J. Indian Inst. Sci., 65 (A), Jan. 1984, pp. 1-10

Steady salt water intrusion into coastal confined aquifer with inclined outflow face

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Received on November 24, 1983

Abstract

The interface that exists between fresh and salt waters in a confined coastal aquifer with an inclined outflow face is analysed. The outflow face behaves in a way analogous to that of a surface of seepage. The *a priori* unknown interface is modified through an iterative process which uses constrained least square fit. The method of finite differences is employed for the solution. The influence of changes in the driving head and inclination of the outflow face on the interface configuration and the quantity of fresh water flow are studied.

Key words: Aquifer, permeability, interface, intrusion.

1. Introduction

When a fresh water aquifer discharges into the sea, the sea water, owing to its comparatively higher density, penetrates the aquifer to some extent forming a salt water wedge underlying the lighter fresh water. The zone of contact between fresh water and salt water takes the form of a transition zone caused by hydrodynamic dispersion. When the transition zone is relatively narrow, a sharp boundary known as the interface is introduced as an approximation, instead of the transition zone. Fresh water flows above this interface towards the sea to maintain dynamic equilibrium.

The phenomenon of salt water intrusion into a fresh water aquifer is analogous to the flow through an earthen embankment [1,2]. The problem of interface has been dealt with previously by analytical and numerical methods. In the analytical approach, the studies using the Dupuit-Forchheimer approximation, Darcy's law, potential flow theory and conformal mapping coupled with hodograph transformation and inversion techniques have resulted in solutions of interface location for a variety of relatively idealised aquifer conditions [3-7]. The numerical methods based on finite difference and finite element techniques have been successfully used for interface problems involving complex boundaries and anisotropy and nonhomogeneity of the aquifer [8-12].

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In most of the investigations, however, the outflow face is assumed to be either vertical or horizontal which is not always true. One can find that most of the coastal aquifers end with an inclined outflow face. Bear and Kaupler[12] have studied a general case but the flow is assumed to be one-dimensional, in their analysis. In the present investigation, a fresh water aquifer with an inclined outcropping surface is considered. The outcropping surface is analogous to a surface of seepage and the analysis takes into account the influence of this on seepage characteristics and on the location of the interface. Further, the flow is considered to be *two-dimensional*. A constrained curve fitting technique based on least squares approach is employed to fix the interface between salt and fresh waters.

2. Formulation of problem

Figure 1 shows the physical situation. *ACD'F* is a confined artesian aquifer carrying fresh water towards the sea. The *aquifer ends with an inclined outcropping surface CD'*. Due to its higher density, the sea water pushes beneath the fresh water forming the wedge *EDD'*. Fresh water flows towards the sea through the outflow face *CD*. It is required to determine the quantity of fresh water seeping towards the sea and the distribution of the fluid pressure throughout the soil as also to locate the position of the interface *DE*.

Assumptions

The porous medium is fully saturated. The fresh water flow is steady, two-dimensional and is governed by Darcy's law. The underlying salt water is assumed to be stationary[13]. The fluids are assumed to be immiscible and hence the interface is a sharply defined line rather than a band of dispersion.

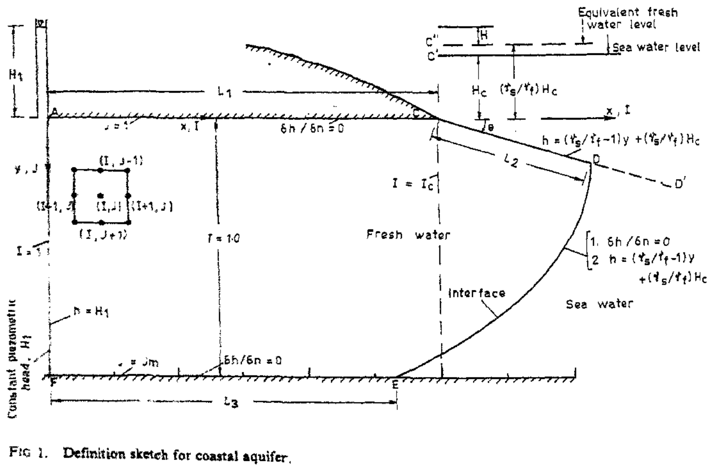

FIG 1. **Definition sketch for coastal aquifer.**

Governing equation and boundary conditions

Referring to fig. 1, the total head at any point in the flow domain is given as

$$h = \frac{p}{\gamma_f} - y \tag{1}$$

where y is measured downward from AC, p is the pressure and γ_f is the specific weight of fresh water.

The differential equation for steady flow in the homogeneous, isotropic region is

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} = 0 \tag{2}$$

A constant fresh water head, H_1, is assumed to act across the vertical face AF. AC and FD' are the impervious boundaries which confine the flow and hence $\partial h/\partial n = O$ along them.

The outcropping face CD across which the fresh water discharges into the sea is analogous to a surface of seepage. At any point on this boundary, the pressure on both the fresh water and salt water sides should be the same. Using this condition, the boundary condition along CD is obtained as

$$h = y\left(\frac{\gamma_s}{\gamma_f} - 1\right) + \frac{\gamma_s}{\gamma_f}H_c \tag{3}$$

where γ_s is the specific weight of salt water and H_c is the depth of salt water above AC (fig. 1).

ED, the interface whose location is initially unknown, is a streamline; the salt water body below it being stationary and the fresh water above it flowing. In addition to the condition for continuity of pressure (*i.e.*, eqn. (3)), the no flow condition across the interface (*i.e.*, $\partial h/\partial n = O$) should also be satisfied along ED. A solution for the flow field is first obtained satisfying the no flow condition (*i.e.*, $\partial h/\partial n = O$) across the interface. The interface is then modified so as to satisfy the second boundary condition, namely, eqn. (3).

In the present study, the improved position of the interface is obtained by a method of constrained curve fitting using the least square approach. The constraint corresponds to the condition that the interface meets the outflow face vertically [14,15]

The finite difference scheme

The finite difference formulation is shown schematically in fig. 1. A suitable finite difference grid covers the entire flow region; for convenience, a square grid system is being used here. However, near the seepage surface CD and the interface DE, partial grids occur. Though zero heads may be assumed at all grid points to commence the iteration, a linear head distribution based on the boundary condition is used to reduce the computational time. The finite difference form of eqn. (2) is applied at each grid point along with the finite difference form of the boundary conditions. In the first cycle of computation, only the no-flow condition across the interface is applied and the solution of Laplace equation is obtained.

Each interface nodal point is now shifted vertically up or down so as to satisfy the second boundary condition (eqn. (3)). As a result, the points on the interface may get scattered (fig. 2) and the curve fitting technique is used to obtain a mean representative curve for the new location of the interface. The process is repeated with the new location of the interface until a specified accuracy is reached. The flow field is now considered solved for the prescribed physical conditions. For the corresponding position of the interface, the necessary quantities of engineering interest such as the quantity of seepage and pore water pressure are obtained.

The *SOR* method of iteration with ω (relaxation factor) as 1.6 is used for the solution of the simultaneous equations. Since the number of trials required in the inner iteration (solution of Laplace equation with the Neumann boundary condition for the interface) reduces considerably after the third or fourth cycle, the *SOR* method requiring minimum computation for one iteration is well suited for the present problem.

3. Curve fitting technique

The constrained curve fitting technique with least squares approach, which is employed in the present studies, is described here. It is assumed that the interface may be represented by a quadratic curve with sufficient accuracy, based on consideration of available solutions for simpler cases.

Referring to fig. 2 ·

$$x_o = \frac{y_o}{m} + L_1 \tag{4}$$

where m is the slope of line CD', x_0, y_0 are coordinates of point D (*i.e.*, the starting point of interface on line CD') and L_1 is the length of AC. A quadratic curve passing through the point D has the form

$$x = Ay^2 + By + C \tag{5}$$

in which A, B, C are arbitrary constants. At point D,

$$x_o = Ay_o^2 + By_o + C \tag{6}$$

From eqns. (4) and (6),

$$y_0 = \frac{1/m - B}{2A} \pm \frac{[(B-1/m)^2 - 4A(C-L_1)]^{1/2}}{2A} \tag{7}$$

The negative sign in eqn. (7) is admissible. The interface has to be tangential to the vertical at point D (fig. 1). In order that the quadratic curve which represents the interface is vertical at point D, from eqn. (5),

$$\frac{dx}{dy}\Big|_{x_o, y_o} = 2Ay_o + B = 0 \tag{8}$$

$$\text{or } y_0 = -B/2A \tag{9}$$

From eqns. (7) and (9), using the negative sign in eqn. (7),

$$B = 1/m - [1/m^2 + 4A(C - L_1)]^{1/2} \tag{10}$$

It is evident that $B = f_1 [A, C]$ where f_1 denotes functional relation given by eqn. (10).

Differentiating eqn. (10) partially, with respect to A and C, one gets

$$\frac{\partial B}{\partial A} = -\frac{2(C - L_1)}{[1/m^2 + 4A(C - L_1)]^{1/2}} \tag{11}$$

and

$$\frac{\partial B}{\partial C} = -\frac{2A}{[1/m^2 + 4A(C - L_1)]^{1/2}} \tag{12}$$

For a given set of n points (x_i, y_i ; $i = 1$ to n), let

$$S = \sum_{i=1}^{i=n} [f(y_i) - x_i]^2 \tag{13}$$

in which

$$f(y_i) = Ay_i^2 + By_i + C ;$$
$$i = 1, 2, \ldots, n. \tag{14}$$

The least square fit of a quadratic curve for the given set of n points requires that (as $B = f$ $[A, C]$),

$$\frac{\partial S}{\partial A} = 0 \tag{15}$$

$$\frac{\partial S}{\partial C} = 0 \tag{16}$$

From eqns. (13) through (16) we obtain the following normal equations.

$$S_1 \cdot \frac{\partial B}{\partial A} + S_2 = 0 \tag{17}$$

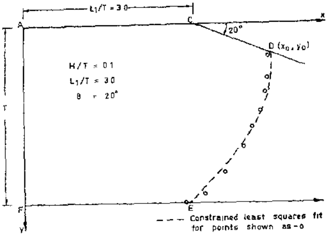

FIG 2. Constrained least square curve fitting.

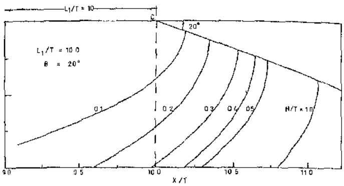

FIG 3. Effect of H/T on interface for $L_1/T = 10.0$ and $\theta = 20°$

and

$$S_1 \cdot \frac{\partial B}{\partial C} + S_0 = 0 \tag{18}$$

in which

$$S_0 = \sum_{i=1}^{i=n} [f(y_i) - x_i] \tag{19}$$

$$S_1 = \sum_{i=1}^{i=n} y_i [f(y_i) - x_i] \tag{20}$$

and

$$S_2 = \sum_{i=1}^{i=n} y_i^2 [f(y_i) - x_i] \tag{21}$$

Equations (11) and (12) give the expressions for $\partial B/\partial A$ and $\partial B/\partial C$.

Since eqns. (17) and (18) are implicit in nature because of eqns. (11) and (12), Newton-Raphson technique is used to solve them simultaneously to obtain A and C. B is then obtained from eqn. (10). With these values of A, B and C, eqn. (5) gives the interface location to be used in the next iteration.

4. Results and discussion

In order to obtain a general soultion, a driving head H is used across AF (fig. 1), where H is the head above the equivalent fresh water surface (passing through C'' in fig. 1) corresponding to the given sea water surface (passing through C' in fig. 1). The solutions thus obtained hold good for any H_c, the depth of sea water above point C (fig. 1), if H is obtained as

$$H = H_1 - \frac{\gamma_s}{\gamma_f} H_c \tag{22}$$

where H_1 is the constant piezometric head across AF (with respect to AC as datum). As the intersection of the sea water level with outflow face is known, and as the numerical solution gives the location of the toe of the interface (point E in fig. 1), the distance between the coast and the toe can be obtained.

The influence of H/T on the shape and position of the interface may be studied from fig. 3. The curves show the shape and position of the interface corresponding to different H/T values, for given $L_1/T = 10.0$ and for the same inclination of the outflow face ($\theta = 20°$). As may be expected, the increase in H/T pushes the interface further towards the sea. The toe of the interface (point E, fig. 1), shifts much more than point D, for an increase of H/T, at low H/T values. Thus, as H/T increases, the slope of the interface at its toe becomes steeper. The amount of the shift in toe with change in H/T reduces with increasing H/T. For $H/T \leq 0.3$, the sea water intrudes to the left of point C (see fig. 1) for the given L_1/T and θ.

A decrease in L_1/T pushes the interface towards the sea, while an increase in L_1/T increases intrusion of sea water due to the shifting of the toe point towards upstream side (fig. 4). This shifting increases with increasing L_1/T for a proportionate change in L_1/T. For example, the toe point shifts upstream by a distance of 0.35 (in terms of T) when L_1/T changes from 1.0 to 2.0, whereas it shifts upstream by 0.46 when L_1/T changes from 2.0 to 4.0 and, by 0.68 for 4.0 to 8.0. For $L_1/T \geq 3.0$, salt water intrusion occurs upstream of point C for the given H/T and θ.

Figure 5 presents results for $H/T = 0.5$ and $\theta = 20°$. It may be seen that for this increased value of H/T, the salt water intrusion extends upstream of C only for $L_1/T > 15$.

Figure 6 shows the effect of variation in θ on the interface while H/T and L_1/T remain unaltered. Increasing θ shifts the interface upstream and the shape of the interface is also affected; the interface becoming steeper with increase in θ. However, the toe of the interface does not shift very significantly for a moderate change in θ. As may be observed from fig. 6, the influence of θ is more pronounced for smaller values of θ and in this particular case, for $\theta > 10°$, the effect of θ on the interface position is negligible. Similar trends were observed for other L_1/T and H/T values also. The results of Henry[3] for the extreme cases of horizontal and vertical outflow faces are in conformity with these trends.

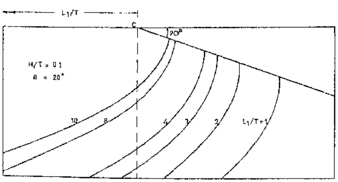

FIG 4. Effect of L_1/T on interface for $H/T = 0.1$ and $\theta = 20°$.

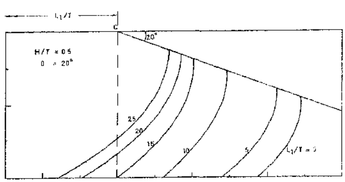

FIG 5. Effect of L_1/T on interface for $H/T = 0.5$ and $\theta = 20°$.

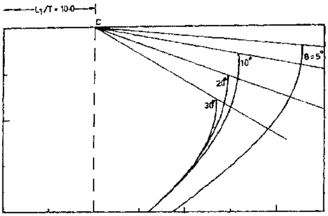

FIG 6. Effect of θ on interface for $H/T = 0.5$ and $L_1/T = 10.0$.

The dependence of the position of the toe point on L_1/T and H/T is shown in fig. 7. Obviously, the positive values of L_3/T-L_1/T indicate that the toe is on the downstream side of point C (*i.e.*, towards sea) and negative values indicate that the toe point is on the upstream of point C (more intrusion of sea water). The possibility of sea water intruding beyond point C (towards AF), is more for smaller H/T than larger ones. For small H/T values, a small increase in L_1/T considerably shifts the toe upstream.

The curves in fig. 8 show the dependence of L_2/T on L_1/T and H/T for a given $\theta = 20°$. L_2/T reduces with increase in L_1/T for all H/T values, and the influence of L_1/T goes on

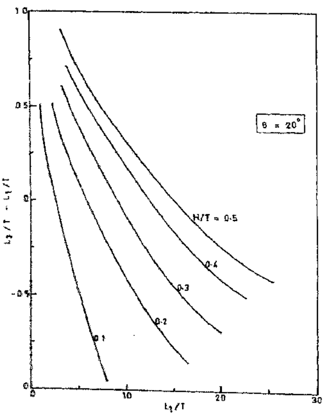

FIG 7. Variation of $(L_3 - L_1)/T$ with L_1/T and H/T for $\theta = 20°$.

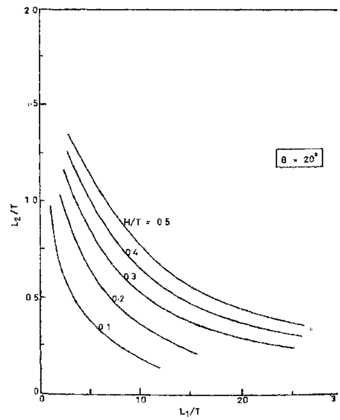

FIG 8. Variation of L_2/T with L_1/T and H/T for $\theta = 20°$.

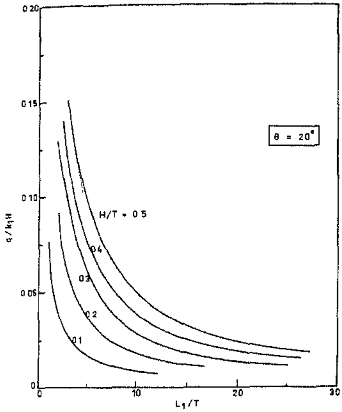

FIG 9. Variation of q/k_1H with L_1/T and H/T for $\theta = 20°$

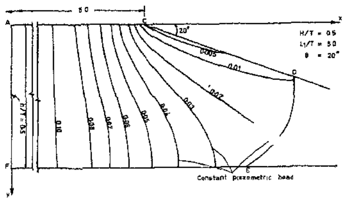

FIG 10. Distribution of piezometric head for $H/T = 0.5$ and $L_1/T = 5.0$.

reducing for larger L_1/T values. On the other hand, L_2/T increases with increase in H/T and the effect of H/T on L_2/T is more pronounced for smaller H/T values. For evidence, L_2/T increases by 100% when H/T increases from 0.1 to 0.2 for given $L_1/T = 10.0$, whereas it increases by 80% when H/T increases from 0.2 to 0.4.

The variation in seepage flow of fresh water with L_1/T and H/T is shown in fig. 9. For a given H/T, as L_1/T increases, q/k_1H (q = discharge, h = hydraulic conductivity) decreases rapidly for smaller L_1/T, and later, for larger L_1/T its influence on q/k_1H considerably reduces. The effect of H/T on q/k_1H is studied by keeping L_1/T constant. It may be observed that with increase in H/T, q/k_1H increases and in the entire range of H/T from 0.1 to 0.5 its influence is quite significant.

Distribution of total head, h/T, for one particular case of $H/T = 0.5$, $\theta = 20°$ and $L_1/T = 5$ is presented in fig. 10. The pore water pressure, p, at any point in the fresh water flow region, corresponding to a given value of H_c, may be obtained by replacing h by $h + (\gamma_s/\gamma_f)\, H_c$ in eqn. (1). In many analyses, one dimensional approach is commonly used in studying the salt water interface problem. It is seen from fig. 10 that the effect of two-dimensionality is significant for some region upstream of the interface, extending up to about $0.5T$ from point C.

5. Conclusions

An aquifer with an inclined outcropping surface discharging fresh water towards sea is analysed, taking into account two dimensional flow and the influence of outflow face behaving analogous to a surface of seepage. The effect of H/T, θ and L_1/T on the shape and position of the interface and on the quantity of seepage has been studied.

An increase in H/T pushes the interface towards the sea and thus reduces the intrusion of salt water into the aquifer. Also, with increase in H/T the interface at the toe becomes steeper. The effect of increasing L_1/T on the interface is similar to that of decreasing H/T. The influence of θ is more pronounced for small values of θ only. An increase in θ shifts the interface upstream, making it steeper.

For given H/T, q/k_1H decreases rapidly with increase of L_1/T for smaller L_1/T, and later, for larger L_1/T its influence considerably reduces. For given L_1/T, q/k_1H increases with increase in H/T. The influence of θ on q/k_1H is of significance only for small values of θ.

The effect of two-dimensionality is significant for some region upstream of the interface, and may extend over a length of $0.5T$ or more (depending on H/T and L_1/T), from point C.

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