqrt{2\gamma}}\cos\frac{1}{\sqrt{2\gamma}} + sh\,\frac{\eta}{\sqrt{2\gamma}}\sin\frac{\eta}{\sqrt{2\gamma}}\, sh\,\frac{1}{\sqrt{2\gamma}}\sin\frac{1}{\sqrt{2\gamma}}\right] \\
& + \frac{\alpha\sigma}{\sqrt{2\gamma}}\left[ch\,\frac{1}{\sqrt{2\gamma}}\cos\frac{1}{\sqrt{2\gamma}}\left(sh\,\frac{\eta}{\sqrt{2\gamma}}\cos\frac{\eta}{\sqrt{2\gamma}} + ch\,\frac{\eta}{\sqrt{2\gamma}}\sin\frac{\eta}{\sqrt{2\gamma}} + ch\,\frac{1-\eta}{\sqrt{2\gamma}}\sin\frac{1-\eta}{\sqrt{2\gamma}}\right.\right. \\
& \left. + sh\,\frac{1-\eta}{\sqrt{2\gamma}}\cos\frac{1-\eta}{\sqrt{2\gamma}}\right) + sh\,\frac{1}{\sqrt{2\gamma}}\sin\frac{1}{\sqrt{2\gamma}}\left(ch\,\frac{\eta}{\sqrt{2\gamma}}\sin\frac{\eta}{\sqrt{2\gamma}}\right. \\
& \left. - sh\,\frac{\eta}{\sqrt{2\gamma}}\cos\frac{\eta}{\sqrt{2\gamma}} - sh\,\frac{1-\eta}{\sqrt{2\gamma}}\cos\frac{1-\eta}{\sqrt{2\gamma}} + ch\,\frac{1-\eta}{\sqrt{2\gamma}}\sin\frac{1-\eta}{\sqrt{2\gamma}}\right) \\
& + ch\,\frac{\eta}{\sqrt{2\gamma}}\cos\frac{\eta}{\sqrt{2\gamma}}\left(sh\,\frac{1}{\sqrt{2\gamma}}\cos\frac{1}{\sqrt{2\gamma}} + ch\,\frac{1}{\sqrt{2\gamma}}\sin\frac{1}{\sqrt{2\gamma}}\right) \\
& \left. + sh\,\frac{\eta}{\sqrt{2\gamma}}\sin\frac{\eta}{\sqrt{2\gamma}}\left(ch\,\frac{1}{\sqrt{2\gamma}}\sin\frac{1}{\sqrt{2\gamma}} - sh\,\frac{1}{\sqrt{2\gamma}}\cos\frac{1}{\sqrt{2\gamma}}\right)\right] \\
& + \frac{\alpha}{\gamma\sigma\sqrt{2\gamma}}\left[ch\,\frac{1}{\sqrt{2\gamma}}\cos\frac{1}{\sqrt{2\gamma}}\left(ch\,\frac{1-\eta}{\sqrt{2\gamma}}\sin\frac{1-\eta}{\sqrt{2\gamma}} - sh\,\frac{1-\eta}{\sqrt{2\gamma}}\cos\frac{1-\eta}{\sqrt{2\gamma}}\right)\right. \\
& \left. - sh\,\frac{1}{\sqrt{2\gamma}}\sin\frac{1}{\sqrt{2\gamma}}\left(ch\,\frac{1-\eta}{\sqrt{2\gamma}}\sin\frac{1-\eta}{\sqrt{2\gamma}} + sh\,\frac{1-\eta}{\sqrt{2\gamma}}\cos\frac{1-\eta}{\sqrt{2\gamma}}\right)\right] \\
& + \alpha^2\sigma^2\left[sh\,\frac{1}{\sqrt{2\gamma}}\cos\frac{1}{\sqrt{2\gamma}}\left(sh\,\frac{\eta}{\sqrt{2\gamma}}\cos\frac{\eta}{\sqrt{2\gamma}} + sh\,\frac{1-\eta}{\sqrt{2\gamma}}\cos\frac{1-\eta}{\sqrt{2\gamma}}\right)\right. \\
& \left. + ch\,\frac{1}{\sqrt{2\gamma}}\sin\frac{1}{\sqrt{2\gamma}}\left(ch\,\frac{\eta}{\sqrt{2\gamma}}\sin\frac{\eta}{\sqrt{2\gamma}} + ch\,\frac{1-\eta}{\sqrt{2\gamma}}\sin\frac{1-\eta}{\sqrt{2\gamma}}\right)\right] \\
& + \frac{\alpha^2}{\gamma}\left[ch\,\frac{1-\eta}{\sqrt{2\gamma}}\sin\frac{1-\eta}{\sqrt{2\gamma}}\, sh\,\frac{1}{\sqrt{2\gamma}}\cos\frac{1}{\sqrt{2\gamma}}\right. \\
& \left. - sh\,\frac{1-\eta}{\sqrt{2\gamma}}\cos\frac{1-\eta}{\sqrt{2\gamma}}\, ch\,\frac{1}{\sqrt{2\gamma}}\sin\frac{1}{\sqrt{2\gamma}}\right]
\end{aligned} \tag{36}$$

$$
\begin{aligned}
\psi = \frac{1}{\gamma}\Big[& ch\frac{\eta}{\sqrt{2\gamma}}\cos\frac{\eta}{\sqrt{2\gamma}}\, sh\frac{1}{\sqrt{2\gamma}}\sin\frac{1}{\sqrt{2\gamma}} - sh\frac{\eta}{\sqrt{2\gamma}}\sin\frac{\eta}{\sqrt{2\gamma}}\, ch\frac{1}{\sqrt{2\gamma}}\cos\frac{1}{\sqrt{2\gamma}}\Big] \\
& + \frac{\alpha\sigma}{\sqrt{2\gamma}}\Big[sh\frac{1}{\sqrt{2\gamma}}\sin\frac{1}{\sqrt{2\gamma}}\Big(ch\frac{\eta}{\sqrt{2\gamma}}\sin\frac{\eta}{\sqrt{2\gamma}} + sh\frac{\eta}{\sqrt{2\gamma}}\cos\frac{\eta}{\sqrt{2\gamma}} + ch\frac{1-\eta}{\sqrt{2\gamma}}\sin\frac{1-\eta}{\sqrt{2\gamma}} \\
& + sh\frac{1-\eta}{\sqrt{2\gamma}}\cos\frac{1-\eta}{\sqrt{2\gamma}}\Big) - ch\frac{1}{\sqrt{2\gamma}}\cos\frac{1}{\sqrt{2\gamma}}\Big(ch\frac{\eta}{\sqrt{2\gamma}}\sin\frac{\eta}{\sqrt{2\gamma}} \\
& - sh\frac{\eta}{\sqrt{2\gamma}}\cos\frac{\eta}{\sqrt{2\gamma}} + ch\frac{1-\eta}{\sqrt{2\gamma}}\sin\frac{1-\eta}{\sqrt{2\gamma}} - sh\frac{1-\eta}{\sqrt{2\gamma}}\cos\frac{1-\eta}{\sqrt{2\gamma}}\Big) \\
& + ch\frac{\eta}{\sqrt{2\gamma}}\cos\frac{\eta}{\sqrt{2\gamma}}\Big(ch\frac{1}{\sqrt{2\gamma}}\sin\frac{1}{\sqrt{2\gamma}} - sh\frac{1}{\sqrt{2\gamma}}\cos\frac{1}{\sqrt{2\gamma}}\Big) \\
& - sh\frac{\eta}{\sqrt{2\gamma}}\sin\frac{\eta}{\sqrt{2\gamma}}\Big(ch\frac{1}{\sqrt{2\gamma}}\sin\frac{1}{\sqrt{2\gamma}} + sh\frac{1}{\sqrt{2\gamma}}\cos\frac{1}{\sqrt{2\gamma}}\Big)\Big] \\
& + \frac{\alpha}{\gamma\sigma\sqrt{2}}\Big[sh\frac{1}{\sqrt{2\gamma}}\sin\frac{1}{\sqrt{2\gamma}}\Big(ch\frac{1-\eta}{\sqrt{2\gamma}}\sin\frac{1-\eta}{\sqrt{2\gamma}} - sh\frac{1-\eta}{\sqrt{2\gamma}}\cos\frac{1-\eta}{\sqrt{2\gamma}}\Big) \\
& + ch\frac{1}{\sqrt{2\gamma}}\cos\frac{1}{\sqrt{2\gamma}}\Big(ch\frac{1-\eta}{\sqrt{2\gamma}}\sin\frac{1-\eta}{\sqrt{2\gamma}} + sh\frac{1-\eta}{\sqrt{2\gamma}}\cos\frac{1-\eta}{\sqrt{2\gamma}}\Big)\Big] \\
& - \alpha^2\sigma^2\Big[sh\frac{1}{\sqrt{2\gamma}}\cos\frac{1}{\sqrt{2\gamma}}\Big(ch\frac{\eta}{\sqrt{2\gamma}}\sin\frac{\eta}{\sqrt{2\gamma}} + ch\frac{1-\eta}{\sqrt{2\gamma}}\sin\frac{1-\eta}{\sqrt{2\gamma}}\Big) \\
& - ch\frac{1}{\sqrt{2\gamma}}\sin\frac{1}{\sqrt{2\gamma}}\Big(sh\frac{\eta}{\sqrt{2\gamma}}\cos\frac{\eta}{\sqrt{2\gamma}} + sh\frac{1-\eta}{\sqrt{2\gamma}}\cos\frac{1-\eta}{\sqrt{2\gamma}}\Big)\Big] \\
& + \frac{\alpha^2}{\gamma}\Big[sh\frac{1-\eta}{\sqrt{2\gamma}}\cos\frac{1-\eta}{\sqrt{2\gamma}}\, sh\frac{1}{\sqrt{2\gamma}}\cos\frac{1}{\sqrt{2\gamma}} \\
& + ch\frac{1-\eta}{\sqrt{2\gamma}}\sin\frac{1-\eta}{\sqrt{2\gamma}}\, ch\frac{1}{\sqrt{2\gamma}}\sin\frac{1}{\sqrt{2\gamma}}\Big]
\end{aligned} \tag{37}
$$

$$
\begin{aligned}
N = \frac{1}{\gamma}\Big[& ch^2\frac{1}{\sqrt{2\gamma}}\cos^2\frac{1}{\sqrt{2\gamma}} + sh^2\frac{1}{\sqrt{2\gamma}}\sin^2\frac{1}{\sqrt{2\gamma}}\Big] \\
& + \alpha^2\sigma^2\Big[ch^2\frac{1}{\sqrt{2\gamma}}\sin^2\frac{1}{\sqrt{2\gamma}} + sh^2\frac{1}{\sqrt{2\gamma}}\cos^2\frac{1}{\sqrt{2\gamma}}\Big] \\
& + \frac{\sqrt{2}}{\gamma}\alpha\sigma\Big[ch\frac{1}{\sqrt{2\gamma}}\, sh\frac{1}{\sqrt{2\gamma}} + \cos\frac{1}{\sqrt{2\gamma}}\sin\frac{1}{\sqrt{2\gamma}}\Big]
\end{aligned} \tag{38}
$$

on the surface $\eta = 0$, we denote $(\chi)_{\eta=0}$ and $(\psi)_{\eta=0}$ by χ_0 and ψ_0 respectively. We get

$$
\begin{aligned}
\chi_0 = \frac{1}{\gamma}\, ch\frac{1}{\sqrt{2\gamma}}\cos\frac{1}{\sqrt{2\gamma}} + \frac{\alpha\sigma}{\sqrt{2\gamma}}\Big[& \sin\frac{1}{\sqrt{2\gamma}}\cos\frac{1}{\sqrt{2\gamma}} + ch\frac{1}{\sqrt{2\gamma}}\, sh\frac{1}{\sqrt{2\gamma}} \\
& + sh\frac{1}{\sqrt{2\gamma}}\cos\frac{1}{\sqrt{2\gamma}} + ch\frac{1}{\sqrt{2\gamma}}\sin\frac{1}{\sqrt{2\gamma}}\Big]
\end{aligned}
$$

$$+\frac{\alpha}{\gamma\sigma\sqrt{2\gamma}}\left[\cos\frac{1}{\sqrt{2\gamma}}\sin\frac{1}{\sqrt{2\gamma}}-ch\frac{1}{\sqrt{2\gamma}}sh\frac{1}{\sqrt{2\gamma}}\right]$$

$$+\alpha^2\sigma^2\left[sh^2\frac{1}{\sqrt{2\gamma}}\cos^2\frac{1}{\sqrt{2\gamma}}+ch^2\frac{1}{\sqrt{2\gamma}}\sin^2\frac{1}{\sqrt{2\gamma}}\right] \tag{39}$$

$$\psi_0=\frac{1}{\gamma}sh\frac{1}{\sqrt{2\gamma}}\sin\frac{1}{\sqrt{2\gamma}}+\frac{\alpha\sigma}{\sqrt{2\gamma}}\left[sh\frac{1}{\sqrt{2\gamma}}ch\frac{1}{\sqrt{2\gamma}}-\sin\frac{1}{\sqrt{2\gamma}}\cos\frac{1}{\sqrt{2\gamma}}\right.$$

$$\left.+ch\frac{1}{\sqrt{2\gamma}}\sin\frac{1}{\sqrt{2\gamma}}-sh\frac{1}{\sqrt{2\gamma}}\cos\frac{1}{\sqrt{2\gamma}}\right]$$

$$+\frac{\alpha}{\gamma\sigma\sqrt{2\gamma}}\left[sh\frac{1}{\sqrt{2\gamma}}ch\frac{1}{\sqrt{2\gamma}}+\cos\frac{1}{\sqrt{2\gamma}}\sin\frac{1}{\sqrt{2\gamma}}\right]$$

$$+\frac{\alpha^2}{\gamma}\left[sh^2\frac{1}{\sqrt{2\gamma}}\cos^2\frac{1}{\sqrt{2\gamma}}+ch^2\frac{1}{\sqrt{2\gamma}}\sin^2\frac{1}{\sqrt{2\gamma}}\right] \tag{40}$$

Slip-velocity is given by

$$u_s=\left(1-\frac{\chi_0}{N}\right)\sin t'+\frac{\psi_0}{N}\cos t'.$$

For $\gamma=100$, $\alpha=\cdot 1$, $\sigma=10$, we have,

$$\chi_0=\cdot 0400001,\qquad \psi_0=\cdot 0001999$$

$$N=\cdot 1059694.$$

Hence, the slip-velocity for Reynolds number ·01 is

$$\cdot 6225316\sin t'+\cdot 0018864\cos t'.$$

In particular, the velocities at the end of time-instants $2n\pi$, $(2n+\frac{1}{2})\pi$, $(2n+\frac{1}{4})\pi$, $(2n+\frac{3}{4})\pi$ where n is a positive integer, are given as follows :

t'	u_s^s
$2n\pi$	·0018864
$(2n+\frac{1}{2})\pi$	·6225316
$(2n+\frac{1}{4})\pi$	·4415352
$(2n+\frac{3}{4})\pi$	·4388574

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References

1. Beavers, G. S. and Joseph, D. D. — Boundary value problems at a naturally permeable wall, *J. Fluid Mech.* 1967, **30**, 197–207.
2. Beavers, G. S., Sparrow, E. M. and Magnuson, R. A. — Experiment on coupled parallel flows in a channel and a bounding porous medium, *J. Basic Engng. Trans. ASME*, 1970, **92 D**, 843–848.
3. Channabasappa, M. N., Umapathy, K. G. and Nayak, I. V. — The effect of the thickness of the porous material on the parallel plate channel flow when the walls are provided with non-erodible porous lining, *Appl. Sci. Res.*, 1976, **32**, 607–617.
4. Channabasappa, M. N. and Ranganna, G. — Flow of viscous stratified fluid of variable viscosity past a porous bed, *Proc. Indian Acad. Sci.*, 1976, **83 A**, 145–155.
5. Channabasappa, M. N. and Ranganna, G. — Flow past a permeable bed with different density and viscosity stratification, *Proc. Indian Soc. Theoretical and Appl. Mech. Twelfth Congress, Varanasi*, 1975, pp. 123–130.
6. Hunt, J. N. — On the damping of gravity waves propagated over a permeable surface, *J. Geophys. Res.*, 1959, **64**, 437.
7. Prakash, Om and Rajbanshi, S. C. — Fluctuating laminar flow past a naturally permeable bed, *Indian J. Pure Appl. Maths.*, 1978, **9**, 728–738.
8. Wiest De Roger, J. M. — *Flow through porous media*, Academic Press, 1969.
9. Hanna, Bishay, S. S. — On the unsteady flow of viscous fluid between two plates, *Acta Mechanica*, 1979, **33**, 199–205.