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Non-linear free vibrations of traiangular plates at elevated temperature

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Abstract

In this paper, non-linear free vibrations of triangular plates at elevated temperature have been analysed using Berger's approximation Criterion for critical buckling temperature has also been deduced as a limiting case

Key words: Non-linear, amplitude, time-period, elevated temperature, thermal buckling, critical temperature

1. Introduction

In a recent publication, Jones *et al*[1], have studied the vibrations of both elastic and visco-elastic plates at elevated temperature. Although they have tried to establish the accuracy of Berger's method, Nowinski and Ohnabe[2] raised some points regarding its inapplicability for movable edges of a plate. Prathap[3] and Prathap and Varadan[4] also criticised the method regarding inaccuracies of Berger's method in certain cases. Yet it was claimed by Banerjee and Sarkar[5] that its applicability may be restricted to the cases of clamped square plates and circular plates with immovable edges, and to some extent, to the simply-supported circular and rectangular plates having smaller aspect ratios

The present paper deals with the dynamic behaviour of a right-angled isosceles triangular plate and an equilateral triangular plate at elevated temperature and having simply-supported boundary conditions with immovable edges. The analysis is based on Berger's method as the plates considered have immovable inplane edge conditions Moreover, Berger's equations are in decoupled form and have wide advantage in the analysis in comparison to the classical von Karman equations which are in the coupled forms and lead to mathematical complexities.

Some numerical computations for the variations of non-dimensional periods *vs* non-dimensional amplitudes and temperature parameters have been presented graphically. Criterion for thermal buckling has been established

2. Governing equations for heated plates

Berger's approximate quasi-linear uncoupled differential equations governing the motion of heated elastic plates are given by[6]

$$D \nabla^4 w + K^2 \nabla^2 w + \rho h \, w_{,tt} + \nabla^2 M_T/(1 - v) = 0 \tag{1}$$

$$N_T/(1 - v) - 12 \, D e_1 / h^2 = K^2 \tag{2}$$

where

$$e_1 = u_{,x} + v_{,y} + 1/2 \, w_{,x}{}^2 + 1/2 \, w_{,y}{}^2 \tag{3}$$

$$N_T = \alpha_t E \int_{-h/2}^{h/2} T(x,y,z) \, \mathrm{d}z, \qquad M_T = \alpha_t E \int_{-h/2}^{h/2} z \, T(x,y,z) \, \mathrm{d}z \tag{4}$$

and u, v, w are displacement components, α_t, co-efficient of thermal expansion, ρ, density per unit mass, v, Poisson's ratio, ∇^2, Laplacian operator, D, flexural rigidity, h, plate thickness, E, Young's modulus, $T(x,y,z)$, temperature distribution within the plate given by

$$T(x,y,z) = \tau_0 (x,y) + z \, \tau (\mathrm{x,y}) \tag{5}$$

in which $\tau_0(x,y)$ and $\tau(x,y)$ satisfy certain temperature distribution differential equations[7] and K^2 is independent of x and y but involves the time t

In the present analysis for free flexural vibrations of heated plates, equation (1) reduces to the form

$$D \nabla^4 w + K^2 \nabla^2 w + \rho h w_{,tt} = 0 \tag{6}$$

as $M_T = 0$.

3. Method of solution

3.1 *Right-angled isosceles triangular plate*

The origin of a simply-supported right-angled isosceles triangular plate is chosen at the vortex containing the right angle with the equal sides of length 'a' along the co-ordinate axes (fig. 1).

For such a plate the inplane and transverse boundary conditions are[8]

$$u = w = w_{,xx} = 0 \quad \text{at } x = 0$$
$$v = w = w_{,yy} = 0 \quad \text{at } y = 0$$
$$w = w_{,\eta\eta} = 0 \quad \text{at} \quad x + y = a \tag{7}$$

where

$$\partial/\partial\eta = 1/\sqrt{2}\,(\partial/\partial x + \partial/\partial y) \tag{8}$$

Compatible with the above boundary conditions u, v and w are chosen in the forms[8]

$$u = \sum_{k=1,3,\ldots}^{\infty} B_k \sin k\pi x/a\,(\cos k\pi y/a + \sin k\pi x/a - k\pi/4)\, H(t) \tag{9}$$

$$v = \sum_{k=1,3,\ldots}^{\infty} B_k \sin k\pi y/a\,(\cos k\pi x/a - \sin k\pi y/a + k\pi/4)\, G(t) \tag{10}$$

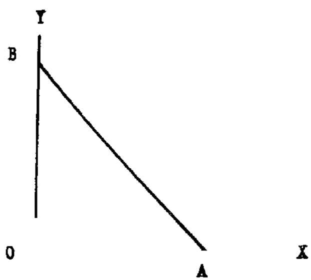

FIG 1 Right-angled isosceles triangular plate of sides $OA = OB = a$; equation of the side AB is $X + Y = a$

$$w = \sum_{m=1,3,}^{\infty} A_m (\sin 2m\pi x/a \sin m\pi y/a + \sin m\pi x/a \sin 2m\pi y/a)\; F(t) \quad (11)$$

Combining equations (6) and (11) one gets

$$\frac{25\, D m^4 \pi^4}{a^4} F(t) - 5\, m^2 \pi^2 K^2 F(t) / a^2 = -\rho h\, F_{,tt} \quad (12)$$

Integrating now equation (2) over the area of the plate and eliminating K^2 with the help of equation (12) one gets the non-linear time-differential equation as

$$F_{,tt}(t) + C_1 F(t) + C_2 F^3(t) = 0 \quad (13)$$

where

$$C_1 = 25\, m^4 \pi^4 D \left(1 - \frac{a^2 N_T^*}{5(1-\upsilon) D m^2 \pi^2}\right) / a^4 \rho h \quad (14)$$

$$C_2 = \frac{75\, m^2 \pi^4 D}{a^4 \rho h} \sum_{m=1,3,}^{\infty} m^2 (A_m/h)^2 \quad (15)$$

and

$$N_T^* = 1/A \iint N_T \,\mathrm{d}x\, \mathrm{d}y$$

is the mean value of N_T over the area A of the plate.

The solution of equation (13) with the initial conditions

$$F(0) = 1, \qquad \mathrm{d}F(0)/\mathrm{d}t = 0 \quad (16)$$

has been given by Nash and Modeer[9] in terms of Jacobian elliptic function of cosine type and obtained the ratio of the time-periods for linear and non-linear vibrations of elastic plates. In the present case such ratio is given by

$$T^*/T = \left(\frac{2\Theta}{\pi}\right) \cdot \left(1 + \frac{C_2}{C_1}\right)^{-1/2} \quad (17)$$

where

$$C_2 / C_1 = \frac{3 \sum_{m=1,3,}^{\infty} m^2 (A_m / h)^2}{m^2 [1 - a^2 N_T^* / 5(1-\upsilon) D m^2 \pi^2]} \tag{18}$$

and T and T^* denote the periods for linear and non-linear vibrations

For free fundamental mode of vibrations without thermal loading equation (17) reduces to the form

$$T^*/T = \frac{2\,\Theta}{\pi} \frac{1}{\sqrt{1 + 3\,(A/h)^2}} \tag{19}$$

as obtained by Banerjee[8]

3.2 Buckling criterion

For the pre-buckling state non-dimensional time-period T^*/T can be obtained from equation (17) by taking values of

$$a^2\, N_T^* / 5\, \pi^2\, (1 - \upsilon)\, D = \lambda \text{ (say)}$$

sufficiently near to unity Buckling occurs when $\lambda = 1$, and the critical buckling temperature $(N_T^*)_{cr}$ is obtained as

$$(N_T^*)_{cr} = 5\, \pi^2\, D\, (1 - \upsilon) / a^2$$

which is in agreeement with the result obtained by Banerjee[10]

3.3 Simply-supported equilateral triangular plate

Analysis of this section shall be carried out with the help of trilinear co-ordinates[11] Let ABC be an equilateral triangle of sides '$2a$'. The centroid O on the undeflected middle surface is taken as the origin and the x and y axes are taken perpendicular and parallel to the side BC. If p_1, p_2, p_3 be the lengths of perpendiculars from any point (x, y) within the triangle on the sides CA, AB and BC respectively and r, the radius of the inscribed circle (fig. 2), then

$$p_1 = r + x/2 - \sqrt{3}/2\, y, \quad p_2 = r + x/2 + \sqrt{3}/2\, y, \quad p_3 = r - x \tag{21}$$

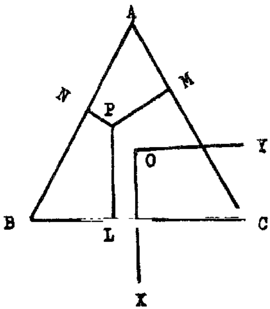

FIG 2 Equilateral triangular plate of sides $2a$

Hence $p_1 + p_2 + p_3 = 3r = \sqrt{3}\, a = k$ (say) (22)

Two-dimensional Laplacian operator shall be obtained as

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} = \frac{\partial^2}{\partial p_1^2} + \frac{\partial^2}{\partial p_2^2} + \frac{\partial^2}{\partial p_3^2} - \frac{\partial^2}{\partial p_2\, \partial p_3} - \frac{\partial^2}{\partial p_3\, \partial p_1} - \frac{\partial^2}{\partial p_1\, \partial p_2} \quad (23)$$

The transverse displacement w satisfying the simply-supported boundary conditions

$$w = \nabla^2 w = 0 \text{ at } p_1 = p_2 = p_3 = 0$$

is assumed in the form

$$w = \sum_{m=1,2,}^{\infty} A_m \left(\sin \frac{2m\,\pi\, p_1}{k} + \sin \frac{2m\,\pi\, p_2}{k} + \sin \frac{2m\,\pi\, p_3}{k} \right) F(t) \quad (24)$$

Also the following forms of u and v

$$u = \sum_{m=1}^{\infty} \sqrt{3}\, B_m \left[\sin \frac{2m\,\pi\,(p_2 + p_3)}{k} + \sin \frac{2m\,\pi\,(p_1 + p_3)}{k} \right] H(t) \quad (25)$$

$$v = \sum_{m=1}^{\infty} B_m \left[\sin \frac{2m\,\pi\,(p_1 + p_3)}{k} - \sin \frac{2m\,\pi\,(p_2 + p_3)}{k} \right] G(t) \tag{26}$$

can be chosen in conformity with the boundary conditions

$$\begin{aligned} u &= 0 \quad \text{at} \quad p_3 = 0 \\ \sqrt{3}\,v + u &= 0 \quad \text{at} \quad p_2 = 0 \\ \sqrt{3}\,v - u &= 0 \quad \text{at} \quad p_1 = 0 \end{aligned} \tag{27}$$

Proceeding in the same way as laid down in the preceding section one arrives at the same type of differential equation (13) where

$$C_1 = \frac{D}{a^4\,\rho h}\;\frac{16\,m^4\,\pi^4}{9\,a^4} \left[1 - \frac{3\,a^2 N_T^*}{4\,(1-v)\,Dm^2\,\pi^2} \right] \tag{28}$$

$$C_2 = \frac{D}{a^4\,\rho h}\;\frac{16\,m^2\,\pi^4}{a^4} \sum_{m=1}^{\infty} m^2\,(A_m / h)^2 \tag{29}$$

Non-dimensional time-periods T^* / T is given by the same equation (17) where C_1 and C_2 are to be replaced by equations (28) and (29)

For free fundamental mode of vibrations without thermal loading one gets

$$T^* / T = \frac{2\,\Theta}{\pi}\,\frac{1}{\sqrt{1 + 9\,(A/h)^2}} \tag{30}$$

as obtained by Karmakar[12]

As in the previous case critical buckling temperature $(N_T^*)_{cr}$ is obtained in the form

$$(N_T^*)_{cr} = \frac{4\,(1-v)\,D\,\pi^2}{3\,a^2} \tag{31}$$

as obtained by Datta[13].

4. *Numerical results and discussion*

Figure 3 shows the variations of non-dimensional time-periods T^* / T for different values of non-dimensional amplitudes A_1 / h and temperature parameter λ. It is seen that the effect of

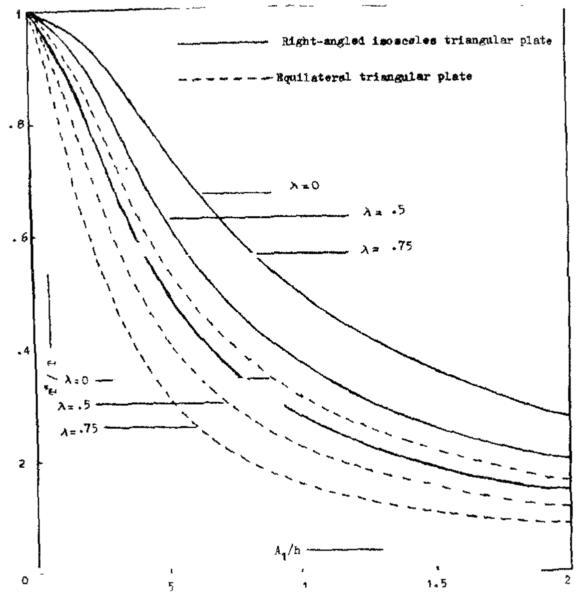

FIG 3 Variations of non-dimensional time-periods T^*/T vs non-dimensional amplitudes A_1/h for the fundamental mode of vibrations ($m = 1$) for different values of temperature parameter λ

N_T^* is to diminish the non-dimensional time-periods Also the circular frequency is given by the expression $\omega_0 = \sqrt{C_1}$ and equations (14) and (31) show that the circular frequency in each case diminishes due to the presence of (N_T^*) It is seen from fig 3 that the non-dimensional time-periods are less for corresponding non-dimensional amplitudes in the cases of plates of more regular shapes As it should be, the non-linear behaviour of the plates due to elevated temperature obtained here, is similar in nature as that of plates subjected to in-plane forces given in Biswas[14]

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