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BOOK REVIEWS

Gabor Szegö Collected papers (Vols 1-3) edited by Richard Askey Birkhauser Verlag, Basel, Switzerland, 1982, S Fr 372

The appearance of the collected works of an eminent and prolific mathematician like Gabor Szegö emphasizes once again the inescapable fact that there is really no substitute for the authentic inspiration of an original genius With his scholarly comments on every major contribution of Szego highlighting the historical aspects, the present state of the art and possibilities for the future, Professor Askey has enriched the volumes immensely A study of these will repay the time spent in many ways 'For example, one can see an important subject develop One can marvel at the brilliant solution of a specfic problem'

The central theme of Szegö's work is Toeplitz matrices, orthogonal polynomials on the unit circle and their applications in the study of extremal problems in geometry and physics. With these topics he has constructed a spring board from which one may plunge into the deep sea of classical analysis

The subject of orthogonal polynomials with respect to a measure on the line has a history of nearly two centuries involving many famous figures like Jacobi, Laguerre, Legendre, Stieltjes, Chebyshev, Markov, Bernstein and so on But orthogonal polynomials on the unit circle is almost completely the creation of one man, Gabor Szego Entering the world of mathematical research with a spectacular result on the asymptotic behaviour of Toeplitz determinants he discovered the famous 'three term forward-backward recursion relations' for orthogonal polynomials on the circle, analysed their zeros and asymptotic properties and found a whole class of deep inequalities, all of which have a tremendous influence in contemporary analysis

For any complex valued integrable function f on the unit circle with Fourier coefficients

$$c_j = (2\pi)^{-1} \int_0^{2\pi} e^{-ij\theta} f(e^{i\theta})\, d\theta, j \in \mathbb{N} \tag{1}$$

the sequence $\{ T_n^f, n = 1,2, \}$ of Toeplitz matrices is given by

$$T_n^f = \begin{pmatrix} c_0 & c_{-1} & \cdots & c_{-n+1} \\ c_1 & c_0 & c_{-1} \cdots & c_{-n+2} \\ \cdots & \cdots & \cdots & \cdots \\ c_{n-1} & c_{n-2} & \cdots & c_0 \end{pmatrix} \tag{2}$$

If $f \geq 0$, T_n^f is positive (semi) definite for all n. Here are some of the remarkable results of Szego in this case

$$\lim_{n \to \infty} (\det T_n^f)^{1/n} = \exp (2\pi)^{-1} \int_0^{2\pi} \log f(e^{i\theta})\, d\theta \tag{3}$$

If, in addition, $df/d\theta$ exists as a Lipschitz's function

$$\lim_{n \to \infty} \frac{\det T_n^f}{G(f)^n} = \exp 4^{-1} \sum_{n=1}^{\infty} n \mid k_n \mid^2 \tag{4}$$

where $G(f)$ denotes the right hand side expression in (3) and

$$k_n = (2\pi)^{-1} \int_0^{2\pi} e^{-in\theta} \log f(e^{i\theta})\, d\theta$$

If $m = \text{ess inf}_\theta f(e^{i\theta})$, $M = \text{ess sup}_\theta f(e^{i\theta})$ and $m \leq \lambda_{n1} \leq \lambda_{n2} \leq \cdots \leq \lambda_{nn} \leq M$ is an enumeration of the eigenvalues of T_n^f in increasing order then for every bounded continuous function F on $[m, M]$

$$\lim_{n \to \infty} \frac{F(\lambda_{n1}) + F(\lambda_{n2}) + \ldots + F(\lambda_{nn})}{n} = (2\pi)^{-1} \int_0^{2\pi} F(f(e^{i\theta}))\, d\theta \tag{5}$$

In particular, the following special case holds: if $0 \leq \alpha < \beta < \infty$ and $N(n, \alpha, \beta)$ denotes the number of eigenvalues satisfying the condition $\alpha \leq \lambda_{nj} \leq \beta$ for each fixed n then

$$\lim_{n \to \infty} N(n, \alpha, \beta)/n = l/2\pi \tag{6}$$

where l denotes the Lebesgue measure of the set $[0, 2\pi] \cap \{\theta : \alpha \leq f(e^{i\theta}) \leq \beta\}$

Toeplitz matrices occur in unexpected places. If $\{X_n, n = 0, \pm 1, \pm 2, \ldots\}$ is a stationary time series (or stochastic process) with spectral density f on the unit circle then the covariance between X_n and X_m is c_{n-m}, c_j being defined by (1). If $\hat{X}_0$ is the best linear predictor of X_0 on the basis of $X_{-1}, X_{-2}, \ldots$ then the mean square error of prediction is given by

$$E \mid \hat{X}_0 - X_0 \mid^2 = \exp (2\pi)^{-1} \int_0^{2\pi} \log f(e^{i\theta})\, d\theta, \tag{7}$$

which is the right hand side of (3)! Suppose $\phi_n(z) = k_{n,n} z^n + k_{n,n-1} z^{n-1} + \ldots + k_{n0}$, $n = 0, 1, 2, \ldots$ is the sequence of normalised orthogonal polynomials with respect to the spectral density f so that

$$(2\pi)^{-1} \int_0^{2\pi} \phi_n (e^{i\theta}) \, \phi_m (e^{-i\theta}) f(-e^{i\theta}) \, d\theta = \delta_{mn}$$

for all m,n Szego 's recursion relations lead to an efficient algorithm for computing $\{\phi_n\}$ and predicting X_0 on the basis of $X_{-1}, \ldots, X_{-(n+1)}$ If $\hat{X}_0(n+1)$ denotes the best linear least squares predictor then

$$\hat{X}_0 (n+1) = \sum_{j=0}^{n} (k_{j,j}\, c_{j+1}, + k_{j,j-1}\, c_j + \quad + k_{j,o}\, c_1)(k_{j,j}\, X_{-(j+1)} + \quad + k_{j\,o}\, X_{-1}) \qquad (8)$$

This regression problem is essentially an inversion problem for the matrices T_n^f Normally the number of computations involved in inverting an $n \times n$ matrix is $O(n^3)$ but the recursion relations of Szego reduce the number of computations to $O(n^2)$. This idea has led to many technological achievements as described nicely in the contributed essay of T Kailath

As a second example, consider the Ising model of a magnet where at each site of a square lattice there is a 'spin variable' $\sigma_{j,k}$ taking only two values ± 1 For each configuration σ there is an interaction energy

$$H = - E_1 \sum_{j,k} \sigma_{j,k}\, \sigma_{j,k+1} - E_2 \sum_{j,k} \sigma_{j\,k}\, \sigma_{j+1,\,k}$$

where the sum is over all sites of an $M \times N$ lattice Let

$$Z = \sum_{\{\sigma\}} e^{-H/kT},$$

$$< \sigma_{00}\, \sigma_{NN} > = Z^{-1} \sum_{\{\sigma\}} \sigma_{00}\, \sigma_{NN}\, e^{-H/kT}$$

where the summations are over all configurations Z is called the 'partition, function', k is the Boltzmann constant and T is the temperature The quantity $< \sigma_{00}\, \sigma_{NN} >$ is called the second order spin correlation Then

$$< \sigma_{00}\, \sigma_{NN} > = \det T_N^g \qquad (9)$$

where T_N^g is the Toeplitz matrix of order N defined by (2) and the weight g is the complex valued function

$$g(e^{i\theta}) = \left(\frac{1 - \alpha \bar{e}^{i\theta}}{1 - \alpha e^{i\theta}} \right)^{1/2} \qquad (10)$$

with

$$\alpha = (\sinh 2E_1/kT \sinh 2E_2/kT)^{-1}$$

In 1944 the Norwegian physicist Lars Onsager, in one of the most remarkable physics papers ever published, calculated the partition function Z and evaluated the limit

$$F = -kT \lim_{\substack{M \to \infty \\ N \to \infty}} (MN)^{-1} \log Z$$

and established the relation (9). For physical reasons he became interested in the asymptotic behaviour of (9) as $N \to \infty$. He posed this problem to S Kakutani who transmitted the same to Szegö. This led Szegö to prove (4) but unfortunately the weight function g in (10) is complex valued. This was settled later by Baxter and Reich For a fascinating account of the history behind this problem along with references leading to other surprising examples of Toeplitz's determinants arising in quantum chromodynamics, the study of Yang-Mills equations and algebraic geometry we refer to the contributed essay by Barry Mc Coy in Vol. 1 If $f \geq 0$ is an integrable function in the interval $[\alpha, \beta] \subset [0, \infty]$ and

$$d_k = \int_{\alpha}^{\beta} t^k f(t)\, dt, \quad k = 0,1,2,..$$

then the sequence $\{ H^f_n, = 1,2, .\}$ of Hankel matrices is defined by

$$H^f_{n+1} = \begin{pmatrix} d_0 & d_1 & . & d_n \\ d_1 & d_2 & .. & d_{n+1} \\ .. & .. & .. & .. \\ d_n & d_{n+1} & .. & d_{2n} \end{pmatrix}$$

Properties analogous to (3), (5) and (6) hold for these matrices.

As one goes through the three volumes in leisure many striking results either due to Szegö himself, or those which arose from his work in subsequent developments, or those which influenced his thinking meet the eyes of the reader. To understand them one has to live with these volumes for atleast a decade However, we present a few glimpses

Theorem (Szegö) A power series with finitely many different coefficients is either a rational function or has the unit circle as a natural boundary.

Theorem (Szegö) For all closed surfaces with a given diameter the capacity is a maximum for the sphere.

Theorem (L. de Branges) If $f(z) = z + a_2 z^2 + a_3 z^3 + ..$ is univalent in the unit disc then $|a_n| \leq 3_n$ for all $n \geq 2$

Theorem [*S Bernstein*] *If f is a trigonometric polynomial of degree n then*

$$\|f'\| \leq An \|f\|$$

where A depends only on the periodicity of f and $\|\ \|$ denotes the sup norm

Theorem (L Fejer) Let f be a continuous point function on the unit sphere and

$$f \sim \sum_{n=0}^{\infty} Y_n$$

its Laplace series, that is, Y_n is a surface harmonic of degree n. Then the Cesaro means of the second order display the same regular behaviour as the Cesaro means of the first order do for the Fourier series

Theorem (Saff and Varga) For the partial sums of e^z there are no zeros in the parabolic region $x > -1$, $y^2 \leq 4\ (x + 1)$.

As late as 1961, Szegö and Turan have interesting new things to say about the nature of convergence of Riemann sums to Riemann integrals and thereby exert an influence on the subject of numerical integration Echoing the sentiments expressed by Hermann Weyl we can only conclude how fruit bearing is the orchard of classical analysis in comparison with the flower gardens of algebra and topology. Needless to say that these volumes are a must for any mathematics library

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L.V. Ahlfors: Collected papers (Vols I and II), Birkhauser, Basel, Switzerland, 1982, pp Vol. I - 520, Vol. II - 515, S. Fr 288 (both the volumes).

These two volumes contain essentially all the papers of Lars V Ahlfors published up to 1979 (Ahlfors, at the age of 77, is still very active).

Ahlfors is known to the Indian mathematical public through his excellent introductory text-book on Complex Analysis; it may be less widely-known that Ahlfors is one of the most eminent mathematicians in his field—the theory of functions of one complex variable with all its ramifications.

The contributions of Ahlfors may be roughly classified under three headings (1) Conformal mappings and Value-distribution theory, (2) Riemann surfaces and Teichmuller theory, (3) Fuchsian and Kleinian groups. Ahlfors' work in each of these domains has been of basic importance. In what follows, I shall briefly discuss one or two of them from each domain.

Ahlfors' very first paper (1929) settled a famous conjecture of Denjoy on the number of asymptotic values of an entire function of finite order. In the subsequent ten years, Ahlfors greatly contributed to the clarification and consolidation of Nevanlinna theory which had at that time just been developed. His efforts in this direction culminated in his paper 'The theory of meromorphic curves' (1941), which may be regarded as a forerunner of the modern higher dimensional value-distribution theory of Stoll, Griffiths *et al*. To this period also belongs his short paper of 1938, in which, from a trivial looking observation, the now famous 'Ahlfors lemma', he derives many truly deep results. The lemma states: if $g(Z)\,|dZ|$ is a metric on the unit disc $|Z| < 1$ whose Gaussian curvature $-g(Z)^{-2}\,\Delta \log g(Z)$ is bounded above by -4, then $g(Z) \leq 1/1-|Z|^2$, *i.e.*, $g(Z)\,|dZ|$ is dominated by the Poincare metric on the disc. This lemma has been very influential in the exploitation of curvature conditions for studying the behaviour of holomorphic maps in arbitrary dimensions.

I turn now to Ahlfors' work on Riemann surfaces, which extended over the years 1945-63. Ahlfors was one of the first to place Teichmuller's brilliant ideas on a firm footing, and confirm Riemann's statement that the space of compact Riemann surfaces (of genus ≥ 2) depends on 3g - 3 complex parameters, by actually constructing a natural complex structure on the topological manifold of 'marked' Riemann surfaces produced by Teichmuller. To the subsequent work of consolidating and perfecting this theory, Ahlfors, together with Lipman Bers, must be regarded as one of the two major contributors. The rather short paper 'Quasi-conformal reflections' (1963) is of decisive importance in this context. But, apart from his work on the moduli problem, Ahlfors also produced during the 50's several important papers on various aspects of abelian integrals on open Riemann surfaces. In particular, the paper 'Open Riemann surfaces and extremal problems ...' (1950) contains a beautiful generalization of Schwarz's lemma to nice bounded domains of open Riemann surfaces.

The study of Kleinian groups, which has won the interest and active participation of leading topologists like Thurston and Sullivan, was initiated by Ahlfors' paper 'Finitely generated Kleinian groups' (1964) and has remained his major interest since then. A Kleinian group is a discrete group of fractional linear transformations (*i.e.* a discrete subgroup of SL (2, C)), which acts discontinuously on some non-empty open subset of the Riemann sphere. Actually this nomenclature goes back to Poincare, but nothing was known about general Kleinian groups till Ahlfors' paper appeared. The major problem occupying workers in this field is the settling of Ahlfors' hypothesis: the discontinuity set of a finitely generated Kleinian group (the smallest closed subset of the sphere stable under the group) has zero area. Special cases have been settled by Ahlfors himself, but the problem is (as far as I know) still open. Since a Kleinian group can also be regarded as a Fuchsian group acting on hyperbolic 3-space, connections with Thurston's work are opened up. Ahlfors himself has several papers on the subject of Kleinian (or Fuchsian) groups in hyperbolic space of higher dimensions.

After this brief survey of Ahlfors' work, I wish to say a few words about his style of writing. His papers are pleasant to read, since he resorts to computations only when geometrical arguments are not adequate by themselves to achieve his objective. For instance, he wrote several papers to show how Nevanlinna's theory gains in transparency and depth when interpreted (differential) geometrically. He writes briefly and elegantly and avoids all padding

Finally, I must draw attention to Ahlfors' own very interesting and illuminating commentaries preceding each article or group of articles They explain the background in which the papers were written, assess their significance, and give credit due to other mathematicians which could not for various reasons be given in the papers themselves Also, throughout the two volumes, there are excellent survey articles, which together with the commentaries referred to above, serve to place Ahlfors' work in proper perspective, and greatly add to the value of these volumes

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Mathematical scattering theory by Helmut Baumgartel and Manfred Wollenberg. Birkhauser Verlag, Basel, 1983, pp 449 S. Fr 85

The authors are well-known researchers in the domain of mathematical theory of scattering and their book is a welcome addition to the already existing texts[1, 2] As mentioned in the preface, unlike the other two texts this book emphasizes the abstract mathematical aspects more than applications Potential scattering and other applications are mostly relegated to the notes of various chapters

The book begins with preliminaries like self-adjoint operators in a Hilbert space and their spectral and multiplicity theory as well as the direct integral representation of a Hilbert space with respect to a self-adjoint operator Operator spectral integrals introduced in Chapter 5, though not of utmost generality, is adequate for applications in Chapter 9. The general theory of asymptotic constants had been extensively studied by the authors and forms the content of the next 3 chapters Some elements of algebraic theory of scattering and of the invariance properties of wave morphisms also appear here Chapters 9 through 11 deal with the abstract scattering theory in two-space formulation. Haag-Ruelle theory of scattering for Quantum fields as well as Lax-Phillips theory are discussed in this framework

Part IV deals with various methods (Kato-Kuroda, Trace-class, smooth perturbations, Cook-type) to prove the existence and completeness of wave operators Unfortunately, there is only a brief discussion of multichannel scattering and the recently developed Enss-Mourre methods find only a passing reference. Finally, Part V contains results on the properties of the S-matrix, scattering amplitude and Krein-Birman formula relating spectral shift with phase shif

The book is well-written though somewhat terse in style, and has an exhaustive bibliography. It is an excellent reference book on the subject of scattering theory and should be part of every mathematics library However, its style and vast contents make it unsuitable for a text-book

1 W O. Amreen, J.M. Janch and K.B Sinha *Scattering theory in quantum mechanics*, W A Benjamin, Reading, Mass , 1977

2 M. Reed and B. Simon *Methods of modern mathematical physics III: Scattering theory*, Academic press, New York, 1979

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Representation theory of reductive groups edited by P C Trombi. Birkhauser Verlag, Basel, Switzerland, 1983, pp. 240, S. Fr 62

This volume comprises research and expository articles presented at a conference on Representation Theory of Reductive Groups held in Utah in April 1982

Casselman's submodule theorem asserts that any irreducible Harish-Chandra module can be imbedded into a principal series representation In the article of Beilinson and Bernstein the above result is generalized as follows For any g-module M (finitely generated) and almost all maximal nilpotent subalgebras n of g the space $M/_{nM}$ is non-zero. The methods employed here are a fine illustration of the techniques of D-modules (sheaf of modules for the sheaf of (germs of) differential operators). For a singular space, it is an interesting question to decide whether the (middle) intersection homology groups can be interpreted as the L^2-cohomology for an appropriate metric on the submanifold of smooth points. The article of Casselman proves this for compactifications of real rank one locally symmetric spaces of finite volume Unitary Harish-Chandra modules which are irreducible quotients of Verma module has been a subject of study in recent years. Even though partial results (which were sufficient for important applications) were known previously a complete classification had been lacking The article of Enright *et al* is a solution to this problem. The well-known 'Kazhdan-Lusztig Conjecture' was formulated to explain the multiplicities in a composition factor of a Verma module. One may also view this as giving an algorithm for computing the coefficients that occur when the character of an irreducible highest weight module is expressed as a linear combination of characters of Verma modules. A related problem is to write down the character of an irreducible Harish-Chandra module as a linear combination of characters of some basic representations (principal series representations). The formulation of the 'Kazhdan-Lusztig Conjectures' and attempts to solve them have been begun by D. Vogan. His article is an expository account of the work in progress on this problem. George Kempf interpreted the BGG resolution of a finite dimensional irreducible g-module as being dual to the Cousin resolution for the associated invertible sheaf on the flag variety with respect to the stratification given by the orbits of a Borel subgroup acting on the flag variety. G. Zuckerman's article develops similar techniques for obtaining resolutions of Harish-Chandra modules which occur as some local cohomology spaces.

These articles (and the others in the book on Weyl group representations, orbit method, analysis on symmetric spaces which I have not mentioned here) amply demonstrate the importance of representation theory of reductive groups

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Spectral theory of hyponormal operators by Daoxing Xia Birkhauser Verlag, CH-4010, Basel, Switzerland, 1983, pp 241, S Fr 54.

In view of the spectacular success for the self-adjoint, unitary and normal operators, new classes were introduced; particularly successful being the hyponormal and the subnormal operators, the major achievement of last decade being the solution of the invariant subspace problem for the subnormal operators and many of its generalizations

The author has devoted this book to the study of hyponormal operators (HN) and semi-hyponormal (SH) operators which were introduced by him in 1980[1] [(SHU) denotes operators in (SH) with equal defect and nullity]. He has mainly presented his results along with those of his collaborators and colleagues and some more concerning related topics. Although, the applications are limited at the moment, new theories are expected to be developed in the years to come.

These operators are unitarily equivalent to singular integral operators which are pseudo-differential operators of *O*-order. The contribution of these operators in problems of mathematical physics, particularly quantum mechanics, is shown.

Using the tools of (i) the Cayley transform (ii) the Berberian technique (iii) symbols (iv) polar symbols, the symbols of HN operators and polar symbols of (SH) operators are discussed. Several results are established to show the similarity with normal operators, *e.g.* rectangular and polar projections. Angular cutting of HN and SH and rectangular cutting of HN is established. Pincus gave a solution of the Riemann-Hilbert problem and introduced operator-valued Riemann-Hilbert problem Polar symbols and general polar symbols and function models are used to develop singular integral models of operators in SHU and HN.

Mosaics and characteristic functions are abstract analytic functions linked with singular integral models. The determining function E $(\cdot,\cdot)$, the role of the analytic function R $(\cdot,\cdot)$, in scattering theory and relation between them w.r.t Riemann-Hilbert problem is presented. The mosaic, B $(\cdot,\cdot)$, of an operator in HN and SH, the characteristic function of SH operator and operator-valued analytic function Y $(\cdot,\cdot)$ are used to consider the Riemann-Hilbert problem and the connection between the spectra of SH operators and some Toeplitz operators is established.

Spectral mapping theorems for HN operators under a special class of functional transformations and for SH operators under another class of functional transformations are established Using the class M (E), of bounded real Baire functions on E, $S(E) = \{ \theta \in M(E), K_\theta \geq 0 \}$ and a special subset P (E) of pick function (analytic in upper half-plane) and Im $\theta(z) > 0$ for Im $Z > 0$ }, spectral mapping theorems for (HN) and (SH) operators are established Estimates of resolvents of operators given by functional transformations of (HN) operators are obtained Using (i) scale functions (ii) modular scale functions and (iii) angular scale functions a new class of operators, called quasi-hyponormal operator, is studied, although some authors use the same term for another class of operators

Using the concept of trace of an operator and the mosaic of an HN operator (B (·,·)) the Pincus principal function is defined By introducing the trace formula for the nearly normal HN (and SH) operators, determinants of some operators can be expressed as the integrals of the principal functions

In the appendix, spectral theory of contractions of Sz-Nagy and Foias is explained The relation between invariant subspaces of c n.u. and factorization of characteristic function is presented.

The results presented in the book have been obtained in the past ten years Some of them have not been published before (*e.g.* Theorems 2 5, 4 3 and corollary 2 6 of chapter VI). Also, Theorem 3 2 and 3 3 of chapter II as well as results about the estimates of spectral radius and resolvents of SH-operators are published here for the first time

Interestingly, Theorems 1.6 and 3 2 dealing with singular integral model of an HN-operator could not be published until 1979, although the manuscript was received in 1965 Zhang[2] contributed his article on symbols of HN-operators in 1966, but this was also published in 1979

The book contains a wealth of information on spectral theory of hyponormal and semi-hyponormal operators. However, without enough expertise and maturity in Operator Theory the material cannot be well appreciated

There are some typographic errors in the book For experts working in hyponormal operators and related topics, this book is a pleasure to read

References

1 Xia Daoxing, On the non-normal operators-Seminormal operators *Sci., Sinica* 1980, **23**, 700-713.

2 Zhang Yinnan, Representation of hyponormal operators and estimation of its spectrum. *J. Fudan University,* 1979, No 4, 76-82.

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Theory of function spaces by Hans Triebel Birkhauser Verlag, P O Box 133, CH-4010, Basel, Switzerland, 1983, pp 284, S. Fr 78

It was S L. Sobolev's original idea to seek solutions of differential equations in a category of 'nice' spaces, that is, if the differential equation admits no solution in a particular space, to change the scale of the space where it becomes possible to find the solutions For example, when f is a continuous function with compact support, solutions of the Laplace equation $\Delta u=f$ need not in general belong to C^2. On the other hand, we can re-scale the space in which a solution is sought to say the Sobolev class W_p^2 where we indeed have solution Also, while regularity results are discussed for partial differential equations, it is more natural to deal with function spaces which are scaled, such as the Holder spaces $C^{k+\alpha}$ $(k \in \mathbb{N}, 0<\alpha<1)$, the Sobolev spaces W_p^s $(s \in R)$, etc Indeed, if $f \in C_0^\alpha$, then any solution of $\Delta u=f$ is in $C^{2+\alpha}$

Just as the gap C^0, C^1, C^2, . is 'filled' up by the Holder spaces, so is the gap between L^P, W_p^1, W_p^2, . is 'filled' up by the spaces W_p^s, note that while the restriction of a C^k function to a hyperplane remains C^k, the restriction of a W_p^m function is in $W_p^{m-1/p}$ For this reason, the category of Sobolev spaces has become a fundamental tool in the study of elliptic boundary value problems, see [1]

The book under review, according to the author himself, is a successor of his previous book [2], but is more self-contained with some new results However, some fundamental results such as the classical maximal inequality have been left unproved ! The principal purpose of the author is to attempt a unified study of the known properties of the classical spaces such as those of Holder and Sobolev, spaces of BMO, etc., in the category of Besov spaces $B_{p,q}^s$, which includes all of the classical spaces. The author has largely achieved his goal Each chapter is well planned with brief historical notes and should attract the discerning reader to make a sincere attempt to know the various function spaces introduced Important applications to elliptic boundary value problems (both degenerate and non-degenerate) have been discussed in the categories of spaces $B_{p,q}^s$ and $F_{p,q}^s$

The book contains in all ten chapters which are essentially interrelated The bibliography is comprehensive and should therefore serve as a reference book as well

Finally, it requires a lot of self-persuasion on the part of the reader to go through the rather small type script used in the text However, once the reader gets the feel of the subject, it is indeed a very pleasant reading.

References

1 Agmon, S, Douglis, A, Nirenberg, L, Estimates near the boundary for solutions of elliptic PDE satisfying general boundary conditions I, *Comm Pure Appl. Math.* 1959, **12,** 623-727.

2 Triebel, H. *Interpolation theory, function spaces, differential operators,* North-Holland, 1978

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Functional integrals in quantum field theory and statistical physics by V N Popov D. Riedel publishing company, P.O. Box 17, 3300 AA Dordrecht, Holland, 1983, pp. 299, D Fl. 150/$ 65 00

The path integral approach to quantum mechanics was invented by Feynman in the early forties Later in that decade he used the same approach to develop a relativistically covariant perturbation theory for quantum electrodynamics. It is sometimes not realised that he arrived at his famous diagram technique along this route. This is largely because of Dyson's brilliant synthesis of Feynman's space-time approach and Schwinger's canonical operator-based methods, and the ensuing elaboration of renormalization theory. Most of the well-known texts of the fifties and sixties - Schweber, Jauch and Rohrlich, Bogoliubov and Shirkov, Bjorken and Drell - relied on the operator approach to relativistic quantum field theory to set up the Feynman diagram technique for perturbation and renormalization theory. However in the intervening years the peculiar beauty and flexibility of the original Feynman path integral method has been more and more widely realised, appreciated and made into a practical tool In this context the book under review is, in certain respects, a welcome addition to the literature It is, in the familiar Russian tradition, a well-organized and lucidly written book Two of the eleven chapters set up the basic formalism in various situations, four deal with applications to specific problems in relativistic quantum field theory, and five with problems drawn from condensed matter physics The author takes a physical and pragmatic attitude to the problem of existence—in the strict mathematical sense — of the Feynman path integral and regards it as a tool 'adjusted to the needs of contemporary physics' The initial chapters describe concisely but adequately the path integral formalism for quantum systems with a finite number of degrees of freedom, for dynamical systems on manifolds, constrained systems, for relativistic quantum field theory and for statistical physics. Then come the applications In most cases it is a fact that satisfactory canonical operator approaches existed prior to application of path integral methods. In a sense the really notable exception is the quantization of nonabelian gauge fields, where the operator approach is quite unwieldy Unfortunately the space devoted to this topic, and then to the case of the gravitational field, is rather small and the material is highly condensed What does come through quite clearly is the power of the path integral method - in a sense as a 'conjugate' to straightforward perturbation theory - since it helps develop an intuitive feeling for a theory as a whole, suggests ways of modifying perturbation theory and setting up and relating new forms of it; efficiently carries out what amount to partial summations of diagrams; helps deal more compactly with collective excitations; and so on All this is possible because one is dealing with changes of integration variable and related operations carried out 'up in the exponent'. A recurrent idea is the separation of fields being integrated over into 'fast' and 'slow' components - small and large wavelengths - integrating over the former in a normal way and then over the latter in an unconventional way to yield a modified perturbation theory -This general method is applied to a variety of problems: infrared limit of QED; the high energy limit in relativistic particle scattering leading to the eikonal approximation; superfluidity in the Bose system and derivation of the hydrodynamical Lagrangian; Bose systems in one and two dimensions; hydrodynamical action for Fermi systems and He_3 in particular; the infrared problem in plasma theory; and so on. The other topics dealt with under condensed matter physics include superconductivity, the Ising model, and the calculation of critical indices for second order phase transitions. This reviewer feels, however, that the value of the book would have been much enhanced if the coverage of gauge field theory and gravitation had been somewhat more detailed and expanded, and if topics like the instanton method were included Considering that this book was published in 1983, this is not an unreasonable expectation As it is, though, this is a very useful book in which the application of path integral methods to problems of statistical physics are particularly comprehensive The language is at times less than felicitous (problem of translation by Czechs from Russian into English), but that causes no real problems

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Special functions of mathematical physics and chemistry by Ian N Sneddon Longman, London, pp 182, £ 5.95.

This is an extremely well-organised and well-written monograph meant for users of special functions in problems of physics and chemistry who are even unacquainted with the basics of several mathematical theories including the theory of functions of a complex variable.

In the present second edition additional topics like general theory of linear ordinary differential equations and one important aspect involving Contour integral representation of the solutions of such equations have been included.

The important special functions that are discussed, in some detail, include the Hypergeometric functions, the Legendre functions, the Bessel functions, the Hermite functions and the Laguerre functions. The treatment is very lucid and is readable by users of these functions of all forms.

The last little chapter on 'the Dirac delta function' is an excellent exposition of the salient features of this well-used generalised function and clarifies some of the very important notions in a very elementary manner to motivate the reader to enquire more about such generalised functions.

The book in its older form itself already has served many important purposes for the students of graduate courses in Engineering and the present form will do more in that direction.

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Bifurcation theory, mechanics and physics: mathematical development and applications edited by C.P. Bruter, A. Aragnol and A. Lichnerowicz. D. Reidel Publishing Company, Dordrecht, Holland, 1983, pp 388, D. Fl. 145.

This book contains a collection of papers on a fairly wide range of topics not entirely unrelated to each other These papers were presented at a colloquium held at Marseille, France

The subjects covered could be classified as follows., Classical mechanics, Quantum mechanics, Partial differential equations, Information theory, Bifurcation theory and, finally, a set of papers on some special real-life problems to which mathematical analysis has been applied.

The first paper in mechanics consists of a review of Hamiltonian, Canonical and Symplectic formulations of dynamics The second paper derives pre-relativistic formalisms of dynamics from a Generalized Energy Gradient Principle as against the traditional approach *via* Hamilton's Principle of Least Action. The third paper studies Poisson structures of Lichnerowicz

In quantum mechanics there is a paper on the theory of deformations and group representations and one on the Schrodinger equation by the eminent mathematician J Leray

Amongst the papers on partial differential equations is the very interesting—though regrettably short—paper of L Nirenberg on variational methods for proving global existence theorems. This paper is part of a larger expository article by the same author

Of the papers in bifurcation theory there is one on bifurcation in systems of ordinary differential equations. The paper of J. Rappaz makes a survey of numerical approximation of problems of bifurcation related to the Lyapunov - Schmidt reduction. Finally the paper of M Golubitsky describes the relationship between symmetry and bifurcation as motivated by the Benard Problem The results are obtained using the machinery of singularity theory and group theory

Amongst the special problems, I would like to mention the papers on tomography and on geometrical and topological problems in liquid crystals.

In conclusion, this volume consists of a number of papers, some of which are expository in nature, others of very specialized interest The topics covered are rather varied and that shows the diversity and power of the applications of mathematics

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Numerical solution of partial differential equations: Theory, tool and case studies edited by D.P. Laurie (Summer Seminar Series held at CSIR, Pretoria, February 8-10, 1982) Birkhauser Verlag, Basel, Switzerland, 1983, pp. 341, S Fr 56

Proceedings of conferences, meetings and smaller seminars form a large part of the scientific publications today The only trouble which editors of these books take is to collect the unconnected review articles and research papers, in a form suitable for quick reproduction and send the collection to an agreeable publisher I had to review one such publication (See *J Indian Inst Sci* (*B*) 1983, **64**(**B**) (11), 335)

However, the present book edited by D P Laurie is different He has picked up ten articles from a vast literature of numerical solutions of PDE which form well connected chapters in which the subject matter has been developed in quite uniform style Of the ten chapters, three are written by Laurie himself, two by T Geveci and the rest of the five chapters have been written by different authors All authors except those of Chapter 6 are from the same institution - National Research Institute for Mathematical Sciences, CSIR, Pretoria (SA) The level of the articles varies from the first year post-graduate in mathematics to the state of art The book is a suitable introductory course for a mature student of mathematics (having a background in functional analysis) It will also be very useful to an engineer (or scientist) who has got some experience in numerical solution of PDE

The first chapter by Geveci contains a brief but excellent introduction to those aspects (especially well-posedness) of the theory of PDE which are necessary for the understanding of numerical analysis of the equations. Basic principles of discretization methods are considered with examples in Chapter 2 Finite difference and finite element methods are lucidly introduced In this chapter, Laurie has achieved to dispel the impression that the two methods are poles apart, in fact, the two methods in some cases lead to almost the same system of linear equations However, he also makes it clear that finite element method is essentially the Galerkin method In Chapter 3, Geveci deals with the analysis of convergence of numerical methods and shows why, unlike in the case of finite difference methods, this analysis is relatively straight forward for finite element methods. The other chapters are: 4 Basic functions in the finite element method (Baart), 5 Time discretization in parabolic equations (Laurie), 6 Parabolic equations with dominating convection terms (Herbst and Schoombie), 7. Solution of large system of linear equations (Laurie), 8 Finite-element mesh partitioning for integration of transient problems (Neishlos), 9 Numerical weather prediction (Riphagen) 10. Conservation laws in fluid dynamics and enforcement of their preservation in numerical discretizations (Navon) Navon discusses, at some length, how to construct conservation laws in some physical systems but does hardly any justice to the full role of conservation laws in numerical solution

The reviewer feels that those interested in numerical solution of PDE should go through this book

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PHOOLAN PRASAD

Classification of algebraic and analytic manifolds edited by K Ueno (Proceedings of the Katata Symposium 1982, sponsored by the Taniguchi Foundation) Birkhauser Verlag, P O Box. No. 133, CH-4010 Basel, Switzerland, 1983, pp. 630, S Fr 86

This is a collection of 15 articles by the participants of the symposium These are articles written by experts for experts and so a novice might find it hard-going. But most of the articles have excellent bibliographies, which often include survey articles to help them along As the title indicates, these are papers on classification of manifolds, which is one of the most active areas of research in geometry today Many different kinds of problems are dealt with and almost any reader with some interest in the subject would find something in this collection to his taste. Also, more often than not, complete proofs are given, which helps a serious student of geometry.

Regarding the papers, it will be very hard to review each one of them individually So I will not attempt to do it, but say a few things in a general vein I wish to emphasize that the results I quote below are chosen almost at random and only with the intention of giving a flavour of what the book contains

There is a nice counter-example by A. Beauville to a conjecture of Bogomolov regarding birational automorphisms of manifolds with trivial canonical bundle There are many papers on 'period maps' of surfaces and several new results, most of them following a very detailed analysis of the type of surfaces considered. These analyses themselves might be of independent interest. Amongst the very few papers on higher dimensional manifolds, there is one by T Fujita, which though has more conjectures than proofs, gives a method of attack for a major problem. whether the canonical ring of a manifold is finitely generated? This, I am sure will be a great help to the researcher. The paper on Torelli's problem by C Peters and J H.M. Steenbrink is more or less a survey article, very well written but a strain on one's eyes since the print is very small There is also a very pleasant result on configurations of curves on smooth hyper-plane sections of a 3-fold in the paper by A J Sommese The result is very surprising and may have more implications than outlined. There is a lone paper on 'open surfaces' by S Tsunoda and M. Miyanishi. They extend the theory of S. Mori to the new situation

There is a collection of open problems at the end, suggested by the participants This is a pointer to the major directions in the theory of classification Also, the problem list has an introduction (presumably by the editor) which is helpful.

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Combinatorics and commutative algebra by R P. Stanley Birkhauser Verlag, Basel, Switzerland, 1983, pp. 357, S. Fr. 64

The study of Stanley Reisner Ring of simplicial complexes and similar rings associated with nonnegative integral solutions to linear equations is a fascinating topic, which is growing rapidly, since the last few years The main contributor to the development of the basic theory of this subject is R.P. Stanley, the author of the book under review. The book is based on a series of eight lectures given by the author at the University of Stockholm.

The book is essentially divided into two chapters Chapter I deals with the graded algebras associated with nonnegative integral solutions to linear equations, while Chapter II is devoted to the study of graded algebra (also known as Stanley Reisner Ring) associated with a simplicial complex The book also has a useful chapter which reviews the basic concepts from combinatorics, algebra and topology needed for this study.

The main result of Chapter I is a reciprocity formula for the number of nonnegative integral solutions to a given set of linear diophantine equations, which was first proved by Stanley in 1973 The basic tool is the study of local cohomology of the associated graded algebra and modules In fact, the reciprocity essentially follows from the fact that these rings are Cohen Macaulay (and Gorenstein in some cases). This was first proved by Hochster in 1972 The book gives a uniform cohesive proof of theorems of Hochster and Stanley. As an application, the truthfulness of some well known conjectures of Anand, Dunoir and Gupta on the number $H_n(r)$ of $n \times n$ matrices with nonnegative integral entries having line sums r, are derived.

The second chapter is essentially devoted to the study of f-vectors of a simplicial complex and related topics The basic tool is again the local cohomology and some results on Cohen Macaulay rings and graded algebras As an application the well known upperbound conjecture for simplicial polytopes and triangulations of spheres is derived. Some recent advancements in these topics are also added in the last sections of the book.

The two topics covered in the book, at first may seem to be unrelated However as one goes through the theory developed in the book, one can get the feeling of the common link *viz.* the associated Cohen Macaulay Ring The book has been quite successful in presenting beautifully the study of this common link

The proofs given in the book are quite clear and the author has tried to be complete as far as possible, however, the style of the book requires certain amount of maturity from the reader in homological algebra and topology The reviewer feels that a more detailed and elementary book on these topics will be quite useful, particularly for the research workers in the field of combinatorics, convex polytopes, etc , where the techniques developed in the book have proved to be a strong tool

The book has a few typographical errors, though all of minor nature

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An introduction to stochastic integration by K.L Chung and R.J. Williams, 1983, pp.191, S. Fr. 49.

Seminar on stochastic processes 1982 edited by E. Cinlar, K.L. Chung and R K Getoor, both by Birkhauser Verlag, CH-4010, Basel, Switzerland, 1983, pp 302, S. Fr. 64.

These two new titles in the Birkhauser series on 'Progress in Probability and Statistics' are a welcome addition to the existing literature on stochastic processes.

The first title, as the name indicates, is intended as an introductory text on stochastic integration Most of the material presented is by now classical and is available in standard texts and monographs such as Liptser-Shiryayev[1], Metivier-Pellaumail[2], Kussmaul[3], to name but a few. However, the book is far from being redundant It is of great pedagogical value as perhaps the first easily digestible account for a beginner of a topic that could easily dount him by its technical difficulties. The authors have avoided going for the maximum generality and have stuck to continuous sample path processes which in itself is a considerable saving in complexity. The topics, spread over the first five chapters and many small subsections, include a quick review of essential probability theory, development of stochastic integrals, the quadratic variation process and its properties and the Ito formula The well-spread material and a generally attractive get-up will contribute a lot towards lessening the teething troubles of a new initiate to the subject

The rest of the chapters are of a more specialized nature and illustrate the use of the stochastic calculus developed in the first five chapters by means of specific applications. They include some interesting applications of the Ito formula and a generalization thereof, a nice treatment of local time and Tanaka's formula not usually found in elementary texts and a chapter on reflected Brownian motion which presents some recent work by the junior author

Unfortunately, the area where the stochastic calculus has proved most useful, *viz.* stochastic differential equations, is completely omitted

The second book under review is a collection of research papers presented at a three-day seminar held at the Northwestern University in March 1982. This is the second such seminar, the proceedings of the first having appeared earlier in the same series[4].

The first paper, by B W Atkinson, considers an increasing filtration of σ-fields with the germ Markov property, which, loosely speaking, amounts to the conditional independence of the σ-fields representing the 'past' and the 'future' information given the 'germ σ-field' at the 'present'. For these, he proves that the random times which preserve this conditional independence property are necessarily optional, thereby establishing a converse to the strong Markov property. The second paper, jointly by Atkinson and Mitro, is devoted to additive functionals of Markov processes satisfying a weak duality relation In another paper concerned with additive functionals, Cinlar and Kaspi give explicit constructions of certain additive functionals for regenerative systems. In another paper, Bass studies additive functionals of semimartingales that generalize the concept of occupation times.

There are five papers in this collection which are devoted to the interaction between probability and potential theory or the theory of partial differential equations Two papers by J. Glover and one by Pop-Stojanovic and K.M Rao are concerned with the former, in particular with the concept of energy. Of the remaining two, the paper by Chung establishes a new inequality for boundary value problems while the paper by Bichteler and Fonken gives a simpler approach to Malliavin calculus in the spirit of Bismut[5].

The rest of the papers in the collection are devoted to various individual topics: Getoor in his paper studies the excursions straddling forward times, which are, in fact, the backward analog of optional times The paper by Monrad is concerned with the p-variation of Gaussian random fields In a note by J W Pitman, he studies the convex minorant of o Brownian motion in the context of path decomposition of Williams Finally, in a lengthy paper, Walsh studies stochastic integration w r t local time, with the space variable playing a role analogous to what the time variable plays in Ito integration, and derives an integral representation result in this set-up

References

1 Lipster R S., Shiryayev A N , *Statistics for random processes I. General Theory*, Springer, 1977

2 Metivier M., Pellaumail J., *Stochastic integration,* Academic Press, 1979.

3 Kussmaul A V , *Stochastic integration and generalized martingales,* Pitman, 1977

4 Cinlar E , Chung K.L , Getoor R.K (Eds), *Seminar on stochastic processes 1981* Birkhauser, 1982

5 Bismut J M , *An introduction to stochastic calculus of variations* in Springer L N Control & Info Sci 1982, **43.**

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The thread by Philip J Davis Birkhauser Verlag, CH-4010, Basel, Switzerland, 1983, pp vii + 26, S Fr 40

This is a book by an applied mathematician but not on mathematics Its second title is 'a mathematical yarn' There is nothing mathematical about it In effect, it starts with an account of the school of mathematicians that developed in Russia Apparently, he is their great admirer, and in particular, Pafnuty Lvovitch Tschebyscheff. He is questioned about the way he spells his surname. This leads to his justification But his real yarn starts when he explores the origin of the first name 'Pafnuty' What follows is an extremely interesting account His contacts with Egyptologists, the discovery of desert saint Paphnutius in the ancient past This naturally leads to the story of Paphnutius the hermit and Thais, the courtesan immortalised in the novel of Anatole France

There are a few stints at rationalism and supernatural and the story of Sikkimese Lama Ted controlling the hurricane Felicia which hit Providence right from the Mount Hope Bridge, Rhode Island, U S A There are interesting episodes of his visit to Tasmania and the old city of Jerusalem

The book deals with personal digressions, partly detective and partly anthropological The style of the book is very entertaining What about its value? It will serve the purpose of light reading, amusing at times with a few anecdotes of unusual and paranormal situations The author is a good yarner

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Scaling and self-similarity in physics *Renormalization in statistical mechanics and dynamics* edited by Jürg Fröhlich. Birkhäuser Verlag, Basel, Switzerland, 1983, pp vi + 426, S Fr. 66.

This book is the seventh volume in the 'Progress in Physics' series published by Birkhauser This series is edited by A Jaffe, G. Parisi, and D Ruelle

The articles in this book are based on seminars given at the Institut des Hautes Etudes Scientifique, Bures-sur-Yvette, France, between spring 1981 and summer 1982 They are devoted to mathematically-rigorous results on scaling and self-similarity in three branches of physics: statistical mechanics, field theory, and dynamical systems. The contributors are well-known mathematical physicists.

The book consists of two parts. Part I is devoted to equilibrium statistical mechanics and field theory and contains the following articles Large fluctuations of random fields and renormalization group Some perspectives, by G. Jona-Lasinio; The Berezinskii-Kosterlitz-Thouless transition (Energy-entropy arguments and renormalization in defect gases), by J Fröhlich and T. Spencer; Interface and surface tension in Ising model, by C.E Pfister, Iterated Mayer expansions and their application to Coulomb gases, by J Z Imbrie, Rigorous results on the critical behaviour in statistical mechanics, by M Aizenman, Non-perturbative methods for the study of massless models, by J-R Fontaine, Rigorous renormalization group and asymptotic freedom, by K Gawedzki and A. Kupiainen; On infrared superrenormalization, by J Magnen and R Seneor; Ultravoilet stability in field theory. The ϕ^4_3 model, by T. Balaban Part II is devoted to dynamical systems and contains the following articles Renormalization group analysis for dynamical systems, by P Collet and H. Koch, Bowen's formula for the Hausdorff dimension of self-similar sets, by D. Ruelle; Perturbation theory for classical Hamiltonian systems, by G Gallavotti

For a variety of reasons, many topics that should have been included in such a collection of articles have been left out. Some of the topics not covered are · Scaling and renormalization in relativistic quantum field theory; the theory of disordered systems, and scaling and self-similarity in mathematics In his foreword, Fröhlich expresses the hope that some of these omissions can be made up for by consulting refs 1-4.

Another shortcoming of this book, also noted by Frohlich, is that the articles are neither complete in themselves nor polished and, indeed, are quite hard to read For this reason, I can only recommend the book to experts in statistical mechanics, field theory, and dynamical systems Even among such experts, I recommend it only to those with a taste for rigorous results

Given the wide range of subjects covered in this book, it is quite unlikely that all the articles will interest any one reader For myself, I found the papers by Frohlich and Spencer, Pfister, Aizenman, Gawedzki and Kupiainen, and Collet and Koch useful and somewhat accessible Frohlich and Spencer show how the energy-entropy argument of Peirls can be extended to two-dimensional XY models (and the related Villain, discrete-Gaussian and solid-on-solid models) to prove the existence of a Berezinskii-Kosterlitz-Thouless transition from a disordered, high-temperature phase with quasi-long-range order The first few sections contain a clear statement of their principal results and methodology; the details of proofs are relegated to subsequent sections, which can be skipped by non-experts Pfister's article contains a good, readable overview of rigorous results on interfaces and interfacial phase transitions in Ising models No details of proofs are given, but these may be found in the list of references that Pfister provides Aizenman shows rigorously that above the upper-critical dimension, certain critical exponents assume their mean-field values The article of Gawedzki and Kupiainen is very interesting because it obtains *rigorous* results for a statistical-mechanical model by *renormalization-group* method, unfortunately, the article is fairly technical and is hard to read. Collet and Koch give a quick and readable account of the period-doubling route to chaos, describe the renormalization group that has been used to understand its universal features, and end with rigorous results on this renormalization group Lastly, Gallavotti's article, which deals with the relation between the Kolmogorov-group ideas, seems very interesting, but is very technical and is very hard to read

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RAHUL PANDIT

Maxwell on Saturn's rings by Stephen G. Brush, C W F Everitt and Elizabeth Garber MIT Press, 28, Carleton Street, Cambridge, Mass. 02142, USA, 1983, pp xii + 199, $ 28.75.

Maxwell on Saturn's rings is a remarkable book describing the circumstances leading to James Clerk Maxwell's monumental work on the stability of rings of Saturn. The central theme is the Adam's prize essay of 1856 on the subject quoted above, but the charm of this book lies in the series of personal letters written by Maxwell to his friends which show the way the mind of a genius had approached a difficult problem In his essay Maxwell had pointed out quite clearly that the existing methods are inadequate to find a total solution to this dynamical problem. His attempts to extend the analysis to the cases where ring particles suffer mutual collisions can be seen in some of his incomplete calculations, these documents read like fragments of manuscripts never completed All these are arranged in this book for students in history of science to find an invaluable collection.

Besides the main Adams Prize Essay, there are several other valuable documents which throw considerable light over the academic deliberations on this topic The excellent review of Maxwell's essay by George Biddell Airy in the Monthly Notices of the Royal Astronomical Society forms one such document, while a heavily annotated introductory chapter presents a historical sketch of the problem underneath in a neat and detailed manner

The first of Maxwell's letter included in the collection is dated July 4, 1856 to his friend R B Litchfield where he casually mentions about his working on the problem of Saturn's ring This is followed by fifteen more letters written to William Thompson, Lewis Campbell, G G Stokes and a few others over the next two years Much of this correspondence might seem irrelevant to a collection of unpublished scientific papers, as considerable personal matters have found their way in these letters, but still it does reveal some aspects of Maxwell not seen otherwise, this is where the students of history of science will find a real glimpse of the academic atmosphere prevalent in those days

The core part of the book is, however, the 1856 Adams Prize winning essay which was published in the final form in 1859 This ninety-page document is remarkable on several counts. To quote from G B. Airy's review "The subject of it is so interesting, the difficulty of treating it in its utmost generality so considerable and the results at which the author arrives so curious .. (The essay) will fully justify the opinion that the theory of Saturn's rings is now placed on a footing totally different from any that it has occupied before and is one of the most remarkable contributions to mechanical astronomy that has appeared for many years".

Nine manuscripts (Documents 20-27) given at the end of the book represent Maxwell's attempts to find an answer to the problem of colliding particles in the Saturn's rings Although incomplete individually, they really display efforts by a mathematical genius to find analytic expression for a vexing problem, on that count I feel they constitute an invaluable set of papers. I am confident, that some day when the total dynamical problem of n-body encounters are solved, we will be in a better position to understand Maxwell's efforts in this direction

To sum up, this is an invaluable book which no Library of science can afford to miss

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J C BHATTACHARYYA

Astronomy from space-*Sputnik to space telescope* edited by James Cornell and Paul Gorenstein The MIT Press, Cambridge, Massachusetts, USA, 1983, pp 248 20

This is an extremely well-written book which should be read not only by the astronomers but also by serious geologists and geophysicists who can get an overview about the planetary and earth sciences and also get very interesting ideas for research. Of course, the general reading community, especially physicists, will enjoy and benefit from the book

The first chapter of the book deals with Astronomy before the space-age where Leo Goldberg beautifully summarises the state of astronomy from 1933 when he started working on astronomy up to the present age This not only gives a historic perspective but also shows clearly how myopic and conservative many scientists can be in their judgement of future possibilities especially in an era where technology is moving very fast This historic fact or clue should be borne in mind by many decision-makers and even scientists who like to propose new experiments Chapter 2 deals with geology of the inner planets and this gives in brief the geology of all the planets which have been explored by space techniques, with good illustrative material and also well-reproduced photographs. In this chapter, all planets are dealt with in totality, the current overall knowledge and lacunae are brought out well. Chapter 3 deals with the exploration of the moon not only in terms of the scientific content but also in terms of events which occurred in its exploration. Chapter 4 deals with voyage to the giant planets and Chapter 5 with the new sun where our new knowledge about our old sun is explained well Chapter 6 deals with the ultraviolet sky, the new exploratory tool which is available mainly because of space techniques Similarly, Chapter 7 deals with an X-ray portrait of our galaxy and Chapter 8 deals with X-rays beyond the Milky Way. Chapter 9 gives a very good summary about the future of space astronomy This is the chapter which would be worth reading and concentrated upon by serious researchers as well, not only in astronomy but also in geophysics Chapter 10 is a beautiful summary about what has happened in earth sciences in the space age. It is titled 'Epilogue: The rediscovery of earth'. This chapter will be a delight to the general reading public and a good source material of idea for serious geologists/geophysicists It is felt that the title should *not* have been *Astronomy in space age* but also included, even in small print, 'Glimpses of planetary and earth sciences from space' The author of Chapter 10 Ursula B Marvin is a geologist.

On the whole, the book is very timely since we are on the threshold of ushering into the new and explosive knowledge in astronomy and planetary sciences with space technology. Giant telescopes will soon be flown in space through Shuttle and Space Platforms

The book does not cover in equal depth some of the Soviet systems though it gives a brief description of the scientific results from the Soviet systems. Results from both the US and Soviet space systems put together lead to a tremendous source of knowledge for humanity and in the forthcoming years many scientists world over would join in this great venture The book does not cover in detail a new area which is emerging *viz*, infrared astronomy, reason perhaps could be that infrared astronomy satellite was flown only in 1983

It is recommended that this book is widely publicised in all our science and technological institutions - at colleges and university level - so that young bright scientists/engineers in all disciplines can get bright ideas early in their career It could form a serious book to be read in institutions conducting research especially by those who are inducted into Ph D or research programme It can also be a source of inspiration as well as ideas since it is a good readable book and gives easily an overview.

An idea that occurs while reading this book is that the same authors or perhaps some others taking the theme from this book, should soon try to write a similar book on earth and planetary science from space since it would be extremely useful to introduce new entrants through such books which give an overview while being scientifically rigorous The style is lucid and in fact in a few cases giving historic data make the book interesting rather than merely giving drab scientific material. Though many authors have contributed to the book, the style looks lucid and uniform.

Space has come to stay in human life, such books will explain people more about what is in store and perhaps prepare humanity to cope up with the space age

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Realm of the long eyes by James E. Kloeppel Univelt, Inc , San Diego, California, USA, 1983, pp 135 $ 15.

It is common knowledge now that the astronomical telescope is one of the most powerful tools in man's quest for understanding the universe Today, there are scores of such telescopes scattered all over the world, their sizes ranging from about six inches to several tens of inches However, during the last few decades, the telescope-making technology made rapid progress and a handful of so-called large-sized telescopes came into being But, the amount of effort that goes into setting up of an observatory with such large telescopes is enormous and almost unimaginable unless one is a part of it The book *Realm of the long eyes* reveals exactly that and brings out the agonies and ecstacies of creating a brand new observatory for astronomical research. Briefly speaking, the book is the story of the Kitt Peak National Observatory in USA, which is operated by the Association of Universities for Research in Astronomy.

This book is divided into twelve chapters and starts with the emphasis on the real need to do great work than the possibility of doing useful work. The second and third chapters deal with how 'in the history of American astronomy, never was a site for an optical astronomical observatory selected with such meticulous care' and how the land was acquired from the Papago Indians, who called the astronomers as the people with the Long Eyes! The fourth and fifth chapters tell about the site development and the early telescopes, including how the engineers succeeded in providing such earthly necessities as adequate water supply to the astronomers at the top of the mountain The next chapter gives a detailed account of the making and installing the second major telescope, the 84-inch, on the Peak The next one is about the gigantic McMath solar telescope, which was described by President Kennedy as 'the largest instrument for solar research in the world' An account of the Observatory's involvement in the US space programmes is given in the eighth chapter. The ninth chapter deals with the 158-inch telescope, the largest on Kitt Peak and with the way in which the complex problems about the building, optical parts and the mechanical structures were solved. The last three chapters are devoted to other astronomical facilities available at the Observatory, the general maintenance and the nightly use of the various telescopes.

The accompanying photographs, all in black-and-white excepting that on the cover, are beautiful and the reproduction is very good They take the reader right on to Kitt Peak and make him walk through its history The photographs of the astronomical objects are excellent and speak well of the telescopes, which obtained them

The reference notes and the index at the end of the book are quite helpful The book is written in simple English and makes an interesting reading without going too much into any technical details The effort put in by the author in collecting the historical information, especially the unpublished material, is commendable It is nice to have this book in one's library - private or public

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Reason in the age of science by Hans-Georg Gadamer (Original in German, translated by Frederick G Lawrence) The MIT Press, Cambridge, Massachusetts, U S A , pp 169, $7.99

It is encouraging to have in English language some of the philosophical works of Western Europe. The Publishers ought to be congratulated for this venture

The author has made a brilliant and valiant attempt to reinstate Reason as an integrating force of humanity and as a means of humanising humanity In this historical period when there is a general pall of gloom and depression about the role of science and technology, when humanity lives in constant danger of a total brutal annihilation thanks to the products of human reason and its grasp of the laws of Nature (nature?), it really requires courage to look forward to Reason itself as the Saviour. Purely from this view point, it is a book that should be read by all who are interested in intellectual pursuits and believe in the role of human reason

Thought it is a small book, and well written, the subject matter is quite tough and it cannot therefore make easy reading The translator's introduction is valuable in giving the reader an 'executive' summary and giving Gadamer's agreements and differences with some salient aspects of theories of Hegel, Marx, Aristotle, etc

The entire thrust of Gadamer's philosophy is to use heremeneutics as a theoretical and practical task of humanity To quote the author " . just as politics as practical philosophy is more than the highest technique, this is true for heremeneutics as well. It has to bring *everything knowable by the sciences into the context of mutual agreement in which we ourselves exist* (underlining ours) . . this context of mutual agreement that links us with the tradition that has come down to us in a unity that is efficacious in our lives, it is not just a repertory of methods . but philosophy It not only accounts for procedures applied by science but also gives an account of the questions that are prior to the application of every science, just as did the rhetoric intended by Plato"

The universality it attempts and the all-pervasive application sought for (as any philosophy, especially of German origin, should attempt !) are through mutual agreement, some sort of a consensus achieved through reasoning, without sacrificing the methodology of modern sciences and the human knowledge existing so far There is a general temperance and humility in the entire approach, since they are analysed as 'the questions that are determinative for all human knowings and doing, the greatest of the questions, that are decisive for human beings as human and their choice of the good' Self-knowledge derived in the context of all human knowledge alone is shown to be 'capable of saving a freedom threatened not only by all rulers but much more by the domination and dependence that issue from everything we think we control'

Three chapters, 'What is practice? The conditions of social reason', 'Heremeneutics as practical philosophy' and 'Heremeneutics as a theoretical and practical task' are crucial to the understanding of Gadamer's thought and thesis In the translator's words, "in summary then, we may say, that for Gadamer's philosophical heremeneutics as a transposition of Aristotle's practical political philosophy enters into the problematics of the relationship between the sciences and the life-world and of the theory of science as an empirically grounded meta theory".

The book naturally addresses only Western philosophic thought and its cultural inheritance, though of course there is one reference asking 'whether in the foreign civilisations that are now being drawn technologically over into the ambit of European-American civilisation - China, Japan and especially India - much of the religious social traditions of their ancient cultures does not still live on under the cover of European furnishings and American jobs, and whether whatever lives on may not perhaps bring about an awareness out of necessity once again of new normative and common solidarities that let practical reason speak again' This is a good question that is answered in the book, and could be a serious topic for scholarly pursuit or research.

While in the book the author systematically argues for reinstatement of reason in modern life, the motivation for such a search is not due to the unemotional scholarship of an academic The following statement by the author is worth quoting "Think about mass murder or about the war machine that by a mere push of a button may be unleashed to do its annihilating work But think too, about the mounting automation of all forms of social life, about the role of planning, say, for which it is essential to make long-range decisions, and that means removing from our disposal a great deal of our freedom to decide, or about the growing power of administration that delivers into the hand of bureaucrats a power not really intended by anyone but no less inevitable for all that In this way even more areas of our life fall under the compulsory structures of automatic processes, and even less does humanity know itself and its spirit within this objectification of the spirit Nevertheless precisely this situation of self-crucifying subjectivism of modernity seems to me to lend significance to another dimension, which has been removed from the modern self-consciousness with its self-aggrandizement to the point of making life anonymous . "

It is this humanism and respect or love for individual freedom, that drives the author to search a solution in rationality. Hegel inspires him in such a search, and he goes back to Aristotle and Plato and puts them in the context of modern sciences and the dilemmas of modern life

The humility of approach (to life) in applying heremeneutics derives from the author's observation from his study of Plato that 'the truth of a single proposition cannot be measured by its merely factual relationship of correctness and congruency, nor does it depend only upon the context in which it stands. Ultimately it depends upon the genuineness of its enrootedness and bond with the person of the speaker in whom it wins its truth potential, for the meaning of a statement is not exhausted in what is stated It can be disclosed only if one traces its history of motivation and looks ahead to its implications'

I have quoted this mainly to explain the complexity of the concept of heremeneutics put forth by the author There appears to be a merger of subjectivity and objectivity of what is commonly understood of these words, a synthesis, if one desires to call it. To this extent, though the author has beautifully brought forth his arguments and described the complexity of life and the role of reason in life, there is a vague feeling about an element of 'mysticism' or 'metaphysics' when one tries to understand 'heremeneutics' as a practical philosophy. Well, this is a view-point and life is perhaps too complex This is where I find some common points of Hindu and especially Buddhist views of life The author perhaps may be annoyed at such a suggestion.

The first three chapters of the book starting from 'On the philosophic element ..' are rather tense but are necessary to describe the context of his philosophy and terminology. There are detailed considerations of roots or words and an extensive consideration of Hegelian philosophy To maintain interest continuously in these parts is difficult and one is well advised to read them patiently It is in going through these difficult phases that the translator's introduction proves especially useful

On the whole it is a well-written, thought-provoking and readable book that addresses itself to the questions of modern life by not just leaving these questions unanswered by particular sciences to metaphysics and religion Whether one has found a solution, a way of life or an approach to life, one is not sure after reading the book Perhaps the strength of the thought put forth by the author is in this. The topic being complex, it requires many readings to understand all the aspects which the author desires to convey, fully Further research in heremeneutics would be greatly worthwhile in view of the experience and thoughts of other human civilisations Whether it will turn out to be an esoteric intellectual exercise, only life will tell

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