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On unsteady MHD flow past a porous plate under pressure gradient

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Abstract

The problem of MHD two-dimensional flow past an infinite porous plate with constant suction moving with arbitrary time dependent velocity, under time dependent pressure gradient when initial distribution of velocity is an exponential form is studied. This problem generalises several earlier works for the case when the motion has started from rest with uniform pressure gradient as a result of the plate movement in various particular ways.

Key words : Unsteady MHD flow, pressure gradient, porous plate, arbitrary time dependent velocity.

1. Introduction

The incompressible laminar viscous fluid flow between two stationary parallel flat plates with an arbitrary time varying pressure gradient and with an arbitrary initial distribution of velocity has been studied by Hepworth and Rice[1]. The same problem is studied by Prakash[2] under the same condition, but with the difference that the flow is in between two stationary coaxial circular cylinders. The problem of viscous incompressible flow past an infinite plate moving parallel to itself with an arbitrary time dependent velocity when the pressure is uniform and the initial distribution of velocity is an exponential form has been discussed by Prakash[3]. Srivastava and Lal[4] extended this problem in case of MHD flow. The present paper is concerned with the study of problem of incompressible laminar viscous electrically conducting fluid flow past an infinite flat porous plate moving parallel to itself with an arbitrary time dependent velocity with uniform suction at the plate, under constant pressure gradient, when the initial distribution of velocity is an exponential form.

2. Formulation of the problem and solution

Consider an unsteady laminar viscous MHD flow past an infinite porous flat insulated plate moving parallel to itself with arbitrary time dependent velocity with uniform suction $V(V > 0)$ under time dependent pressure gradient, with initial distribution of velocity being in exponential form. We take x and y axes along and normal to the plate and assume a uniform magnetic field H_0 acting along y-axis. Then the governing equation of motion for this problem is

$$\frac{\partial u}{\partial t} - V\frac{\partial u}{\partial y} + mu = \nu\frac{\partial^2 u}{\partial y^2} - \frac{1}{\rho}\frac{\partial p}{\partial x} \tag{2.1}$$

where $m = \frac{\sigma}{\rho}\mu_e^2 H_0^2 =$ constant. ν is the kinematic viscosity and p is the pressure.

The initial and boundary conditions are

$$t = 0;\ u = A\exp(-By) \text{ for } y \geqslant 0 \tag{2.2}$$

$$t > 0 : u = g(t) \qquad \text{for } y = 0 \tag{2.3}$$

$$t > 0 : u = 0 \qquad \text{as } y \to \infty \tag{2.4}$$

Here A, B are non-negative constants and $g(t)$ is bounded continuous or piecewise continuous arbitrary function of y.

Now if we assume $-\frac{\partial p}{\partial x} = f(t)$, (2.1) reduces to

$$\frac{\partial u}{\partial t} - V\frac{\partial u}{\partial y} + mu = \nu\frac{\partial^2 u}{\partial y^2} + \frac{f(t)}{\rho}. \tag{2.5}$$

We solve (2.1) with initial and boundary conditions (2.2)–(2.4), with Laplace transform techniques and the solution, after assuming pressure gradient constant, *i.e.*,

$$-\frac{\partial p}{\partial x} = f(t) = C,$$

where C is constant, is given by

$$u = \exp\left(-\frac{Vy}{2\nu}\right)\int_0^t g(t-T)\frac{y}{\sqrt{4\pi\nu T^3}}\exp\left\{-\frac{y^2}{4\nu T} - \frac{V^2 T}{4\nu} - mT\right\}dT$$

$$-\frac{A}{2}\exp\left\{-\frac{Vy}{2\nu} + \nu B^2 t - VBt - mt\right\}\left[\exp\left\{y\left(B\frac{V}{2\nu}\right)\right\} erfc\left\{\frac{y}{\sqrt{4\nu t}}\right.\right.$$

$$\left.\left.+\left(B - \frac{V}{2\nu}\right)\sqrt{\nu t}\right\} + \exp\left\{-y\left(B - \frac{V}{2\nu}\right)\right\} erfc\left\{\frac{y}{\sqrt{4\nu t}} - \left(B - \frac{V}{2\nu}\right)\sqrt{\nu t}\right\}\right]$$

$$+\frac{C}{2\rho m}\exp(-mt)\left[\exp\left(-\frac{Vy}{\nu}\right) erfc\left\{\frac{y}{\sqrt{4\nu t}} - \frac{V\sqrt{t}}{\sqrt{4\nu}}\right\}\right.$$

$$\left.+ erfc\left\{\frac{y}{\sqrt{4\nu t}} + \frac{V\sqrt{t}}{\sqrt{4\nu}}\right\}\right] - \frac{P}{2\rho m}\exp\left(-\frac{Vy}{2\nu}\right)$$

$$\times \left[\exp\left\{-\frac{y}{\sqrt{\nu}}\left(\frac{V^2}{4\nu}+m\right)^{1/2}\right\} erfc\left\{\frac{y}{\sqrt{4\nu t}}-\sqrt{\left(\frac{V^2}{4\nu}+m\right)t}\right\}\right.$$

$$\left.+\exp\left\{\frac{y}{\sqrt{\nu}}\left(\frac{V^2}{4\nu}+m\right)^{1/2}\right\} erfc\left\{\frac{y}{\sqrt{4\nu t}}+\sqrt{\left(\frac{V^2}{4\nu}+m\right)t}\right\}\right]$$

$$+\frac{C}{\rho m}(1-e^{-mt})+A\exp\{-By+\nu B^2 t-VBt-mt\},\quad (y>0) \tag{2.6}$$

$$u=g(t)\quad (y=0). \tag{2.7}$$

The steady state solution is obtained by taking limit of eqn. (2.6) as $t\to\infty$.

$$u=\exp\left(-\frac{Vy}{2\nu}\right)\int_0^\infty g(t-T)\frac{y}{\sqrt{4\pi\nu T^3}}\exp\left\{-\frac{y^2}{4\nu T}-\frac{V^2 T}{4\nu}-mT\right\}.dT$$

$$-\frac{C}{\rho m}\exp\left(-\frac{Vy}{2\nu}\right).\exp\left\{-\frac{y}{\sqrt{\nu}}\left(\frac{V^2}{4\nu}+m\right)^{1/2}\right\}+\frac{C}{\rho m} \tag{2.8}$$

3. Discussion

We find that solution (2.8) is valid for both $y\geqslant 0$. However, this solution is derived from solution (2.6) which is only valid for $y>0$. This is due to discontinuity in the flow at $y=0$ since the start of motion.

From the solution (2.6) we note that velocity field depends on the initial distribution of velocity, motion of plate and on the pressure gradient, whereas the steady state solution does not depend on the initial distribution of velocity but on plate motion and pressure gradient. To see the effect of suction and magnetic field on the velocity profile we take the plate to be uniformly accelerated, *i.e.*, $g(t)=at$. By giving the values to constants, A, B, C, and a, as unity ($=1$) and taking $\rho=1$ (*e.g.*, water) the solution for velocity profile, eqns. (2.6) and (2.7) become

$$u=\frac{1}{2}\left[t+\frac{\eta}{2\times\sqrt{\frac{V_1^2}{4}+m}}-\frac{1}{m}\right]\times\exp\left(-\frac{V_1\eta}{2}\right)$$

$$\times\left[\exp\left\{-\eta\left(\frac{V_1^2}{4}+m\right)^{1/2}\right\} erfc\left\{\frac{\eta}{2\sqrt{t}}-\sqrt{\left(\frac{V_1^2}{4}+m\right)t}\right\}\right.$$

$$\left.+\exp\left\{\eta\left(\frac{V_1^2}{4}+m\right)^{1/2}\right\} erfc\left\{\frac{\eta}{2\sqrt{t}}+\sqrt{\left(\frac{V_1^2}{4}+m\right)t}\right\}\right]$$

$$-\frac{1}{2}\exp\left\{-\frac{V_1\eta}{2}+t-V_1t-mt\right\}\left[\exp\left\{\eta\left(1-\frac{V_1}{2}\right)\right\}\right.$$

$$erfc\left\{\frac{\eta}{2\sqrt{t}} + \left(1 - \frac{V_1}{2}\right)\sqrt{t}\right\} + \exp\left\{-\eta\left(1 - \frac{V_1}{2}\right)\right\}$$

$$\times\ erfc\left\{\frac{\eta}{2\sqrt{t}} - \left(1 - \frac{V_1}{2}\right)\sqrt{t}\right\}\Bigg] + \frac{1}{m}(1 - e^{-mt}) + \frac{1}{2m}\exp(-mt)$$

$$\times\left[\exp(-V_1\eta)\, erfc\left\{\frac{\eta}{2\sqrt{t}} - \frac{V_1\sqrt{t}}{2}\right\} + erfc\left\{\frac{\eta}{2\sqrt{t}} + \frac{V_1\sqrt{t}}{2}\right\}\right]$$

$$+ \exp(-\eta + t - V_1 t - mt) \qquad (\eta > 0) \tag{3.1}$$

$$u = t \qquad (\eta = 0) \tag{3.2}$$

where

$$\eta = \frac{y}{\sqrt{\nu}}\ V_1 = \frac{V}{\sqrt{\nu}} \text{ and } B_1 = \sqrt{\nu}\, B = 1.$$

We plot the velocity profiles using eqns. (3.1) and (3.2). Figure 1 (*a*) and (*b*) show the velocity profiles for $t = 0.5$ and $t = 1$ respectively.

We find from the figures that u is just t as given by eqn. (3.2) for $\eta = 0$. And for increasing value of η the velocity decreases and for large value of η, u attains a steady value determined by the magnetic field parameter m. For higher value of time t, the steady value is attained quickly compared with lower value of time t. With increasing value of suction the value of u decreases before it attains steady state value. The effect of magnetic field is more prominent; it decreases the velocity field and the

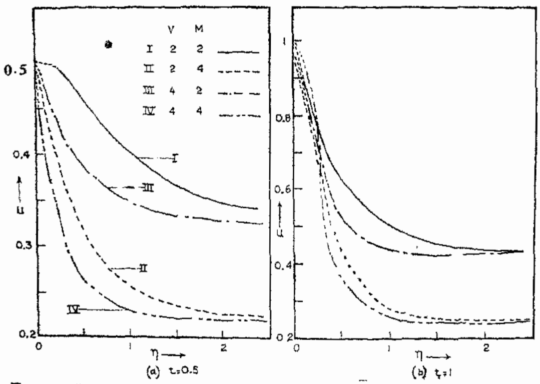

FIG. 1. Velocity Profiles.

decrease in the value of the velocity at a point is more for higher magnetic field for the same difference in the value of magnetic field strength. This is also true for higher value of time t.

4. Special cases

(*a*) Solution for ordinary hydrodynamic flow $(m = 0)$:

If

$$m = \frac{\sigma}{\rho} \mu_e^2 \mathrm{H}_0^2 = 0,$$

then eqn. (2.6) becomes

$$\begin{aligned} u = {} & \exp\left\{-\frac{Vy}{2\nu}\right\} \int_0^t g(t-T) \frac{y}{\sqrt{4\pi\nu T^3}} \exp\left\{-\frac{y^2}{4\nu T} - \frac{V^2 T}{4\nu}\right\} dT \\ & - \frac{A}{2} \exp\left\{-\frac{Vy}{2\nu} + \nu B^2 t - VBt\right\} \left[\exp\left\{y\left(B - \frac{V}{2\nu}\right)\right\} erfc\left\{\frac{y}{\sqrt{4\nu t}} \right. \right. \\ & \left. + \left(B - \frac{V}{2\nu}\right)\sqrt{\nu t}\right\} + \exp\left\{-y\left(B - \frac{V}{2\nu}\right)\right\} erfc\left\{\frac{y}{\sqrt{4\nu t}}\right. \\ & \left.\left. - \left(B - \frac{V}{2\nu}\right)\sqrt{\nu t}\right\}\right] - \frac{C}{2\rho}\left[\frac{y}{V} + t\right] . \exp\left\{\frac{yV}{2\nu}\right\} . erfc\left\{\frac{y}{\sqrt{4\nu t}} + \frac{V\sqrt{t}}{\sqrt{4\nu}}\right\} \\ & + \frac{C}{2\rho}\left[\frac{y}{V} - t\right] \exp\left\{-\frac{yV}{2\nu}\right\} erfc\left\{\frac{y}{\sqrt{4\nu t}} - \frac{V\sqrt{t}}{\sqrt{4\nu}}\right\} \\ & - A\exp\{-By + \nu B^2 t - VBt\} + \frac{Ct}{\rho}, (y > 0). \end{aligned} \tag{4.1}$$

In the absence of pressure gradient this corresponds to the solution for hydrodynamic flow given by Srivastava[5].

(*b*) An infinite porous flat plate moving in non-conducting fluid with time dependent velocity $U(t)$ with uniform suction V on the fluid at rest.

The solution for this problem is obtained by putting $g(t) = U(t), m = 0, A = 0, P = 0$,

$$u = \exp\left\{-\frac{Vy}{2\nu}\right\} \int_0^t u(t-T) \frac{y}{\sqrt{4\pi\nu T^3}} \exp\left\{-\frac{y^2}{4\nu T} - \frac{V^2 T}{4\nu}\right\} . dT (y > 0) \tag{4.2}$$

This corresponds to the expression given by Hasimoto[6], (*c*) An infinite porous flat plate oscillating (linear harmonic) parallel to itself with velocity $U \cos nt$ with uniform suction V in the fluid at rest.

The solution for this problem for large times is obtained by putting $g(t) = U \cos nt$ $A = 0$, $m = 0$, $p = 0$,

$$u = \frac{2U}{\sqrt{\pi}} \exp\left\{-\frac{Vy}{2\nu}\right\} \int_0^\infty \cos\left\{n\left(t - \frac{y^2}{4\nu\eta^2}\right)\right\} \exp\left\{-\eta^2 - \frac{V^2}{16\eta^2\nu^2}\right\} . d\eta$$

with $\eta = y/\sqrt{4\nu T}$. (4.3)

This solution can be compared to the solution obtained by Srivastava and Lal[7], namely,

$$u = U_0 \exp\left\{\left[\frac{V_0}{2\nu_0} - \left(r^{1/2} \sin\frac{\alpha}{2}\right)\right] . y\right\} \cos\left\{\left(r^{1/2} \cos\frac{\alpha}{2}\right) y - nt\right)\right\}$$

where

$$r = \left[\left(\frac{n}{\nu_0}\right)^2 + \left(\frac{V_0}{4\nu_0}\right)^2\right]^{1/2}, \; \alpha = \pi - \tan^{-1}\left(\frac{4\nu_0 n}{V_0}\right).$$

(*d*) Stokes first problem

The classical Stokes first problem can be obtained by putting, $g(t) = U$, $A = 0$, $m = 0$, $V = 0$, and $p = 0$.

which is the same as Schlictching[8] solution (Page 72, eqn. 5.22).

(*e*) Stokes second problem

Solution for Stokes second problem can be obtained with $g(t) = U \cos nt$, $A = 0$, $m = 0$, $V = 0$, $p = 0$, for large times, we have from (3.4),

$$u = \frac{2U}{\sqrt{\pi}} \int_0^\infty \cos\left\{n\left(t - \frac{y^2}{4\nu\eta^2}\right)\right\} \exp(-\eta^2) . d\eta \tag{4.4}$$

which is equivalent to Schlichting[8] solution, (Page 75, eqn. 5.26).

5. Conclusions

(1) There is discontinuity in the flow at $y = 0$, since the start of motion.

(2) The velocity decreases with increase in the magnetic field strength and this decrease is more with higher value of time t.

(3) The increase in the value of suction decreases the transient velocity profile.

(4) The solution obtained is generalisation of several earlier works such as Stokes problems,

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